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KTH Engineering Sciences

New insight on the

Odd-Even mass Staggering

and pairing correlation in drip-line nuclei

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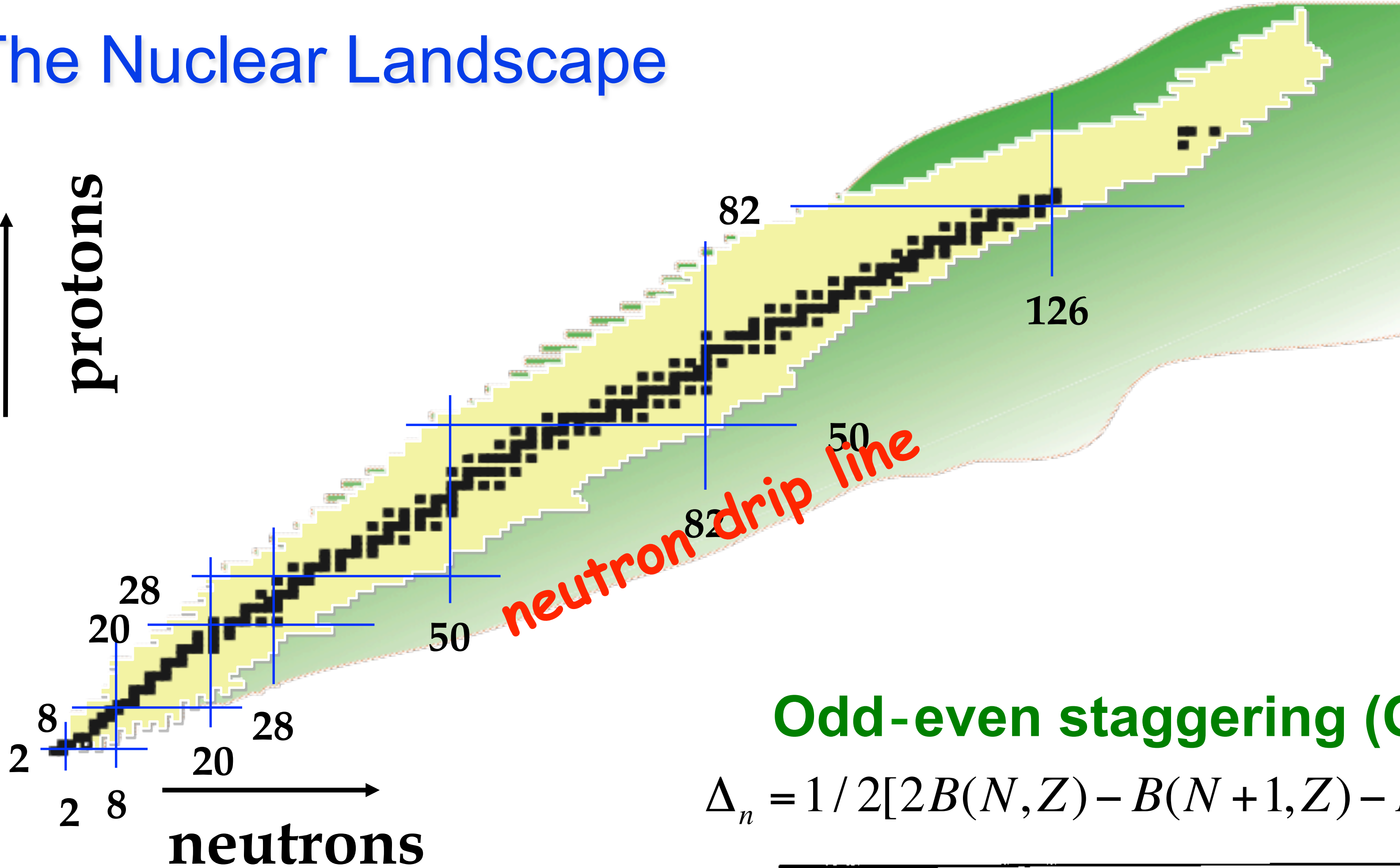
Outline

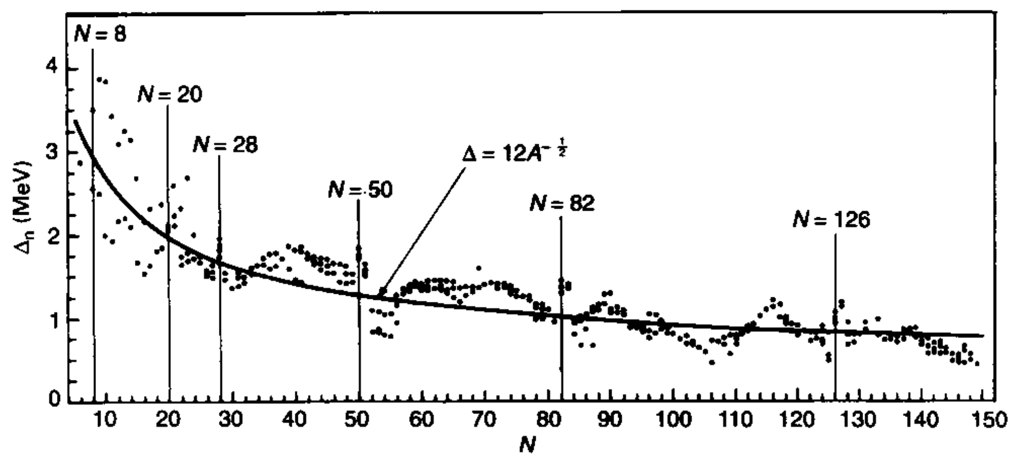
- **Brief introduction to odd-even staggering (OES) in nuclear binding energy**
- **Exact solution of the pairing Hamiltonian and comparison with the BCS and Richardson approaches**
- **OES and the residual pairing correlation**
- **Alpha cluster formation amplitudes in heavy nuclei and their relation with the pairing collectivity;**
- **Summary**

The Nuclear Landscape



Odd-even staggering (OES)

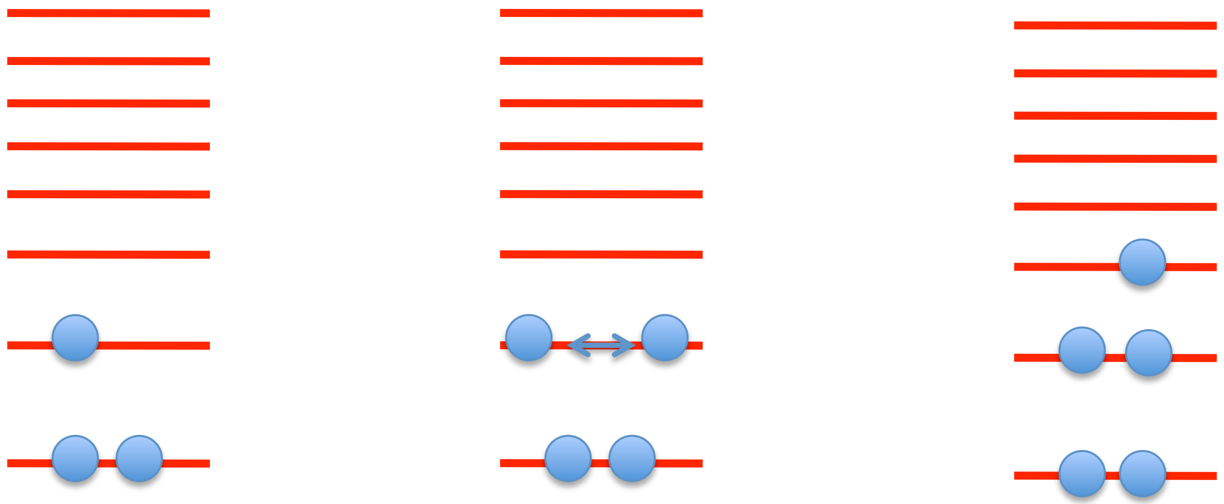
$$\Delta_n = 1/2[2B(N,Z) - B(N+1,Z) - B(N-1,Z)]$$



OES may be attributed to:

- **Pairing correlation effect/pair energy**
- **Deformation effect/mean field effect**

$$\Delta_n = 1/2[2B(N,Z) - B(N+1,Z) - B(N-1,Z)]$$



A. Bohr, B. R. Mottelson, Nuclear structure, vol. 1, World Scientific, 1998.
W.Satula,J.Dobaczewski,W.Nazarewicz,Phys.Rev.Lett.81 (1998) 3599.
W. Friedman, G. Bertsch, E. Phys. J. A 41 (2009) 109.

