Aspects of structure of exotic nuclei from low and intermediate energy scattering

S. Karataglidis
University of Johannesburg

MCAS Collaboration

K. Amos, D. van der Knijff, *University of Melbourne*
L. Canton, *INFN/University of Padova*
P. R. Fraser, *Curtin University*
J. P. Svenne, *University of Manitoba*

Introduction

The use of existing, and construction of new, exotic beam facilities allows for the formation and study of new species, particularly beyond the drip lines.

This talk will review the study of exotic nuclei with the MCAS (Multi-Channel Algebraic Scattering) Theory, with comparisons to shell model results.

There will also be discussion on structure studies from intermediate energy scattering.

Low-energy scattering: MCAS

We seek to obtain *S* matrices and evaluate:

Total elastic scattering cross sections:

$$\sigma_{\mathrm{el}} = \frac{\pi}{k^2} \sum_{l=0}^{\infty} \left\{ (l+1) \left| S_{ll+\frac{1}{2}}(k) - 1 \right|^2 + l \left| S_{ll-\frac{1}{2}}(k) - 1 \right|^2 \right\}$$

Total reaction cross sections:

$$\sigma_{\mathrm{R}} = \frac{\pi}{k^2} \sum_{l=0}^{\infty} \left\{ (l+1) \left[1 - \left| S_{ll+\frac{1}{2}}(k) \right|^2 \right] + l \left[1 - \left| S_{ll-\frac{1}{2}}(k) \right|^2 \right] \right\}$$

The MCAS approach is built upon:

1. Finite-rank separable representations of realistic interactions;
2. Scattering matrices for separable Schrödinger interactions;
3. Sturmian functions (Weinberg states) to define form factors.

Multi-channel T matrices

Solution of coupled Lippmann-Schwinger equations:

$$T_{cc'}(p,q;E) = V_{cc'}(p,q) - \mu \sum_{c'=1}^{\text{closed}} \int_0^\infty V_{cc''}(p,x) \frac{1}{h_{c''}^2 + x^2} T_{c''c'}(x,q;E)\, x^2 dx$$

$$+ \mu \sum_{c'=1}^{\text{open}} \int_0^\infty V_{cc''}(p,x) \frac{1}{k_{c''}^2 - x^2 + i\varepsilon} T_{c''c'}(x,q;E)\, x^2 dx$$

Expand the potential matrix:

$$V_{cc'}(p,q) \sim V_{cc'}^{(N)}(p,q) = \sum_{n=1}^{N} \hat{\chi}_{cn}(p) \eta_n^{-1} \hat{\chi}_{c'n}(q)$$

Optimal functions, $\hat{\chi}_{cn}(q)$, involve Sturmians $|\Phi_{c'n}\rangle$:

$$|\hat{\chi}_{cn}\rangle = \sum_{c'} V_{cc'} \, |\Phi_{c'n}\rangle$$

$$\sum_{c'} G_c^{(0)} V_{cc'} \, |\Phi_{c'n}\rangle = -\eta_n \, |\Phi_{cn}\rangle$$

Multi-channel S matrices

Separable expansion of multi-channel $V_{cc'}$ ⇒ multi-channel S matrix (c,c' are open channels, specified by J^π):

$$S_{cc'} = \delta_{cc'} - i\pi\mu\sqrt{k_c k_{c'}}\,T_{cc'}$$
$$= \delta_{cc'} - i\pi\mu \sum_{n,n'=1}^{N} \sqrt{k_c}\,\chi_{cn}(k_c) \left([\boldsymbol{\eta} - \mathbf{G}_0]^{-1} \right)_{nn'} \chi_{c'n'}(k_{c'})\sqrt{k_{c'}}$$

Matrix elements (Sturmian basis) $[\boldsymbol{\eta}]_{nn'} = \eta_n \delta_{nn'}$, with:

$$[\mathbf{G}_0]_{nn'} = \mu \sum_{c=1}^{\text{open}} \int_0^\infty \hat{\chi}_{cn}(x) \frac{x^2}{k_c^2 - x^2 + i\varepsilon} \hat{\chi}_{cn'}(x)\, dx$$
$$- \mu \sum_{c=1}^{\text{closed}} \int_0^\infty \hat{\chi}_{cn}(x) \frac{x^2}{h_c^2 + x^2} \hat{\chi}_{cn'}(x)\, dx$$

MCAS and the Pauli Principle

The Pauli Principle can be satisfied in the coupled-channel calculations by:

Either $V_{cc'}(\mathbf{r}, \mathbf{r}')$ defined using shell model wave functions;

Or by using an Othogonalising Pseudo-Potential (OPP) to specify the Sturmians:

Recall:

$$|\hat{\chi}_{cn}\rangle = \sum_{c'} V_{cc'} |\Phi_{c'n}\rangle$$

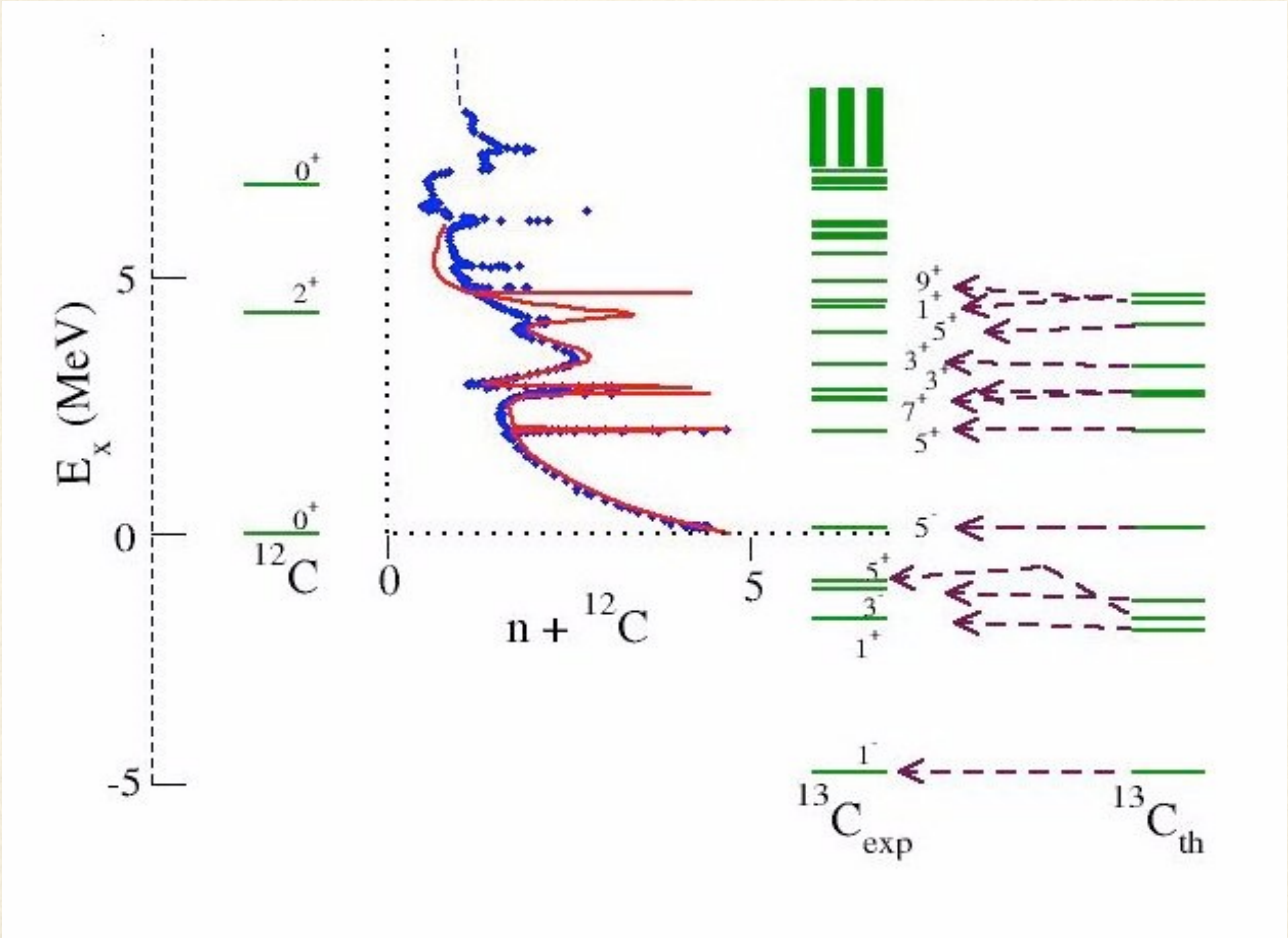
$$\sum_{c'} G_c^{(0)} V_{cc'} |\Phi_{c'n}\rangle = -\eta_n |\Phi_{cn}\rangle$$

