CLASSICAL ASPECTS OF THE NUCLEAR SYMMETRY ENERGY

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Outline

Symmetry energy

Considerations at higher energies

Computational model

Symmetry energy

Conclusions
Adding more neutrons must affect asymmetry term

Asymmetry term penalizes nuclei with $N \neq Z$

Quantum origin of the empirical asymmetry term
How to generalize to neutron-rich nuclei?

Extend liquid drop formula to non-symmetric isospin values

\[ E(\rho, \alpha) = E(\rho, \alpha = 0) + E_{\text{sym}}(\rho)\alpha^2 + O(\alpha^4) \]

Taylor expansion in terms of \( \alpha \) about isospin symmetric \( \alpha = 0 \)
Odd-terms excluded due to p-n exchange symmetry.

Obtain \( E_{\text{sym}} \) through:

\[ E_{\text{sym}}(\rho) = \frac{1}{2} \frac{\partial^2 E(\rho, \alpha)}{\partial \alpha^2} \bigg|_{\alpha=0} \]

\( E \) can be calculated with many body techniques
RELATIVISTIC MEAN-FIELD MODELS

The nonlinear RMF model

The density-dependent RMF model

The nonlinear point-coupling RMF model

The density-dependent point-coupling RMF model

Previous studies

All calculations at T=0

Tremendous work of Bao-An Li
Relativistic and Nonrelativistic Hartree Fock approximations:


Relativistic and Non-relativistic Mean Field Theories:

In summary, the knowledge we have about the symmetry energy is as good as the techniques used for solving the nuclear many-body problem, which are far from perfect.

How about \( T > 0 \)?

Can we learn about the symmetry energy at higher energies \( (T > 0) \)?

At intermediate energies, colliding nuclei fragment

Remember the liquid-gas phase transition
It is known that:

- Nuclear density is \( \sim 0.15 \text{ fm}^{-3} \)
- Nuclear compressibility is \( \sim 200 \text{ MeV} \)
- At low densities nuclear matter is a non-interacting Fermi gas

Can we extract an equation of state out of these data?
Behavior pressure-density resembles a liquid

Liquid-gas phase transition is possible in nuclear matter
Collision trajectory

Fragmentation reflects conditions in liquid-gas region
But at such conditions

1. Nucleons in a nucleus are not so restricted by Pauli principle

2. Nucleons and fragments obey classical dynamics

Let’s check these quantum caveats
QUANTUM CAVEATS I

Take nucleons in fragments as particles in a box

Number of levels available at a given energy

\[ \Phi(\epsilon) = \frac{\pi V}{6} \left( \frac{8 M \epsilon}{\hbar^2} \right)^{3/2} \]

Compare to number of particles \( N \)

\[
\frac{N}{\Phi(\epsilon)} = \sqrt{\frac{\pi}{6}} \frac{\rho h^3}{(2\pi MT)^{3/2}} \ll 1
\]

There are more states available than nucleons

Pauli blocking is not restrictive at

\[ 1 < T < 5 \text{ MeV} \]

FIG. 9. Values of \( N/\Phi(\epsilon) \) as a function of temperature for \( x = 0.3, 0.4, \) and 0.5 and for protons (p) and neutrons (n).
In the liquid-gas phase the inter-particle distance is larger than de Broglie wavelength for all cluster sizes for $1 < T < 5$ MeV.
Then, since
1. Nucleons in a nucleus are not so restricted by Pauli principle and
2. Nucleons and fragments obey classical dynamics

One can use classical mechanics to study the symmetry energy at nuclear fragmentation energies!

Problem solved?
No, it is still a many-body problem
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Classical molecular dynamics

- Inter-particle potential
- Uses protons & neutrons
- Proper dynamics and geometry
- Does not use “test particles”
- Does not use gaussian density distributions
- Produces fragments without external aid
- Deexcites fragments naturally
- Uses a unique set of parameters

But . . . is classic and not quantum
Classical Molecular Dynamics

- Potential
- Solve equation of motion
- Recognize clusters
- Track evolutions in space-time
CMD can determine

• Mass distributions
• Critical phenomena
• Caloric curves
• Isoscaling
• Nuclear “Pasta”
Procedure to study infinite nuclear matter