Examples of OES formulae

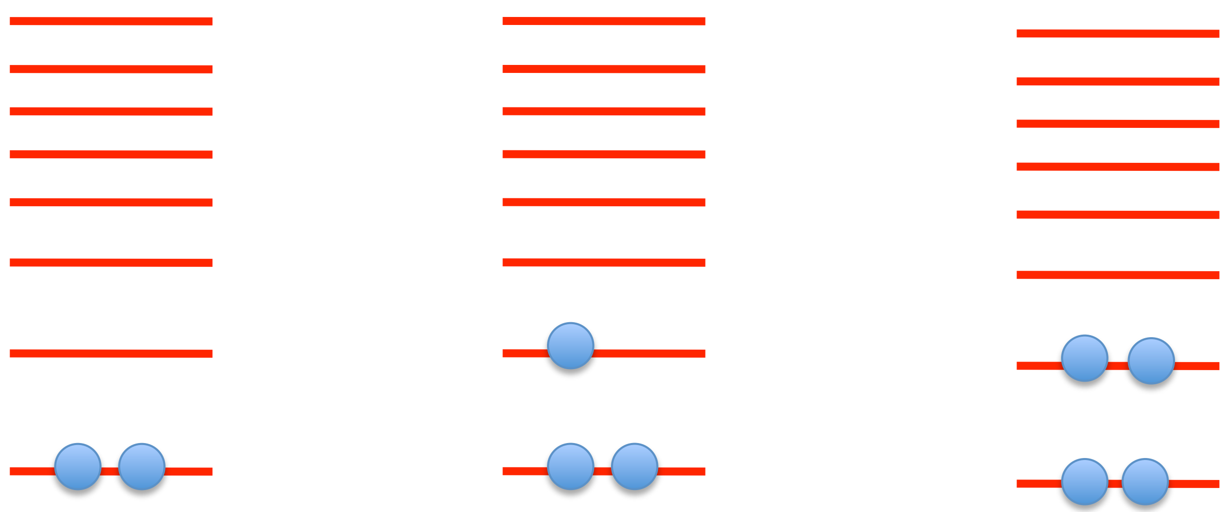
$$\Delta^{(3)}(N) = -\frac{1}{2}\left[B(N-1,Z) + B(N+1,Z) - 2B(N,Z)\right]$$
$$= -\frac{1}{2}[S_n(N+1,Z) - S_n(N,Z)] \quad (1)$$

$$\Delta_C^{(3)}(N) = \frac{1}{2}[S_n(N,Z) - S_n(N-1,Z)]$$
$$= \frac{1}{2}\left[B(N,Z) + B(N-2,Z) - 2B(N-1,Z)\right]$$
$$= \frac{1}{2}[S_{2n}(N,Z) - 2S_n(N-1,Z)]$$
$$= \Delta^{(3)}(N-1),$$

$$\Delta^{(4)}(N) = \frac{1}{4}[-B(N+1,Z) + 3B(N,Z) - 3B(N-1,Z) + B(N-2,Z)]$$
$$= \frac{1}{2}[\Delta^{(3)}(N) + \Delta_C^{(3)}(N)].$$

$$\Delta^{(5)}(N) = \frac{1}{8}[B(N+2,Z) - 4B(N+1,Z) + 6B(N,Z) - 4B(N-1,Z) + B(N-2,Z)]$$
$$= \frac{1}{4}[\Delta_C^{(3)}(N+2) + 2\Delta^{(3)}(N) + \Delta_C^{(3)}(N)].$$

$\Delta_C^{(3)}$



Examples of OES formulae

$$\Delta^{(3)}(N) = -\frac{1}{2}\left[B(N-1,Z) + B(N+1,Z) - 2B(N,Z)\right]$$
$$= -\frac{1}{2}[S_n(N+1,Z) - S_n(N,Z)] \quad (1)$$

$$\Delta_C^{(3)}(N) = \frac{1}{2}[S_n(N,Z) - S_n(N-1,Z)]$$
$$= \frac{1}{2}\left[B(N,Z) + B(N-2,Z) - 2B(N-1,Z)\right]$$
$$= \frac{1}{2}[S_{2n}(N,Z) - 2S_n(N-1,Z)]$$
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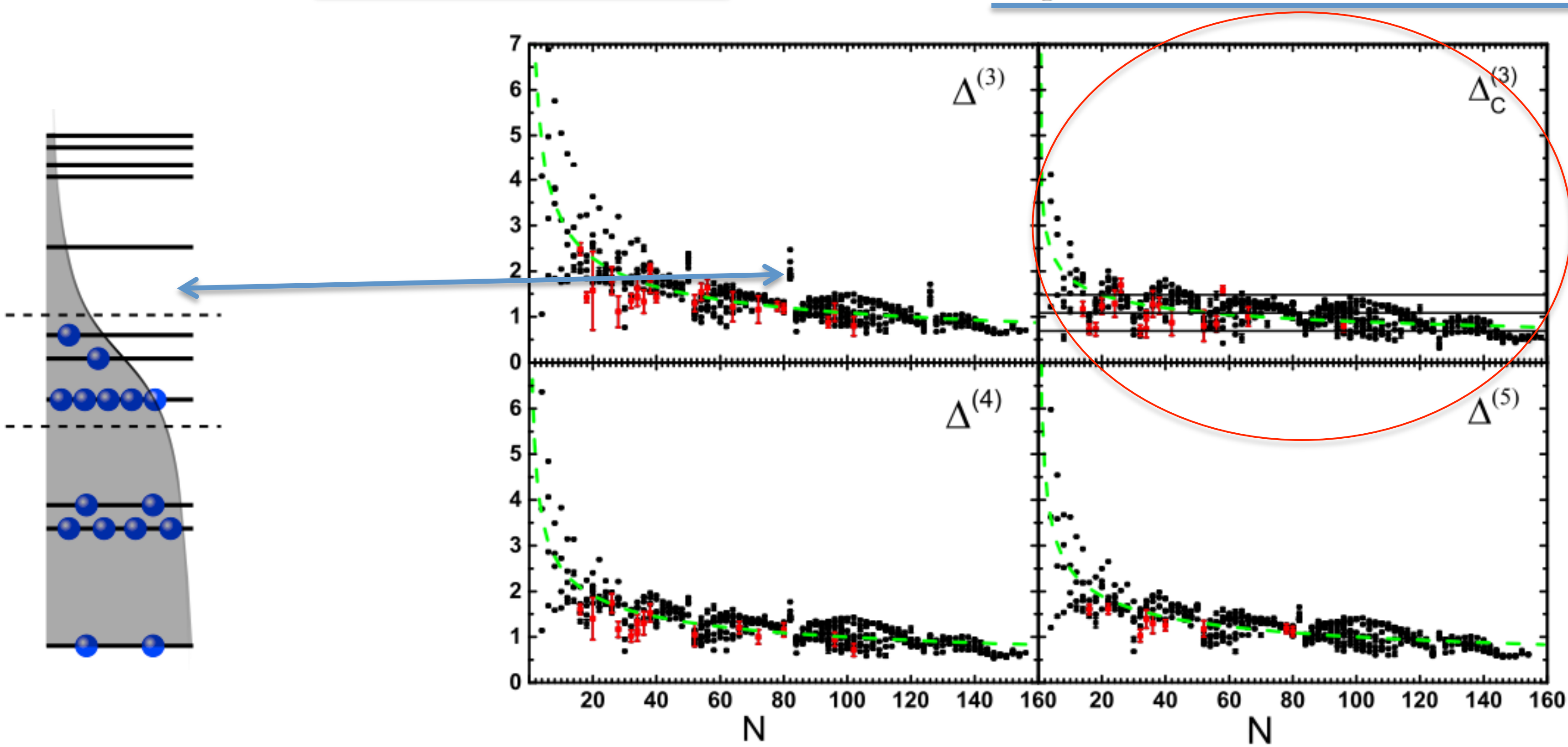
$$\Delta^{(4)}(N) = \frac{1}{4}[-B(N+1,Z) + 3B(N,Z)$$
$$- 3B(N-1,Z) + B(N-2,Z)]$$
$$= \frac{1}{2}[\Delta^{(3)}(N) + \Delta_C^{(3)}(N)].$$