Use instead: $V_{cc'}(r) + \lambda A_c(r) A_{c'}(r')$

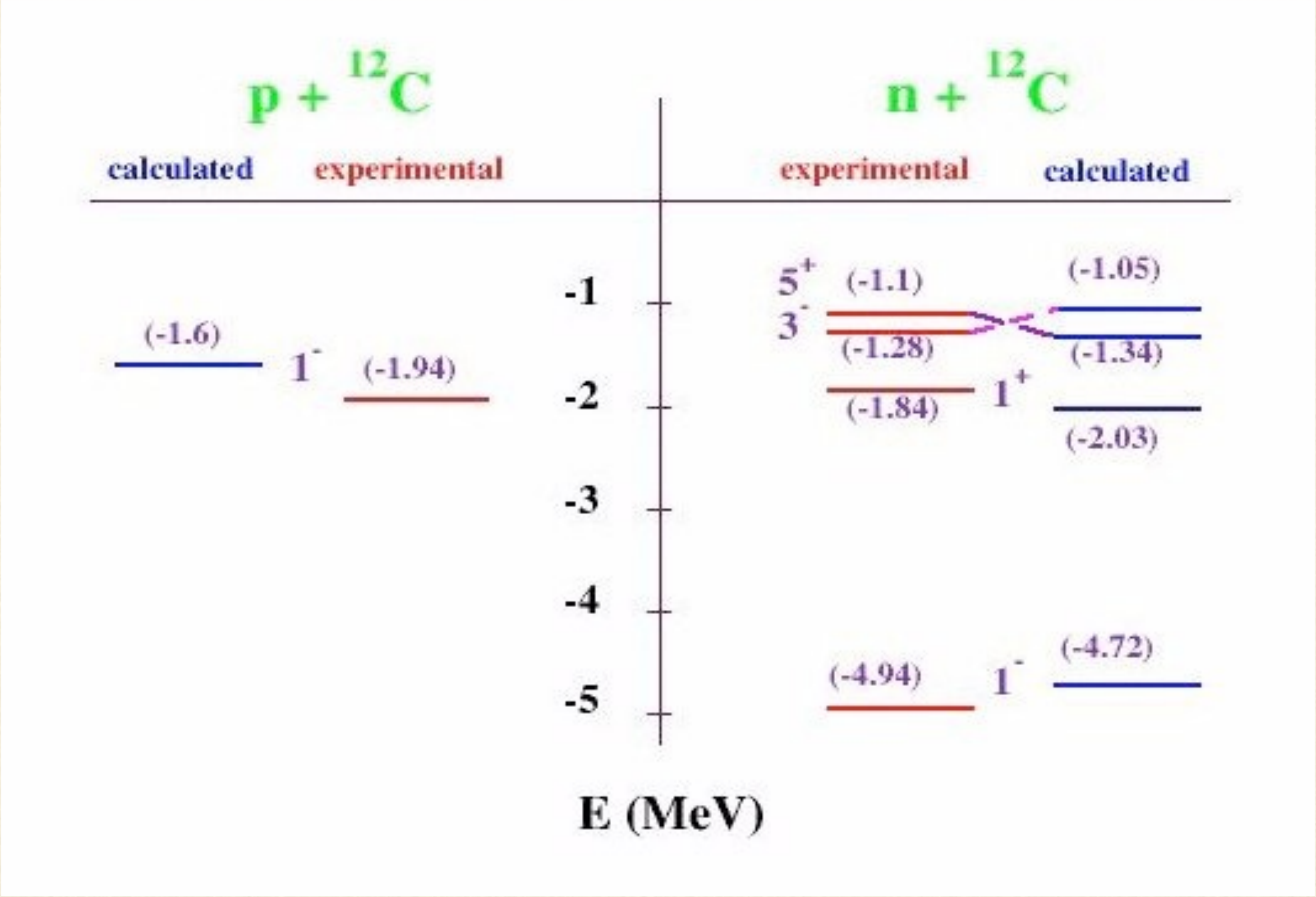
to specify the Sturmians. $A_c(r)$ are bound state functions for (partially) occupied orbits.) Thus Sturmians and factors $\chi_{cn}(k)$ are orthogonal to Pauli-blocked states.