- Create an infinite system
  - Select density $\rho$
  - Select Temperature
- Equilibrate
- Measure
  - Binding energy $E(\rho,T)$
  - Pressure $p(\rho,T)$
  - Compressibility $K(\rho,T)$
- Obtain equation of state
- Study other properties of system
INFINITE MATTER

- Method 1: Lattices (SC, FCC and BCC)
- Method 2: Molecular dynamics
- Symmetric matter (1,000 p & 1,000 n)
- \(0 < \rho < \rho_0, \ 0 < T < 1.0 \text{ MeV}\)
- Periodic boundary conditions
- Andersen thermostat
- Minimum spanning tree to identify clusters
- Potentials:
  - Pandharipande medium
  - Pandharipande stiff
  - Horowitz
Procedure

Energy per nucleon

Symmetry Energy

 CMD

ρ

Compressibility

Pressure

α

T
Statistical averages were obtained out of ensembles of 200 systems at every combination of \((\rho, T, x)\).

The average of all standard deviations of \(E/A\) was 0.036 MeV.
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**Energy per nucleon**

**Saturation density varies with T**

**Symmetric matter** $x = \frac{Z}{A}$

**Bound**

**Unbound**

**Homogeneous system for** $\rho > 0.1 \text{ fm}^{-3}$

**Non-homogeneous system for** $\rho < 0.1 \text{ fm}^{-3}$

**Saturation density**

**Phase change**
Energy per nucleon: variation with isospin

Phase change

Unbound since very low T

Bound only at very low T

Lower saturation density

Phase change?

Very low saturation density
How to extract the symmetry energy from CMD simulations

\[ E_{\text{Sym}} = \frac{1}{2} \left. \frac{\partial^2 E(\rho, \alpha)}{\partial \alpha^2} \right|_{\alpha=0} \]

Procedure: pick a temperature

Obtain fits

\[
E(\rho, \alpha) = E_1(\alpha) \rho + E_2(\alpha) \rho^2 + E_3(\alpha) \rho^3
\]

Extract coefficients

\[
E = -1.15 \rho - 254.00 \rho^2 + 1396.38 \rho^3 \quad \alpha = 0
\]

\[
E = 10.05 \rho - 171.11 \rho^2 + 1226.08 \rho^3 \quad \alpha = 0.2
\]

\[
E = 94.64 \rho - 581.32 \rho^2 + 2775.69 \rho^3 \quad \alpha = 0.4
\]
E(ρ, α) = E_1(α) ρ + E_2(α) ρ^2 + E_3(α) ρ^3

Fit coefficients

E_1(α) = E_{00} + E_{02} α^2 + E_{04} α^4
E_2(α) = E_{10} + E_{12} α^2 + E_{14} α^4
E_3(α) = E_{20} + E_{22} α^2 + E_{24} α^4

E(ρ, α) = E_0 + E_2 α^2 + E_4 α^4

α = 0
E = 1.15 ρ - 254.00 ρ^2 + 1396.38 ρ^3

α = 0.2
E = 10.05 ρ - 171.11 ρ^2 + 1226.08 ρ^3

α = 0.4
E = 94.64 ρ - 581.32 ρ^2 + 2775.69 ρ^3
In general:

\[ E(\rho, \alpha) = \left[ E_{00} + E_{02} \alpha^2 + E_{04} \alpha^4 \right] \rho + \left[ E_{10} + E_{12} \alpha^2 + E_{14} \alpha^4 \right] \rho^2 + \left[ E_{20} + E_{22} \alpha^2 + E_{24} \alpha^4 \right] \rho^3 \]

\[
E_{\text{Sym}} = \left. \frac{1}{2} \frac{\partial^2 E(\rho, \alpha)}{\partial \alpha^2} \right|_{\alpha=0}
\]

\[ = E_{02} \rho + E_{12} \rho^2 + E_{22} \rho^4 \]

For each temperature
Mostly classical behavior at low densities


Rel. and non-rel. HF approximations. Rel. and non-rel. Mean Field theories
CMD – RMFT
Agreement at low densities?
ρ ≤ 0.08 fm$^{-3}$
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CONCLUSIONS

- Nuclear symmetry energy can be studied with CMD
- Symmetry energy is mostly classical at low $\rho$
Symmetry energy and phase diagram of warm asymmetric matter
Current work:
• Isospin diffusion

Future work:
• Connection with reactions

Funding from NSF and DOE
Thanks to Lawrence Berkeley Lab for hospitality