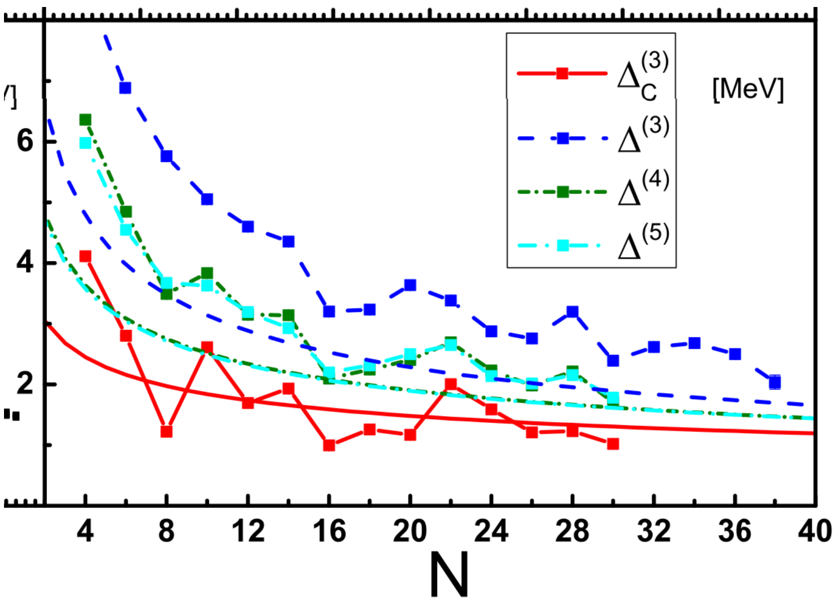
$$\Delta^{(5)}(N) = \frac{1}{8}[B(N+2,Z) - 4B(N+1,Z)$$
$$+ 6B(N,Z) - 4B(N-1,Z) + B(N-2,Z)]$$
$$= \frac{1}{4}[\Delta_C^{(3)}(N+2) + 2\Delta^{(3)}(N) + \Delta_C^{(3)}(N)].$$

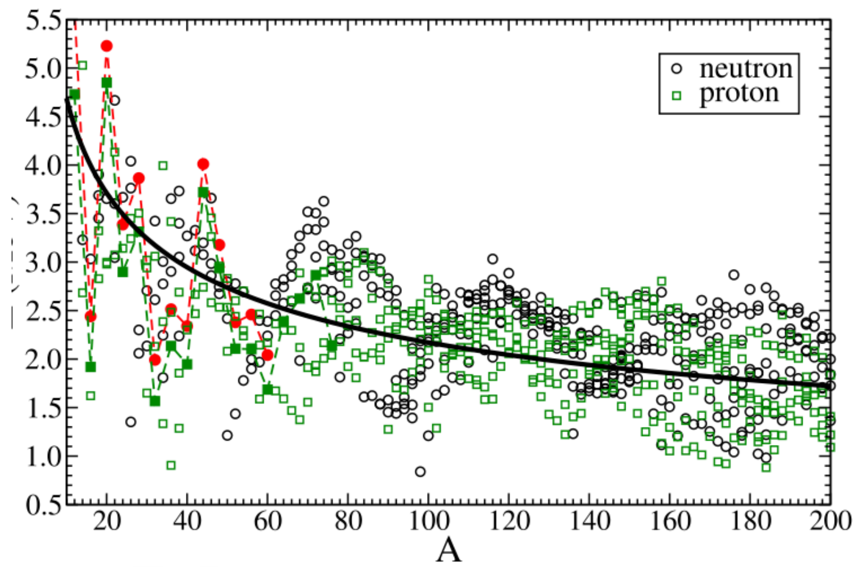


Wigner effect

We hope that $\Delta_C^{(3)}$ contains **minimal contribution from the mean field** and is **'free' from the Wigner effect.**

$$\begin{aligned}\Delta_C^{(3)}(N) &= \frac{1}{2}[S_n(N,Z) - S_n(N-1,Z)] \\ &= \frac{1}{2}[B(N,Z) + B(N-2,Z) - 2B(N-1,Z)] \\ &= \frac{1}{2}[S_{2n}(N,Z) - 2S_n(N-1,Z)] \\ &= \Delta^{(3)}(N-1),\end{aligned}$$





Neutron gaps for Z = N nuclei in comparison with their corresponding fitted curves/average behavior.