OPP correction for Pauli

L. Canton, et al., Phys. Rev. Lett. ***94****, 122503 (2005)*



Mass-13 bound states



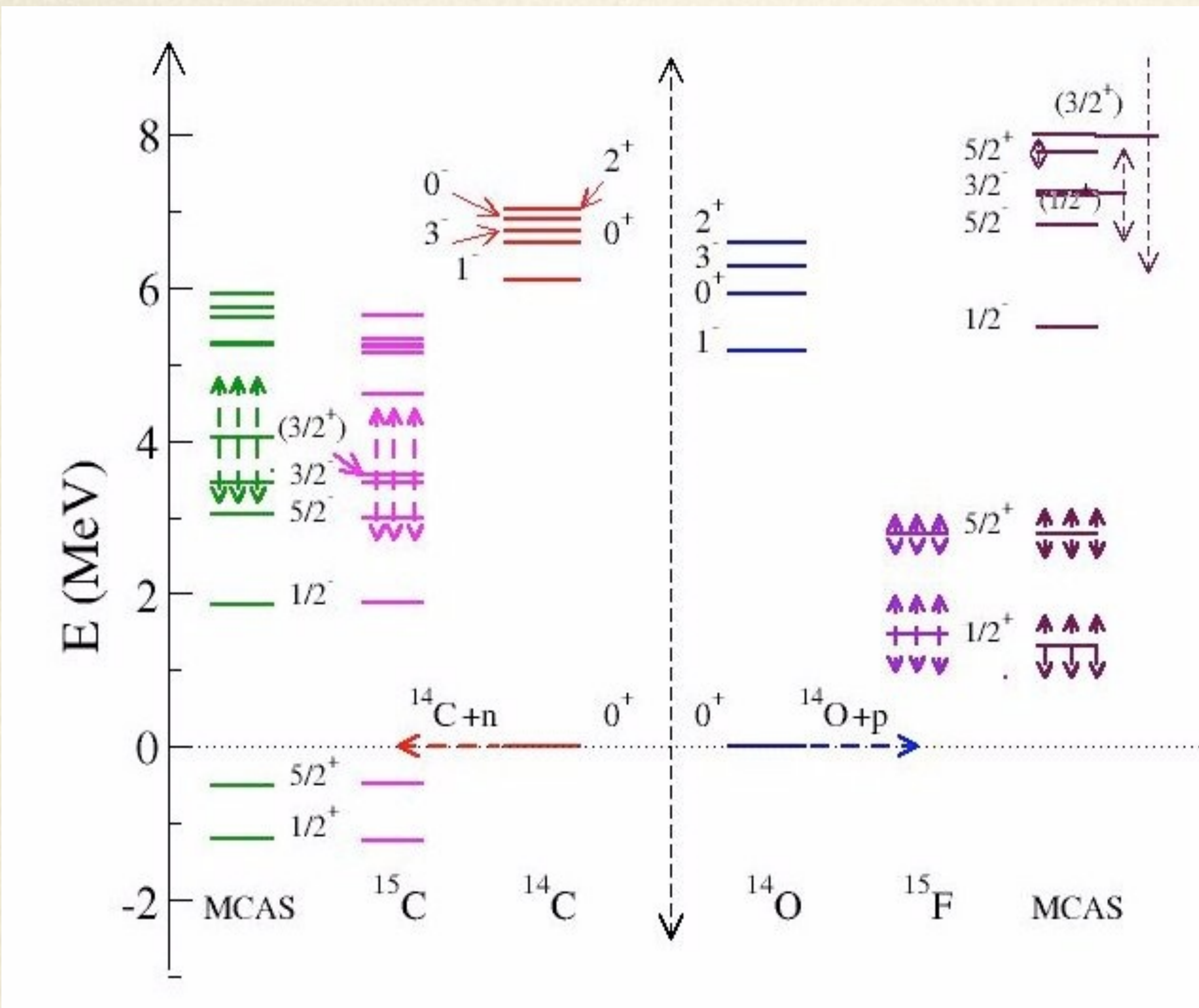
Energies in reference to p+^{12}C and n+^{12}C thresholds.

Structure of nuclei beyond the drip lines: ^{15}F

L. Canton, et al. Phys. Rev. Lett. ***96****, 072502 (2006)*

Structure of nuclei beyond the drip lines: ^{15}F

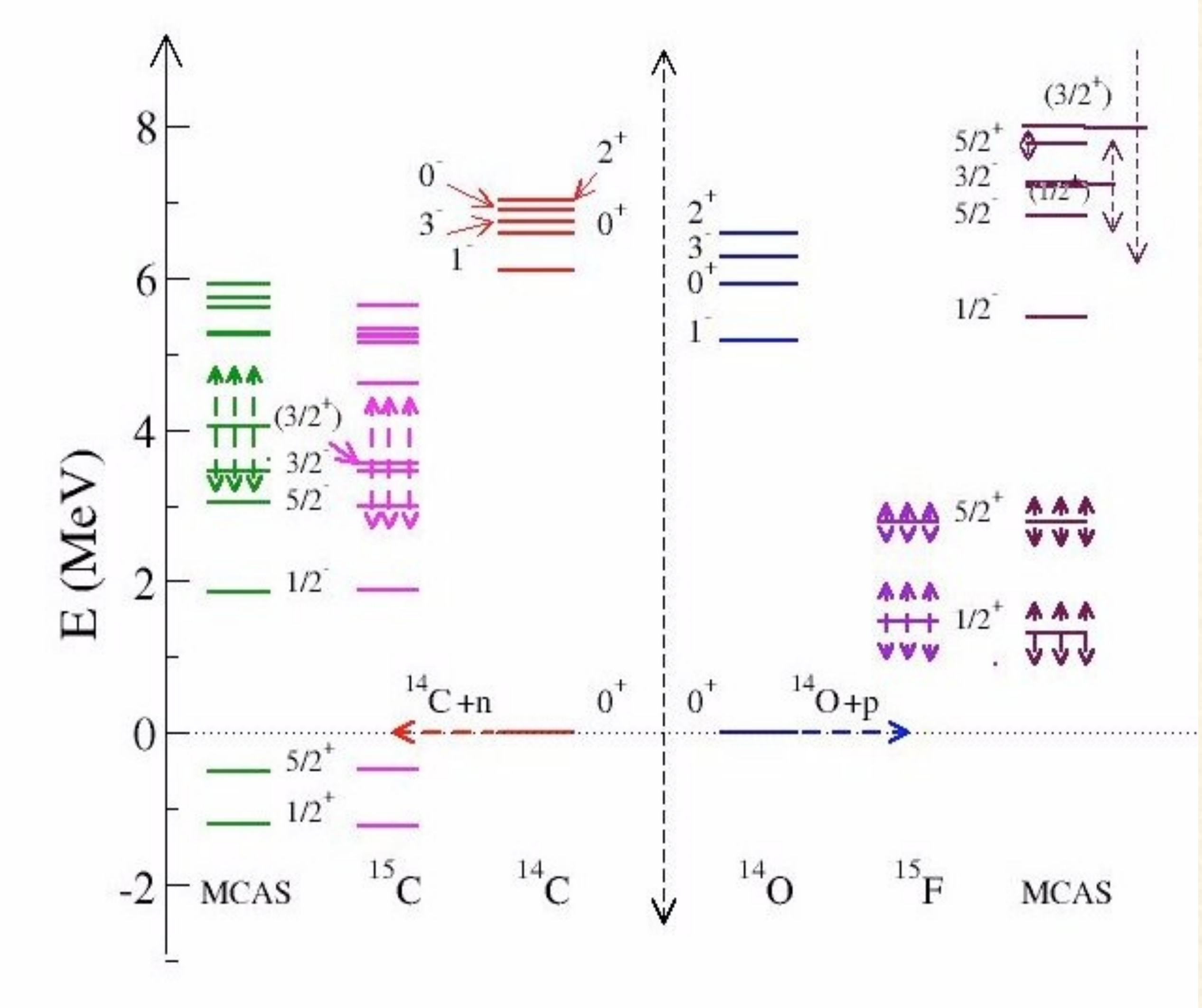
L. Canton, et al. Phys. Rev. Lett. ***96****, 072502 (2006)*