Influence of the symmetry energy

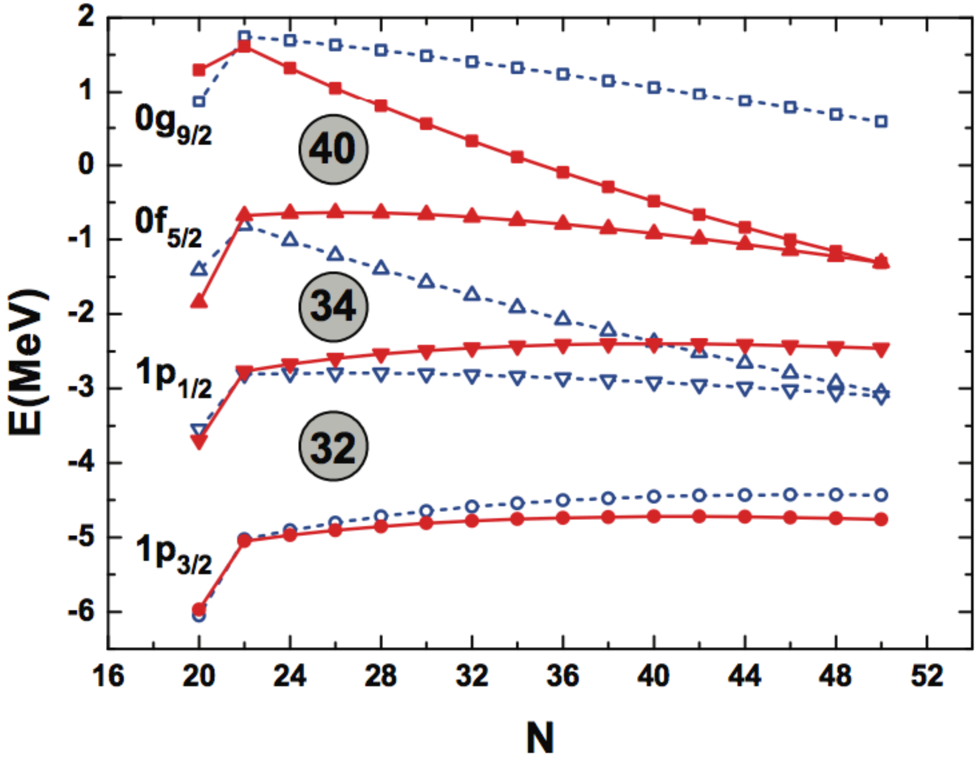
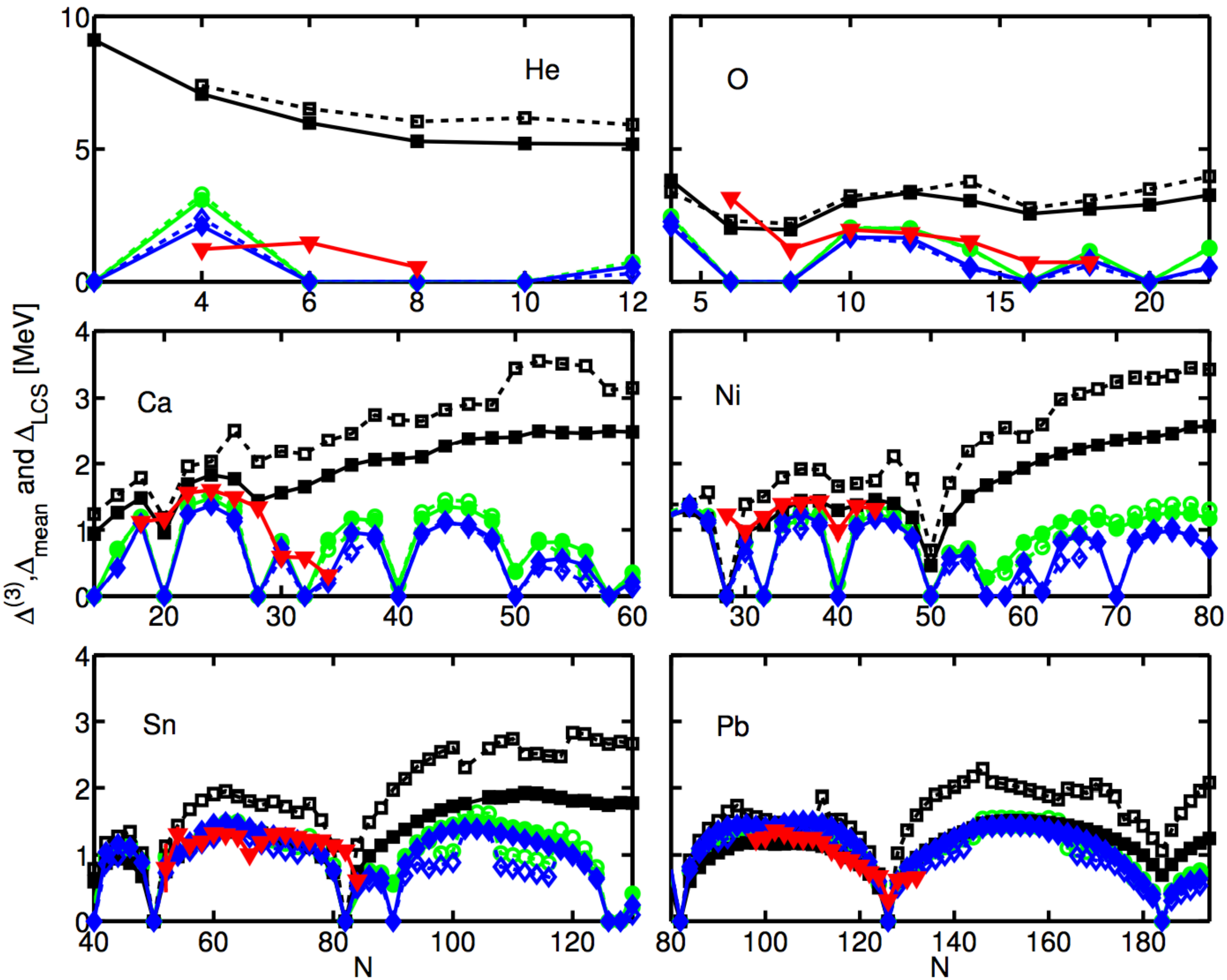


FIG. 3. (Color online) Evolution of the single-particle energies of the $1p_{3/2}$, $1p_{1/2}$, $0f_{5/2}$ and $0g_{9/2}$ orbitals in Ca isotopes as a function of the neutron number N for calculations with the standard Woods-Saxon parameters and $\kappa_{SO} = \kappa$ (open sym-

$$V = V_0\left(1 + \frac{4\kappa}{A}\mathbf{t}\cdot\mathbf{T}_d\right),$$

Z.X. Xu, C. Qi.Phys. Lett. B 724, 247 (2013).

OES in semi-magic nuclei and comparison with the HFB calculations



Pairing in a single-*j* shell

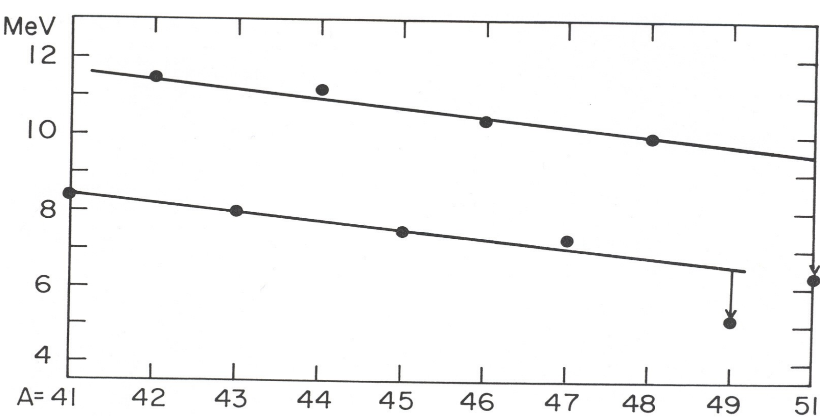
$$\hat{V} = a + b\mathbf{t}_1 \cdot \mathbf{t}_2 + GP_0,$$

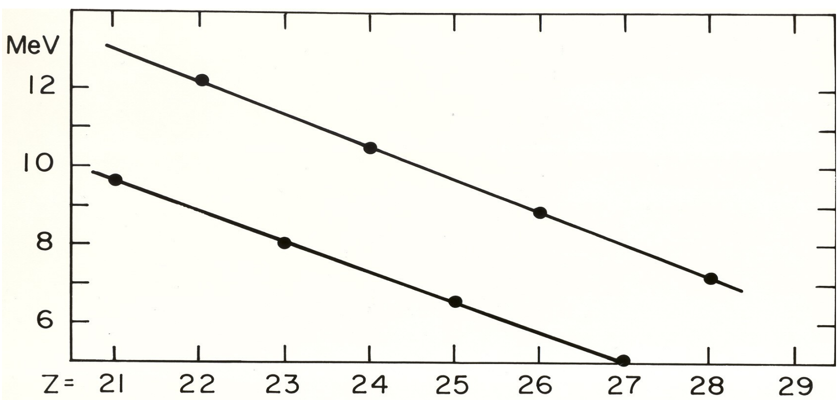
$$E\left(j^n\right) = \frac{1}{4}n(n-1)G - \left[\frac{1}{2}n\right](j+1)G$$

First term takes into account the Pauli principle effect. It gives a minor contribution to the three-point OES formulae.

Seniority coupling in semi-magic nuclei

1943 Racah
1949 Goeppert-Mayer



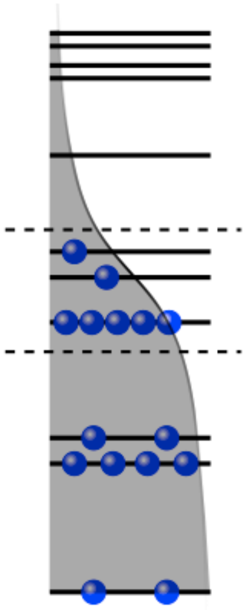


Neutron separation energies from Ca isotopes

Proton separation energies from *N*=28 isotones

I. Talmi, Simple models of complex nuclei (Harwood, Chur, Switzerland, 1993)

The ‘competing’ BCS scheme



$$E_\nu = \sqrt{(\varepsilon_\nu - \lambda)^2 + \Delta_\nu^2}$$

$$\Delta = G \sum_i u_i v_i,$$

uv measures the correlation of the wave function/collectivity of the stat.

For a single-j shell

$$\Delta = \frac{G}{2}\sqrt{n(2j+1-n)},$$

BCS:

- *Does not conserve particle number*
- *Collapse at closed shell*
- *May have problems when applied to drip line nuclei*

Two neutron transfer

Seniority

$$\langle N+2, \upsilon, \alpha | P^\dagger | N, \upsilon, \alpha \rangle = \tfrac{1}{2}\sqrt{(2\Omega - N - \upsilon)(N - \upsilon + 2)},$$

$$\langle N-2, \upsilon, \alpha | P | N, \upsilon, \alpha \rangle = \tfrac{1}{2}\sqrt{(N - \upsilon)(2\Omega - N - \upsilon + 2)}$$

BCS

$$\langle \mathrm{BCS} | P^\dagger | \mathrm{BCS} \rangle = \frac{\Delta}{G}\,.$$

$$\Delta = G \sum_i u_i v_i,$$

Exact Solution of the pairing Hamiltonian for systems with many shells

Richardson's approach

ıme 3, number 6 PHYSICS LETTERS 1 February 1963

A RESTRICTED CLASS OF EXACT EIGENSTATES OF THE PAIRING-FORCE HAMILTONIAN *

R. W. RICHARDSON
H. M. Randall Laboratory of Physics,
University of Michigan, Ann Arbor, Michigan

Received 23 November 1962

$$H_P = \sum_k \varepsilon_k n_k + g \sum_{k,k'} c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} c_{-k'\downarrow} c_{k'\uparrow}$$

$$|\Psi\rangle = \prod_{\alpha=1}^{M} \Gamma^\dagger_\alpha |0\rangle, \quad \Gamma^\dagger_\alpha = \sum_k \frac{1}{2\varepsilon_k - E_\alpha} c^+_{k\uparrow} c^+_{-k\downarrow}$$

Richardson equation

$$1 + g \sum_{k=0} \frac{1}{2\varepsilon_k - E_\alpha} + 2g \sum_{\beta(\neq\alpha)=1}^{M} \frac{1}{E_\alpha - E_\beta} = 0, \qquad E = \sum_{\alpha=1}^{M} E_\alpha$$

J. Dukelsky, S. Pittel, and G. Sierra, Rev. Mod. Phys. **76**, 643 (2004)

Richardson equation

•A set of M nonlinear coupled equations with M unknowns (E_α) and it is very difficult to solve.

•The pair energies are either real or complex conjugated pairs and do not have clear physical meaning.

•The wave function is not given directly

$$1 + g\sum_{k=0} \frac{1}{2\varepsilon_k - E_\alpha} + 2g \sum_{\beta(\neq\alpha)=1}^{M} \frac{1}{E_\alpha - E_\beta} = 0, \qquad E = \sum_{\alpha=1}^{M} E_\alpha$$

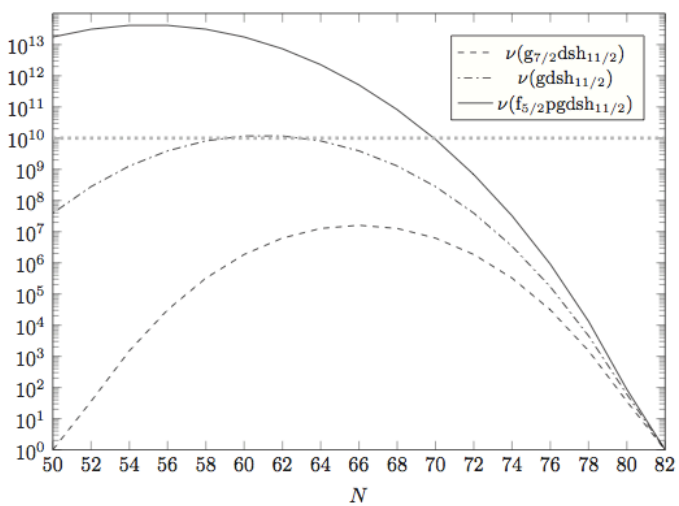
For two pairs in a single-j shell

$$E_\alpha = -(\Omega - 1)g \pm i\sqrt{\Omega - 1}g$$

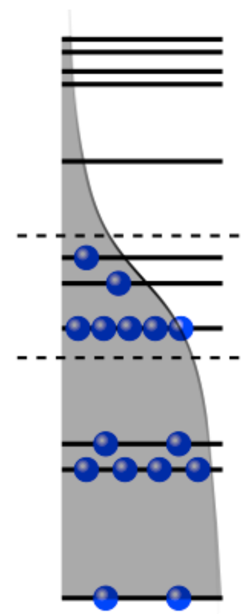
Seniority coupling for many shells

- Shell model calculations restricted to the v=0 subspace
- There are as many independent solutions as states in the v=0 space.
- Valid for any forms of pairing.

$$\begin{aligned}&\langle \{s_j\}, \dots N_j + 2, \dots N_{j'} - 2, \dots | \\ &\quad \times H | \{s_j\}, \dots N_j, \dots N_{j'}, \dots \rangle \\ &= \frac{G_{jj'}}{4} \big[(N_{j'} - s_{j'})(2\Omega_{j'} - s_{j'} - N_{j'} + 2) \\ &\qquad \times (2\Omega_j - s_j - N_j)(N_j - s_j + 2) \big]^{1/2}.\end{aligned}$$



10^2 in seniority space
Easier to include many shells



What is the pairing correlation energy?