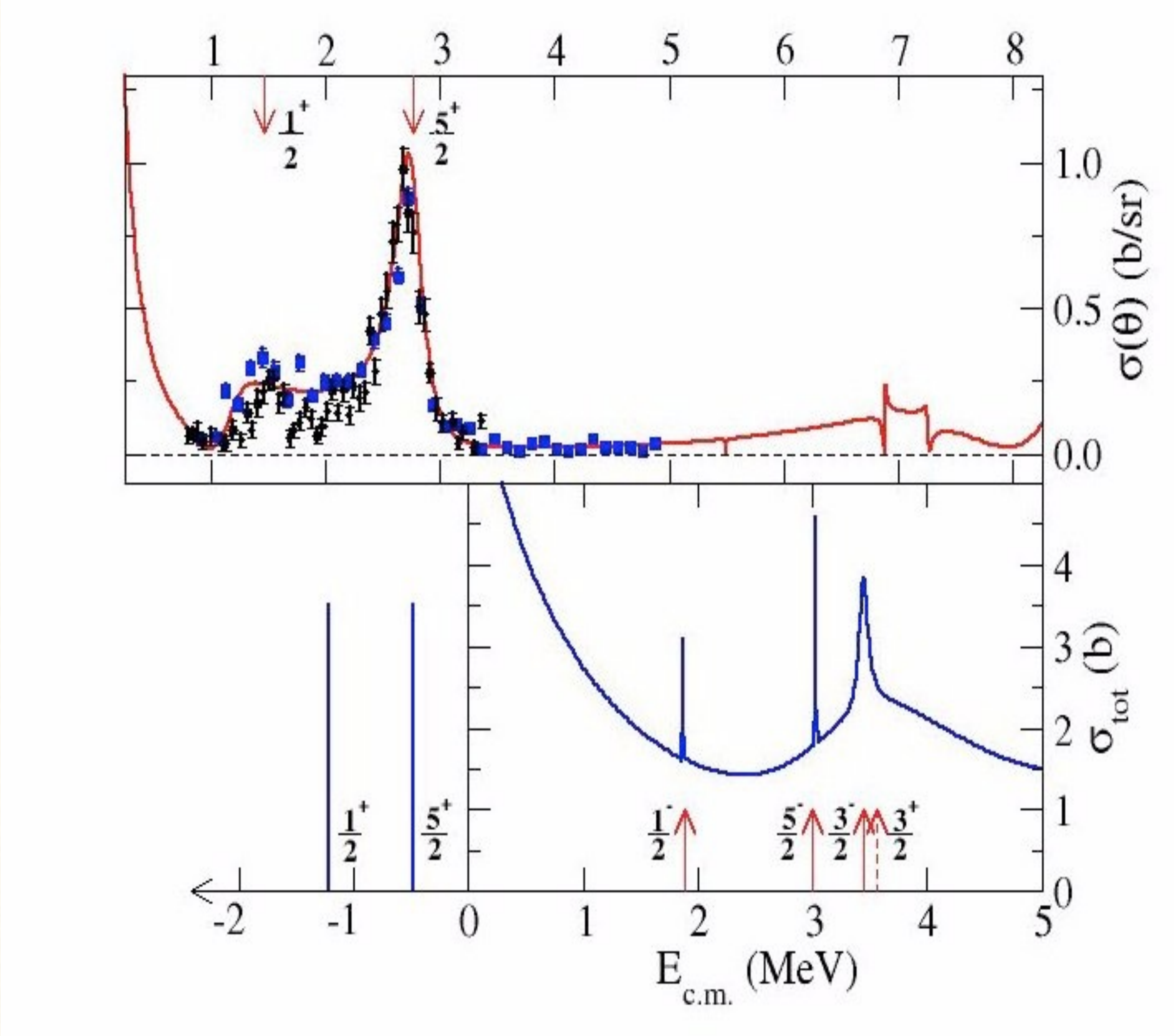


Mass-15 spectra.

Structure of nuclei beyond the drip lines: ^{15}F

L. Canton, et al. Phys. Rev. Lett. ***96****, 072502 (2006)*





Mass-15 spectra.

Cross section to states in ^{15}F.

rms radii for ^{17,19}C

S. K., et al., Nucl. Phys. ***A813****, 235 (2008)*

Nucleus	r_c (fm)	r_T (fm)	r_T (fm) (Exp)
^{17}C	2.425	2.500	2.72±0.04
^{19}C	2.422	2.556	3.23±0.08 (1)
^{19}C	2.699	3.244	3.13±0.07 (2)

[Exp: A. Ozawa et al., NPA **691**, 599 (2001)]
(1): Glauber assuming few-body; (2) Glauber.

rms radii for $^{17,19}C$

S. K., et al., Nucl. Phys. ***A813****, 235 (2008)*

Nucleus	r_c (fm)	r_T (fm)	r_T (fm)
			(Exp)
^{17}C	2.425	2.500	2.72±0.04
^{19}C	2.422	2.556	3.23±0.08 (1)
^{19}C	2.699	3.244	3.13±0.07 (2)

[Exp: A. Ozawa et al., NPA **691**, 599 (2001)]
(1): Glauber assuming few-body; (2) Glauber.

Halo, ^{19}C: r_T=3.982 fm

rms radii for $^{17,19}C$

S. K., et al., Nucl. Phys. ***A813****, 235 (2008)*

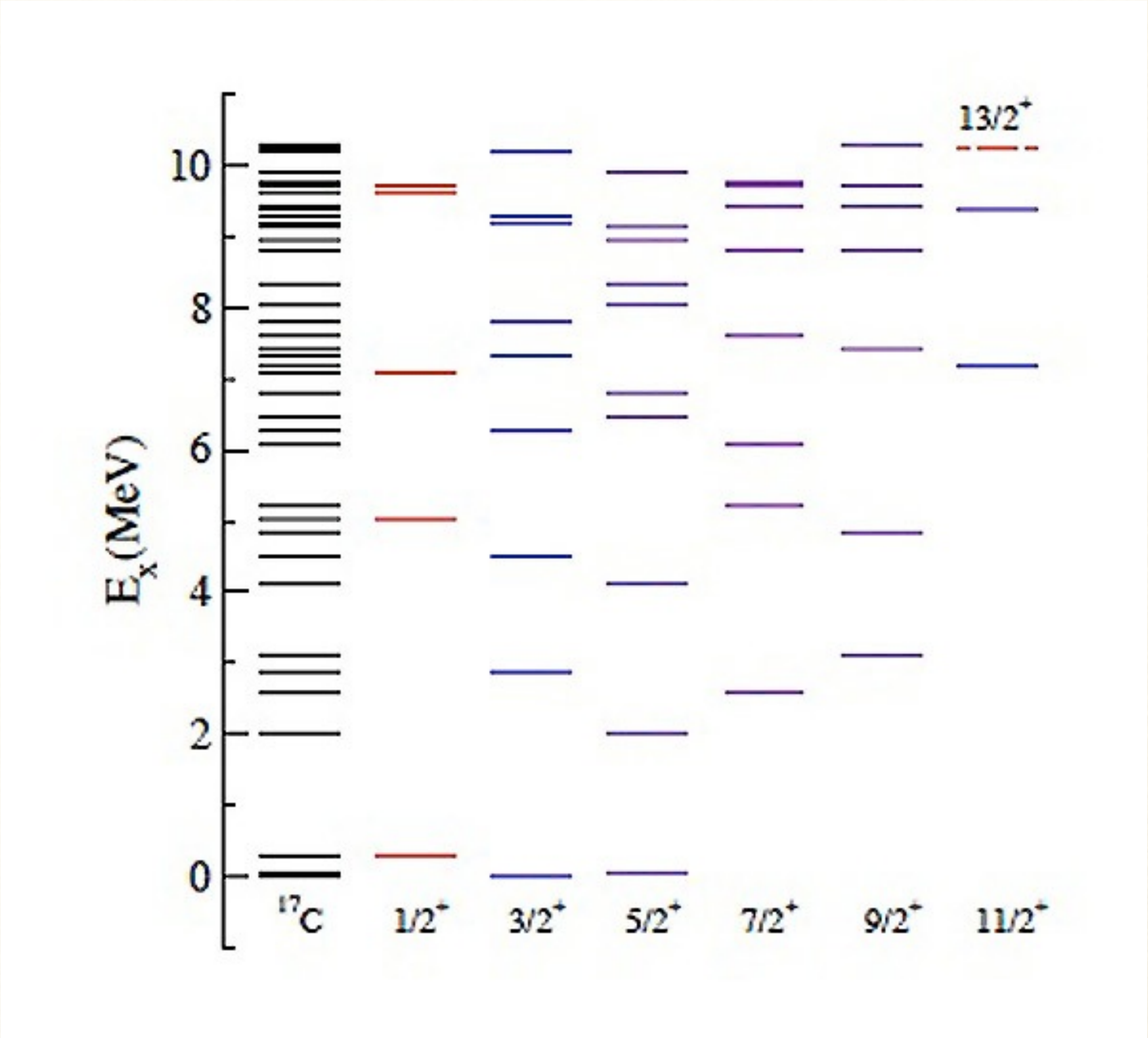
Nucleus	r_c (fm)	r_T (fm)	r_T (fm)
			(Exp)
^{17}C	2.425	2.500	2.72±0.04
^{19}C	2.422	2.556	3.23±0.08 (1)
^{19}C	2.699	3.244	3.13±0.07 (2)