If one removes the self-energy, which may have been taken into account by the mean field, the binding energy of a single j system can be rewritten as

$$E\left(j^{n}\right)=\frac{1}{4}n(n-1)G-\left[\frac{1}{2}n\right](j+1)G+\left[\frac{1}{2}n\right]G$$

A. Volya et al. / Physics Letters B 509 (2001) 37

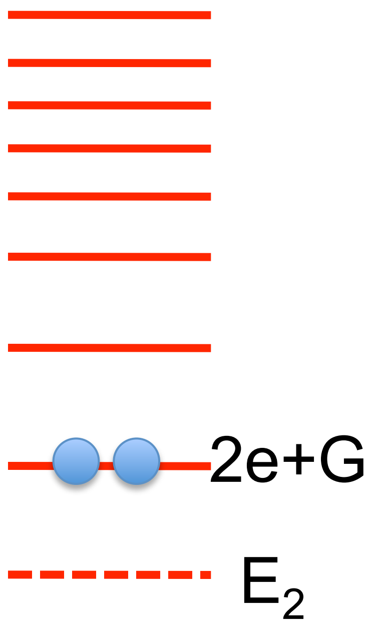
$$E_{\mathrm{corr}}=E-\sum_{j}\epsilon_{j}\overline{N}_{j}-\sum_{j}\frac{G_{jj}}{2\Omega_{j}-1}\frac{\overline{N}_{j}(\overline{N}_{j}-1)}{2},$$

For two particles in a non-degenerate system with a constant pairing, the energy can be evaluated through the well known relation,

$$G \sum_i \frac{2j_i + 1}{2\varepsilon_i - E_2} = 2. \tag{10}$$

The corresponding wave function amplitudes are given by

$$X_i = N_n \frac{2j + 1}{2\varepsilon_i - E_2} \tag{11}$$



The correlation energy induced by the monopole pairing corresponds to the difference

$$\Delta = \varepsilon_\delta - \frac{1}{2} E_2, \tag{12}$$

where δ denotes the lowest orbital. As the gap Δ increases the amplitude X_i becomes more dispersed, resulting in stronger two-particle correlation.

Seniority coupling involving many shells

- A way to solve the pairing Hamiltonian exactly
- Low-seniority configurations are dominant

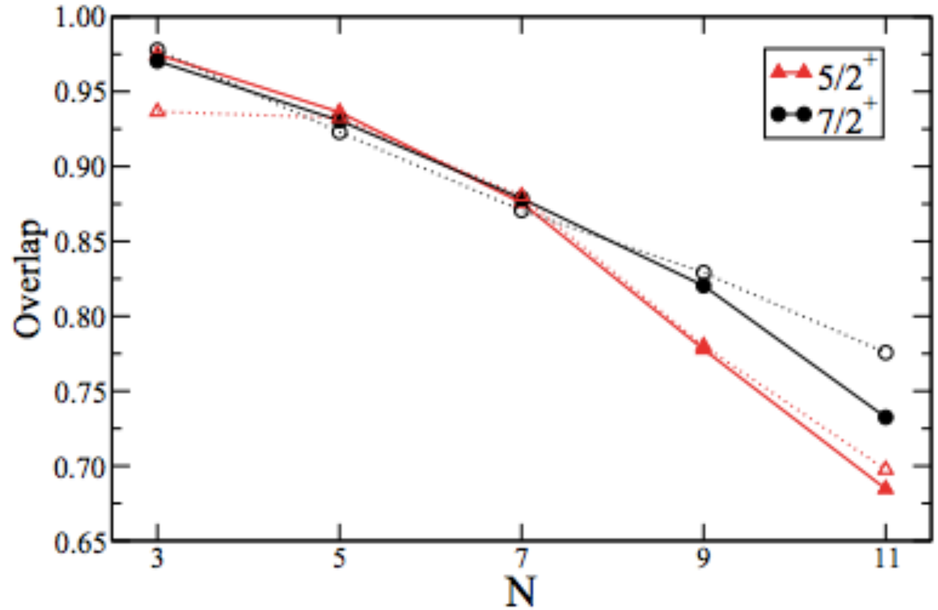


FIG. 11. (Color Online) Solid symbol: The overlaps between the wave functions $|\Psi_I\rangle$ of the full Hamiltonian H and those of the pairing Hamiltonian with $J_{\rm max} = 0$ for the first $5/2^+$ and $7/2^+$ states in light odd-A Sn isotopes. Open symbol: Same as above but only the non-diagonal pairing matrix elements are considered.

CQ, Z. Xu, Phys. Rev. C 86, 044323 (2012)

Binding energies of Ca isotopes

WS + constant pairing

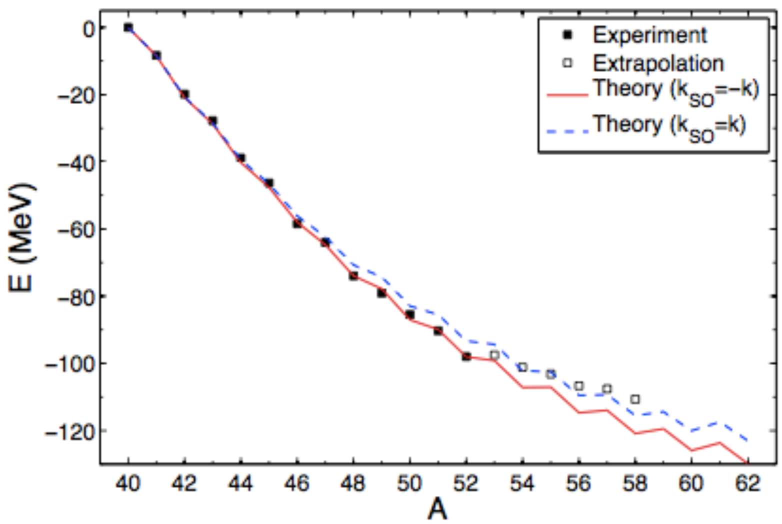
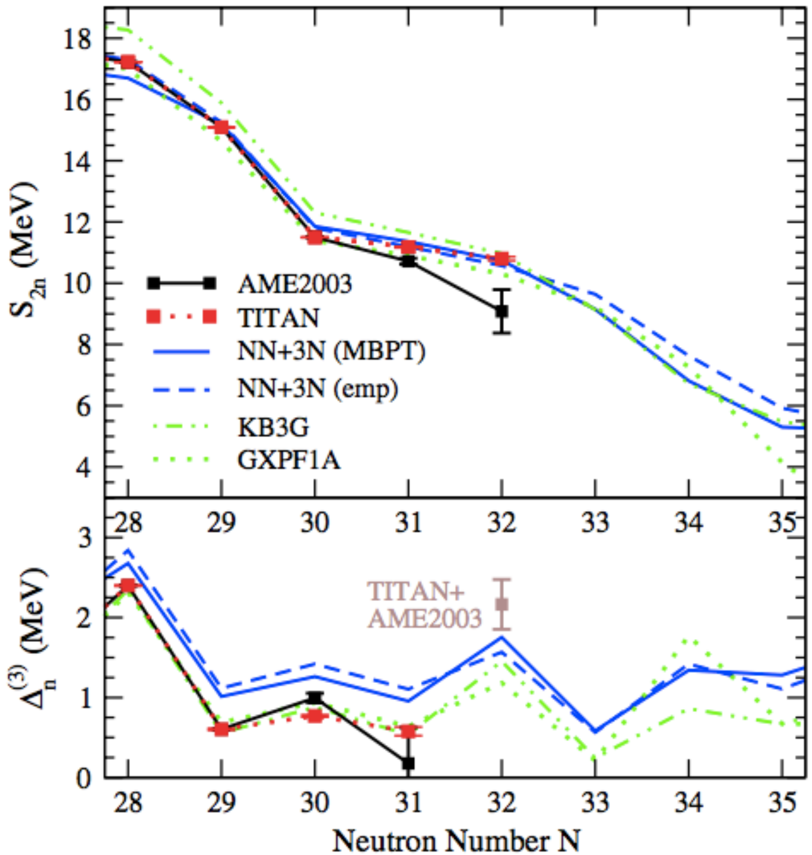


FIG. 6. (Color online) Experimental [34, 37] and calculated ground-state energies of Ca isotopes, relative to that of ^{40}Ca, as a function of mass number A.