[Exp: A. Ozawa et al., NPA **691**, 599 (2001)]
(1): Glauber assuming few-body; (2) Glauber.

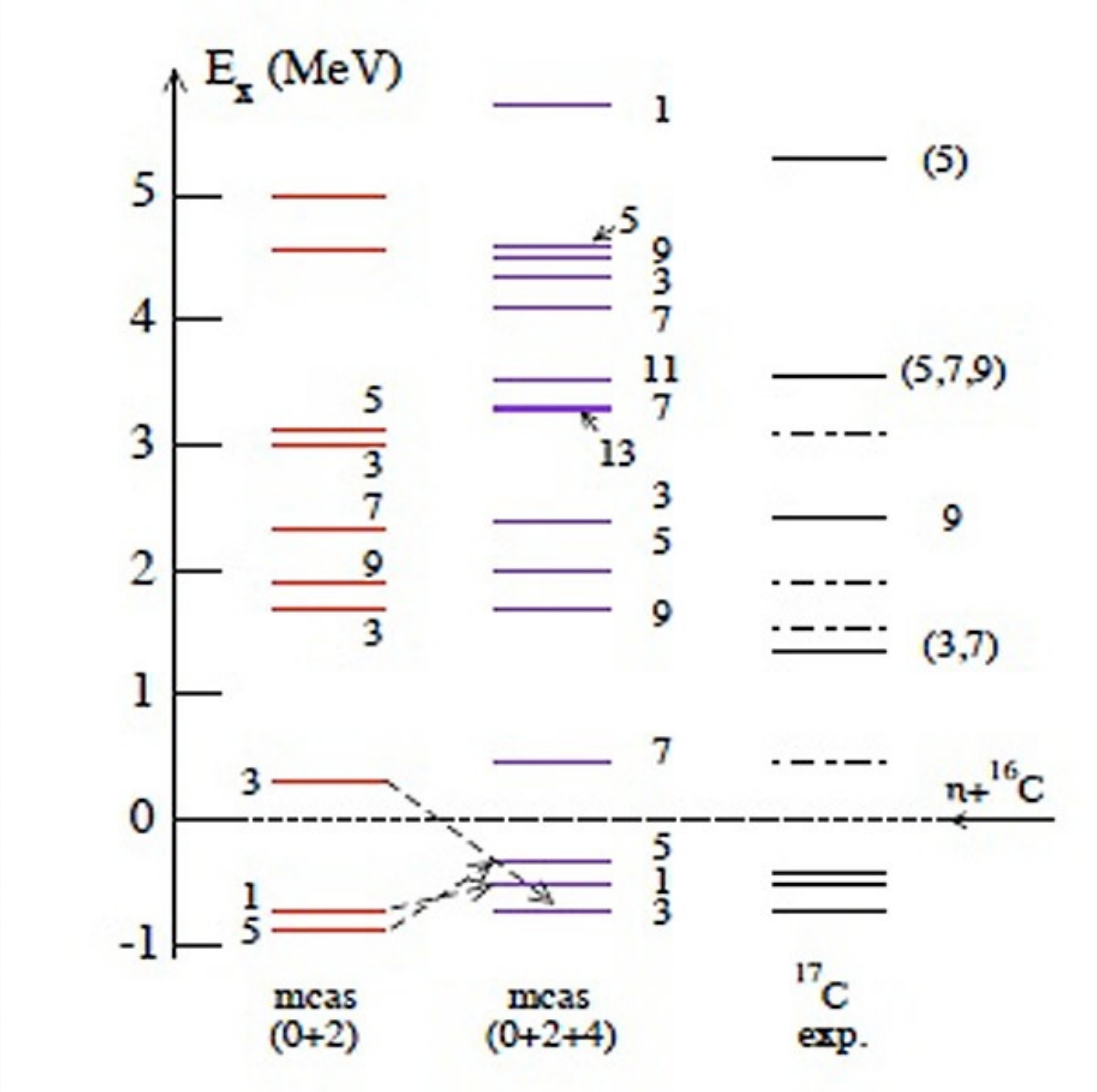
Halo, ^{19}C: r_T=3.982 fm

Halo result consistent with result of Lassaut and Lombard [Z. Phys. A 341, 125 (1992)].