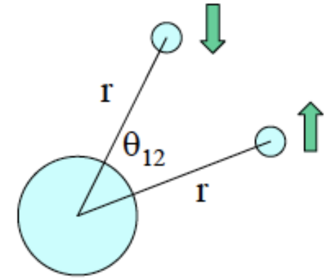


A. T. Gallant et al., PRL 109, 032506 (2012)

Calculations with three-body interaction

J.D. Holt, T. Otsuka, A. Schwenk, and T. Suzuki, J. Phys. G 39, 085111 (2012).
G. Hagen, M. Hjorth-Jensen, G.R. Jansen, R. Machleidt,
T. Papenbrock, Phys.Rev.Lett. 109, 032502 (2012).

Two-body clustering



Configuration mixing from higher lying orbits is important for clustering at the surface

$$\Psi_2(\mathbf{r_1},\mathbf{r_2}) = (\chi_1\chi_2)_0\Phi_2(r_1,r_2,\theta_{12}) = (\chi_1\chi_2)_0\frac{1}{4\pi}\sum_p\sqrt{\frac{2j_p+1}{2}}X_p\phi_p(r_1)\phi_p(r_2)P_{l_p}(\cos\theta_{12}),$$

r_1=9fm

^{210}Pb

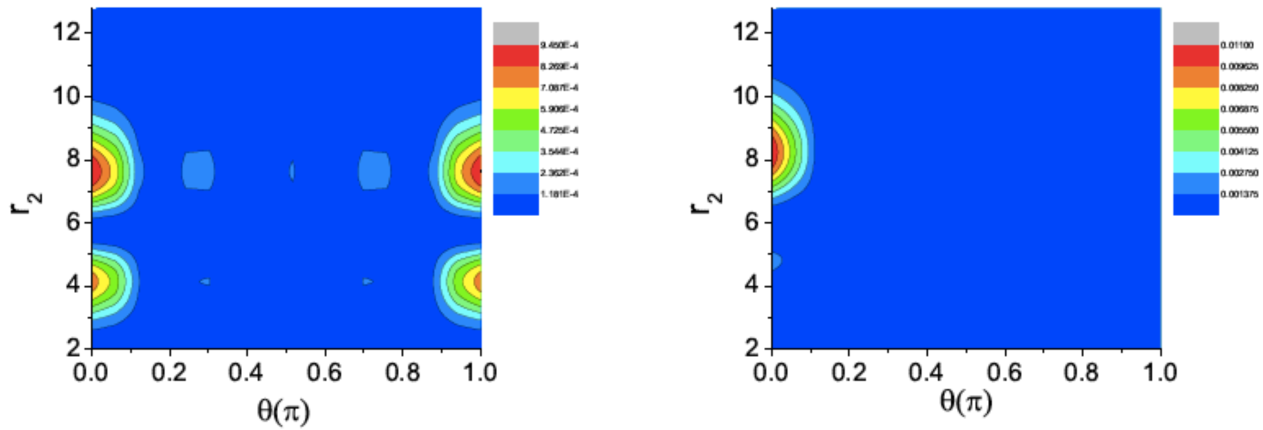
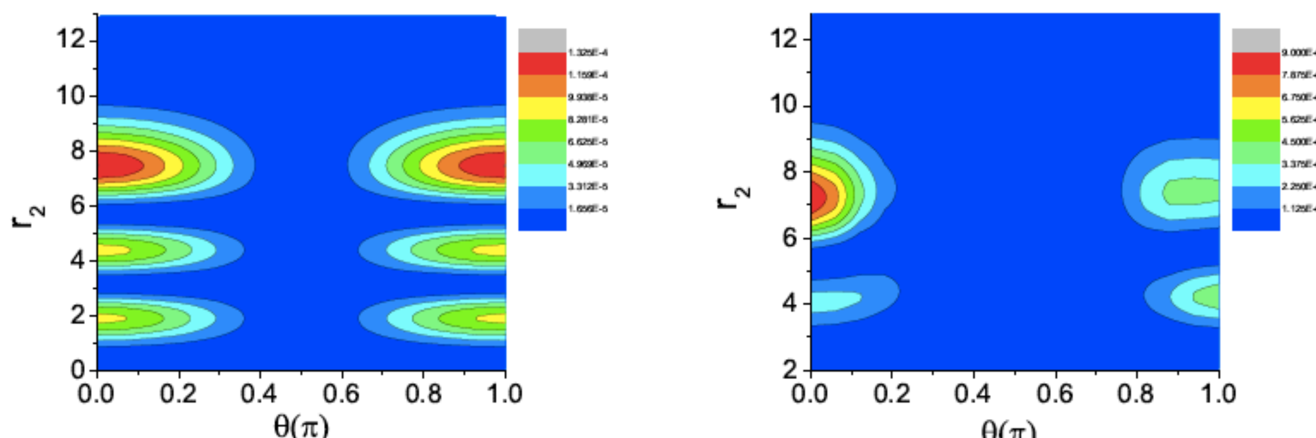
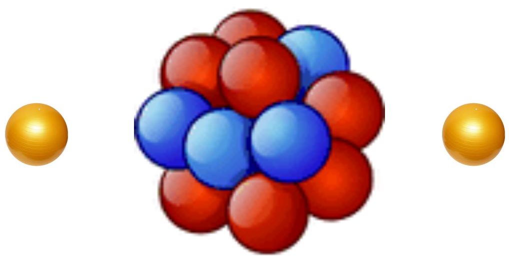
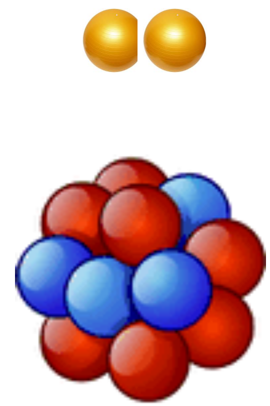


FIG. 10: (color online). The square of the two-neutron wave function $|\Psi_{2\nu}(r_1,r_2,\theta)|^2$ with r_1 = 9 fm as a function of r_2 and θ. Left: the leading configuration; Right: 4 major shells