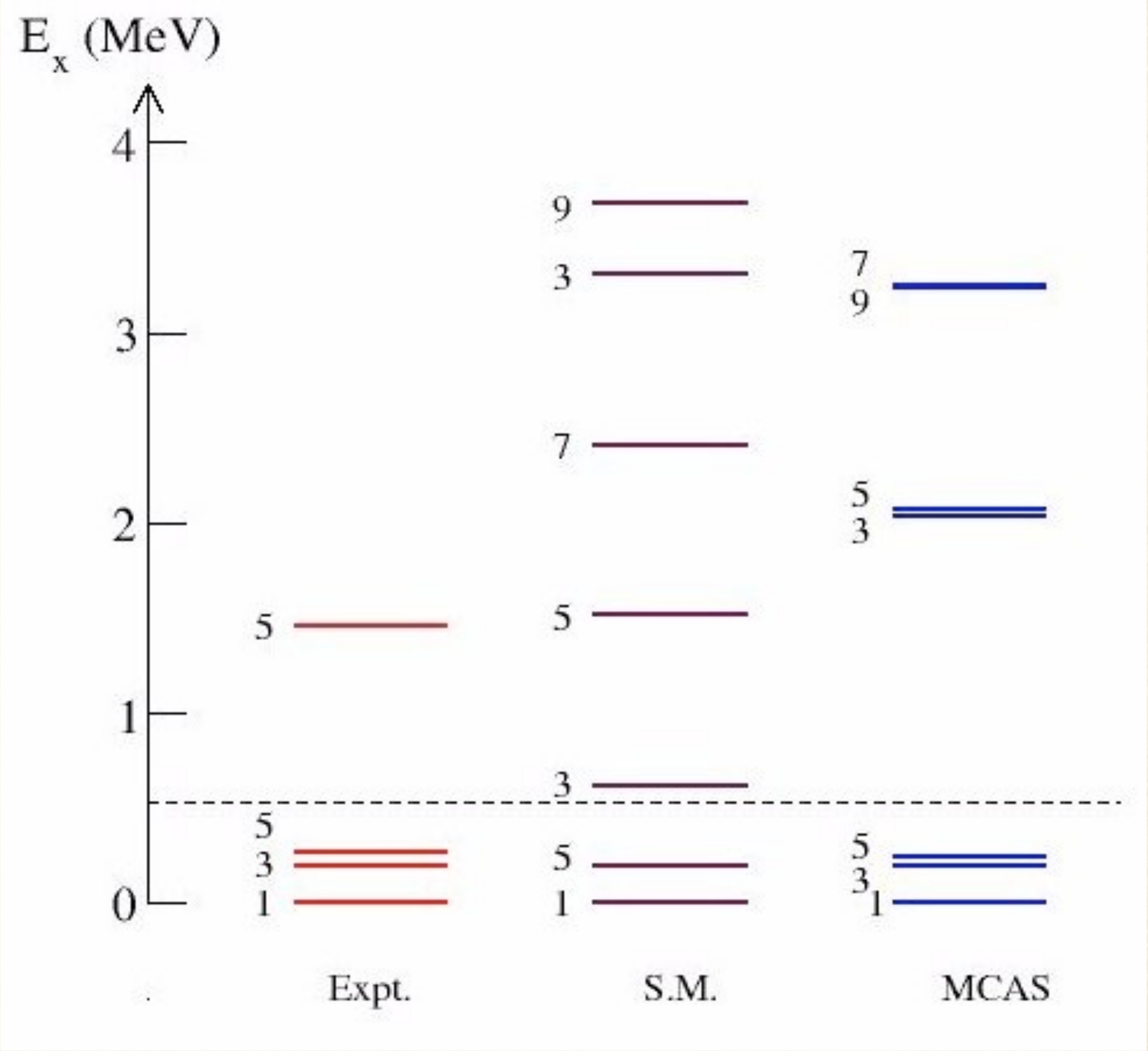
^{17}C spectrum from the shell model



^{17}C from n+^{16}C coupling



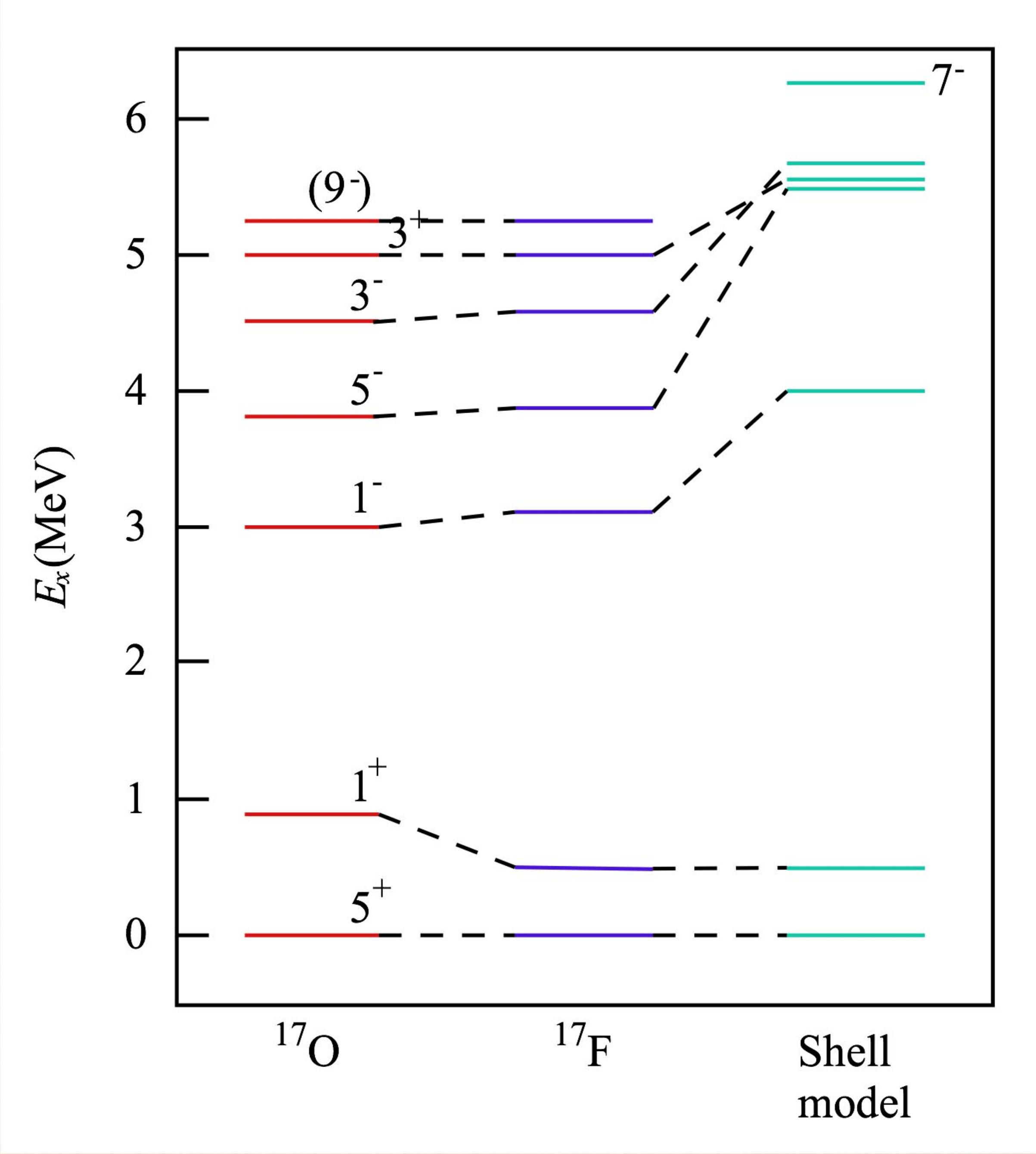
^{19}C spectrum from n+^{18}C coupling



Shell model, mass 17

Positive parity: (0+2)ħω;
Negative parity: (1+3)ħω.

WBP interaction of Warburton and Brown.

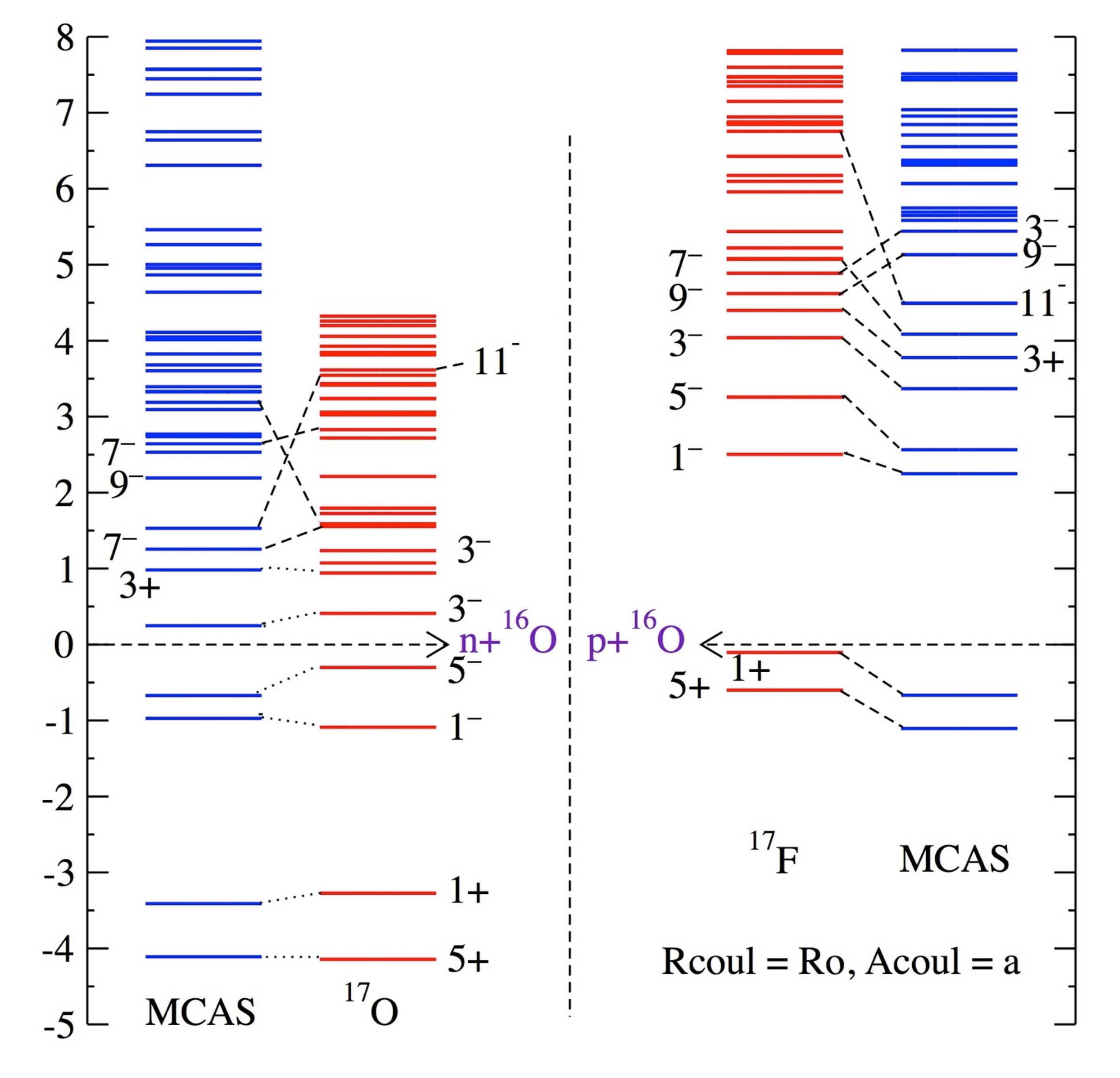


MCAS, mass 17

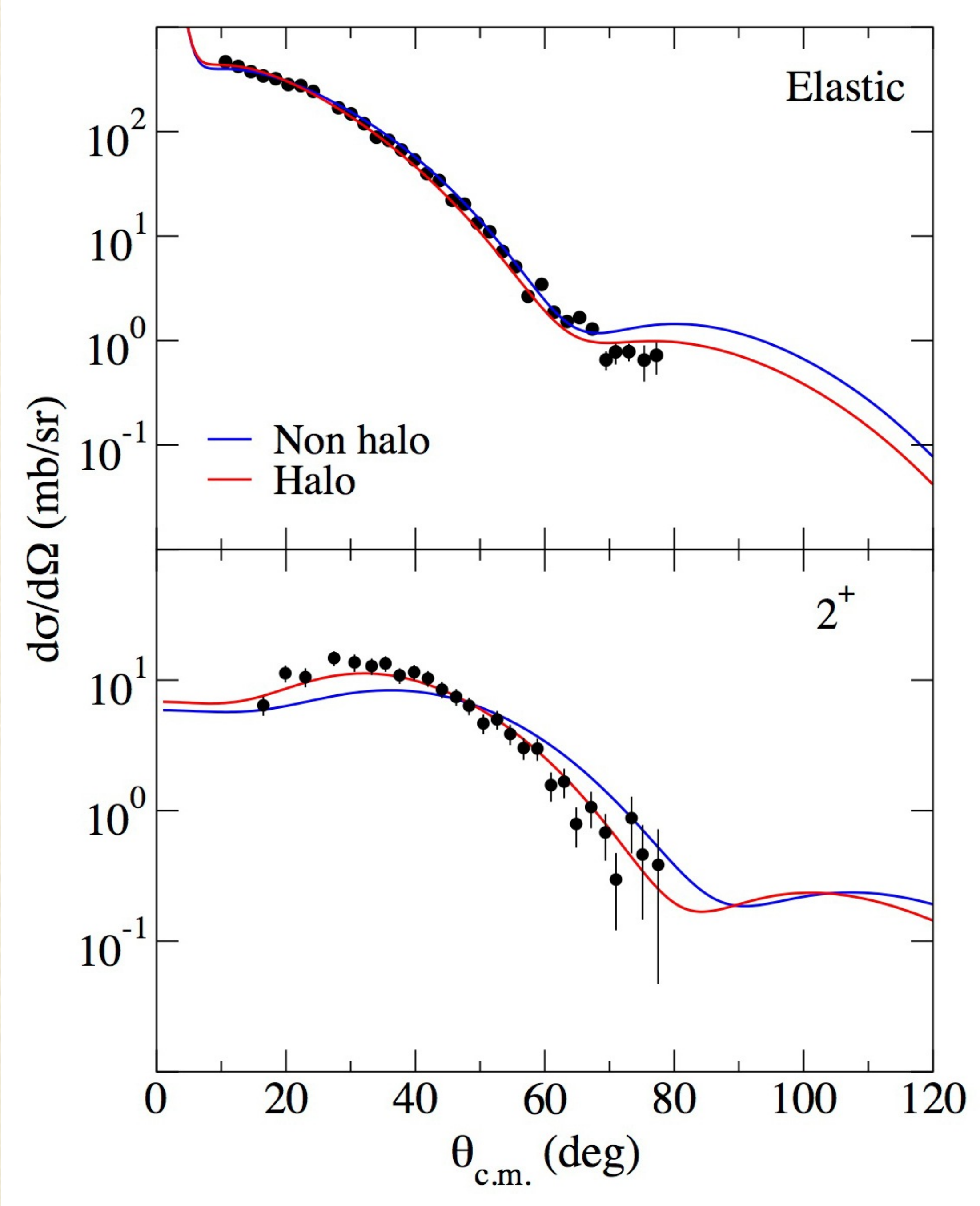
Vibrational model:

0+ (gs), 0+ (6.05 MeV),
3- (6.13 MeV), 2+ (6.92 MeV)
1- (7.12 MeV).

(Preliminary results.)



Intermediate energy scattering - ^{6}He-p elastic scattering



Reaction cross section:

Predicted:

$\sigma_R = 353$ mb (nonhalo)

$= 406$ mb (halo)

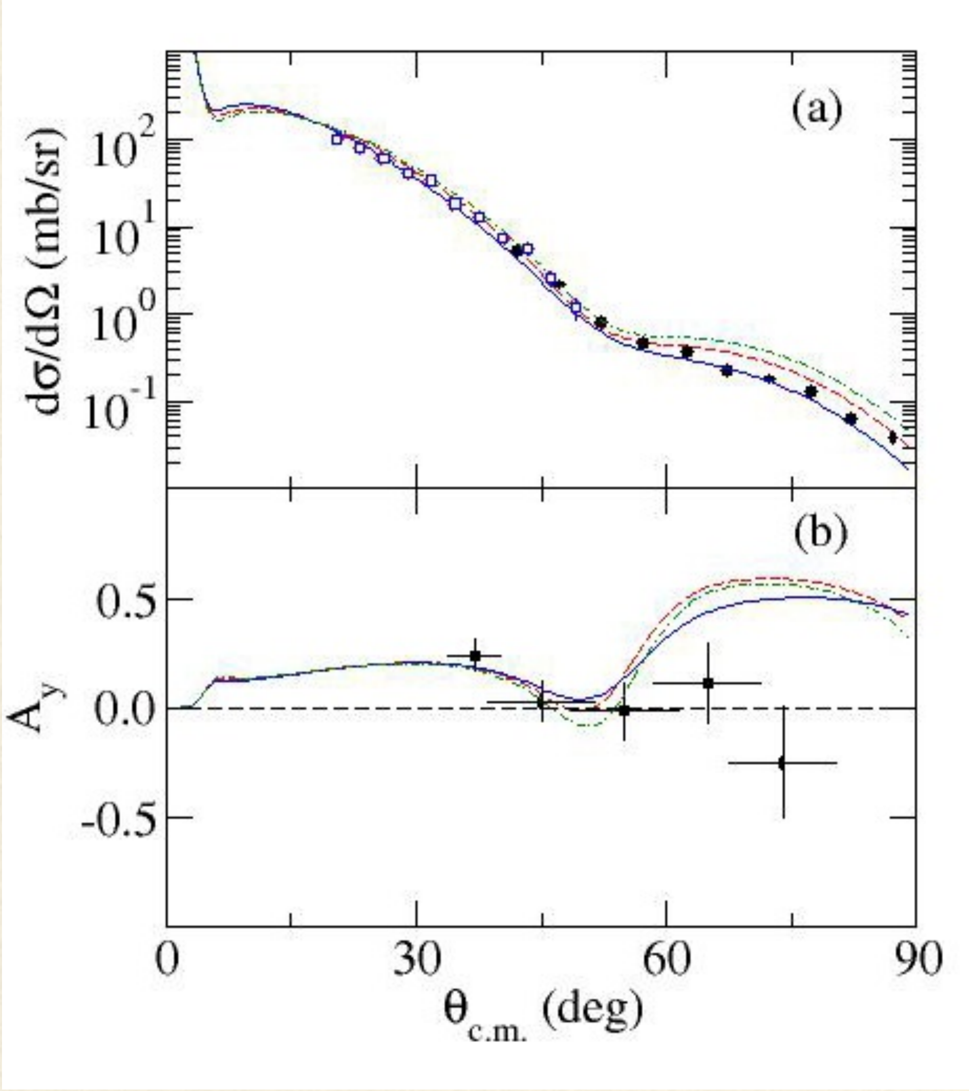
Measured:

$\sigma_R = 409 \pm 22$ mb

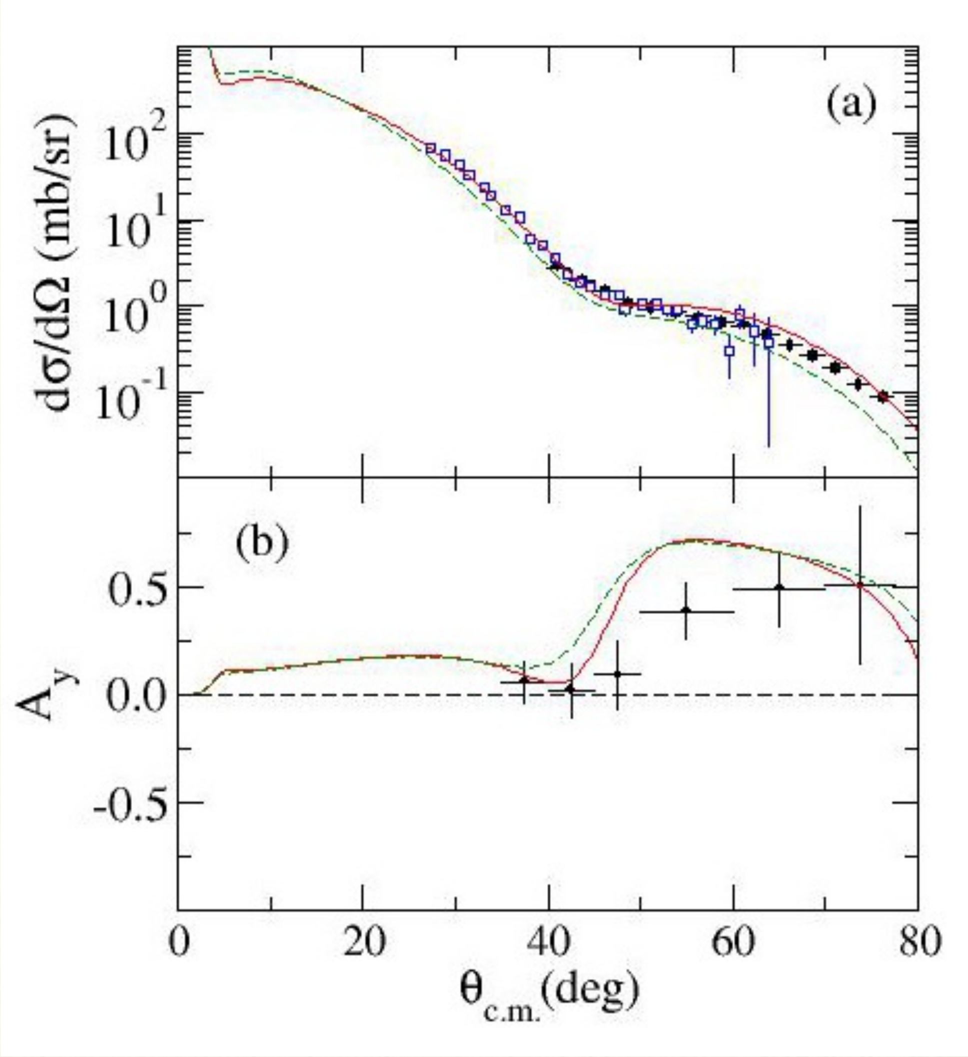
(A. Lagoyannis, *et al.*, Phys. Lett. B 518, 27 (2001)).

Spin observables

(SK and K. Amos, Phys. Rev. C **87**, 054623 (2013))



^{6}He-p



^{8}He-p

Conclusions

Structures of exotic nuclei can be studied from both low and intermediate energy scattering from hydrogen. That information is either from the formation of a compound nucleus or direct information from the ion projectile.

- For low-energy scattering, the MCAS is suited to elicit those details, including those nuclei beyond the drip line;
  - That requires detailed handling of Pauli.
  - Comparison to results from the shell model is good.
- For intermediate energy scattering, the Melbourne *g* folding model is suited.
  - This requires that nuclear structure information be handled directly at the level of density matrix elements (amplitudes).