^{206}Pb



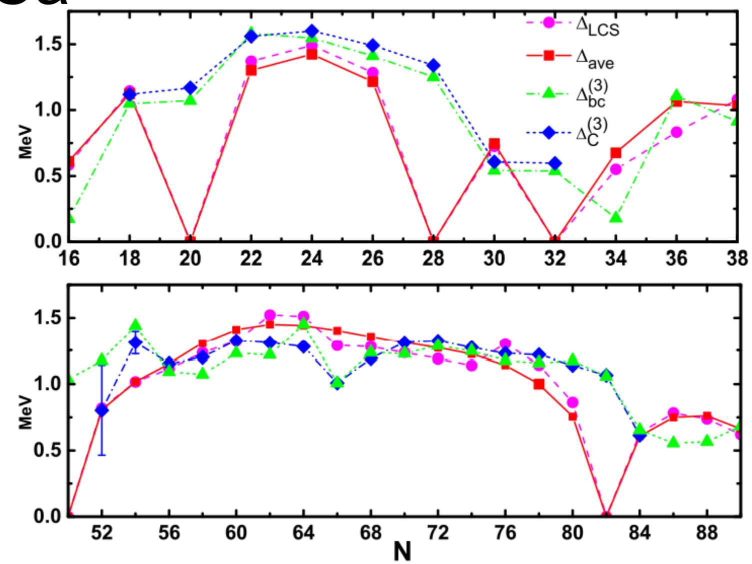




No Pairing **'Strong' pairing**

Pairing gap

Ca



Sn

Two-body wave function from HFB

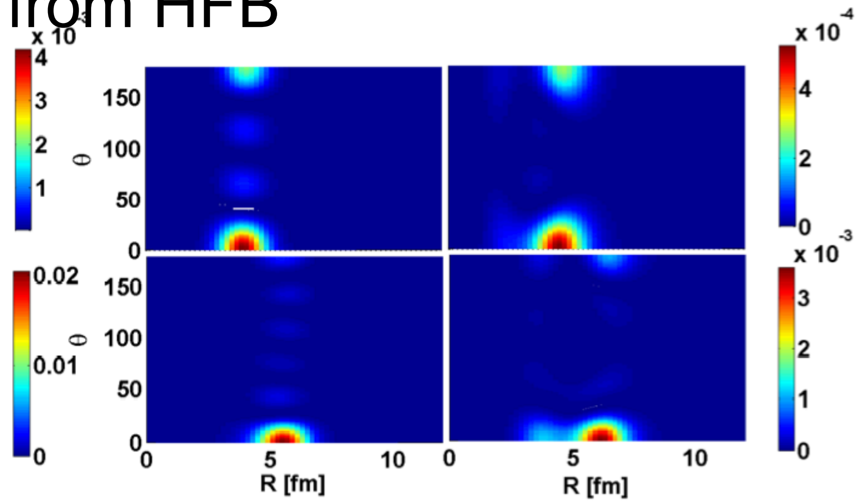


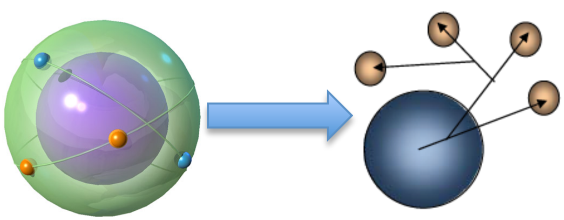
Figure 5: Upper: The two-neutron correlation plots for ^{46}Ca (left) and ^{54}Ca (right). Lower: Same as upper but for ^{128}Sn (left, 4 holes) and ^{136}Sn (right, 4 particles). Notice that the scale is different.

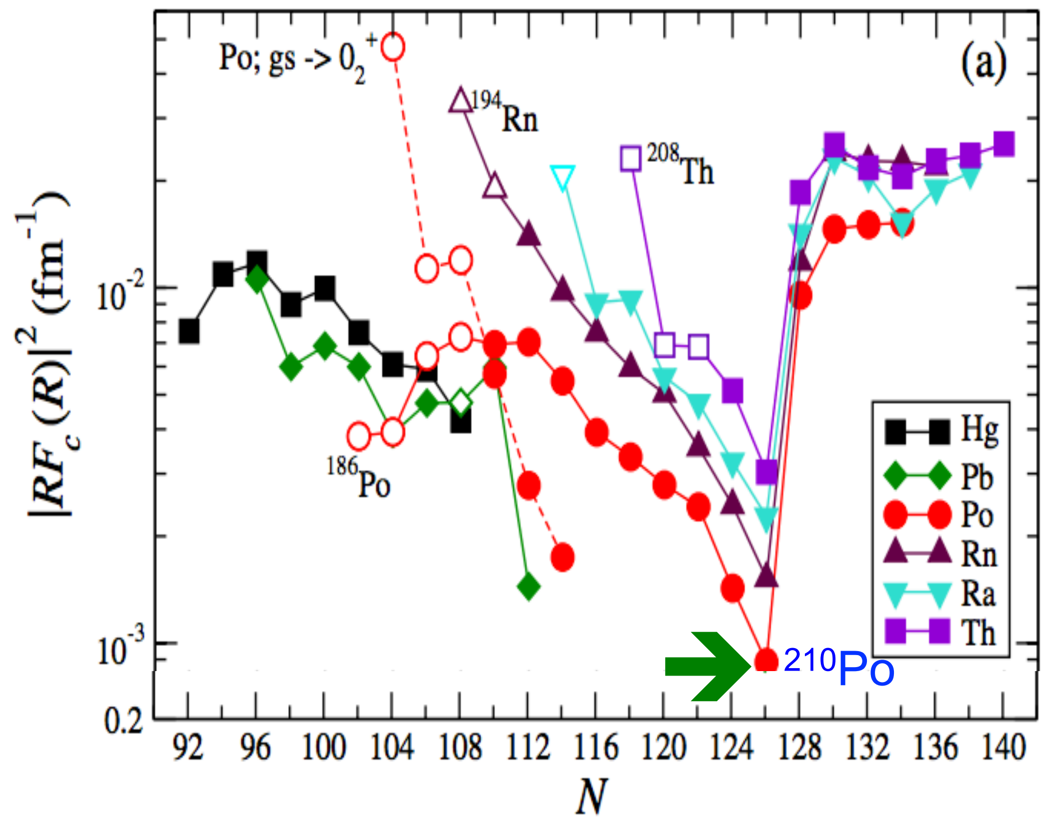
Alpha formation probability from experiments

$$\log|RF(R)|^{-2} = \log T_{1/2}^{\text{Expt.}} - \log\left[\frac{\ln 2}{\nu}|H_0^+(\chi,\rho)|^2\right],$$

R should be large enough that the nuclear interaction is negligible, i.e., at the nuclear surface.

$R=1.2(A_d^{1/3}+A_c^{1/3})$





CQ et al, Phys.Rev.C80,044326 (2009); 81,064319 (2010).

210Po vs 212Po (The later is the textbook example of alpha emitter)

$$|^{212}\mathrm{Po}(\alpha_4)\rangle = \sum_{\alpha_2\beta_2} X(\alpha_2\beta_2;\alpha_4)|^{210}\mathrm{Pb}(\alpha_2)\otimes^{210}\mathrm{Po}(\beta_2)\rangle$$

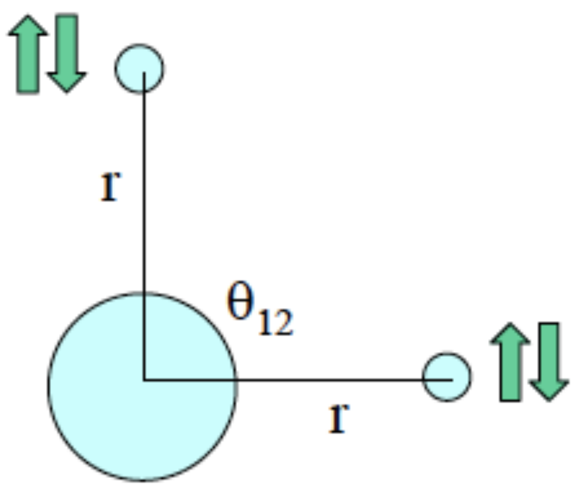
If we neglect the proton-neutron interaction

$$|^{212}\mathrm{Po}(\alpha,\mathrm{g.s.})\rangle = |^{210}\mathrm{Pb}(2\nu,\mathrm{g.s.})\otimes|^{210}\mathrm{Po}(2\pi,\mathrm{g.s.})\rangle,$$

$$|^{210}\mathrm{Po}(\alpha,\mathrm{g.s.})\rangle = |^{206}\mathrm{Pb}(2\nu^{-1},\mathrm{g.s.})\otimes|^{210}\mathrm{Po}(2\pi,\mathrm{g.s.})\rangle.$$

$$\mathscr{F}_\alpha(R;^{212}\mathrm{Po(gs)}) = \int d\mathbf{R}d\xi_\alpha\phi_\alpha(\xi_\alpha)\Psi(\mathbf{r_1r_2};^{210}\mathrm{Pb(gs)})\Psi(\mathbf{r_3r_4};^{210}\mathrm{Po(gs)}),$$

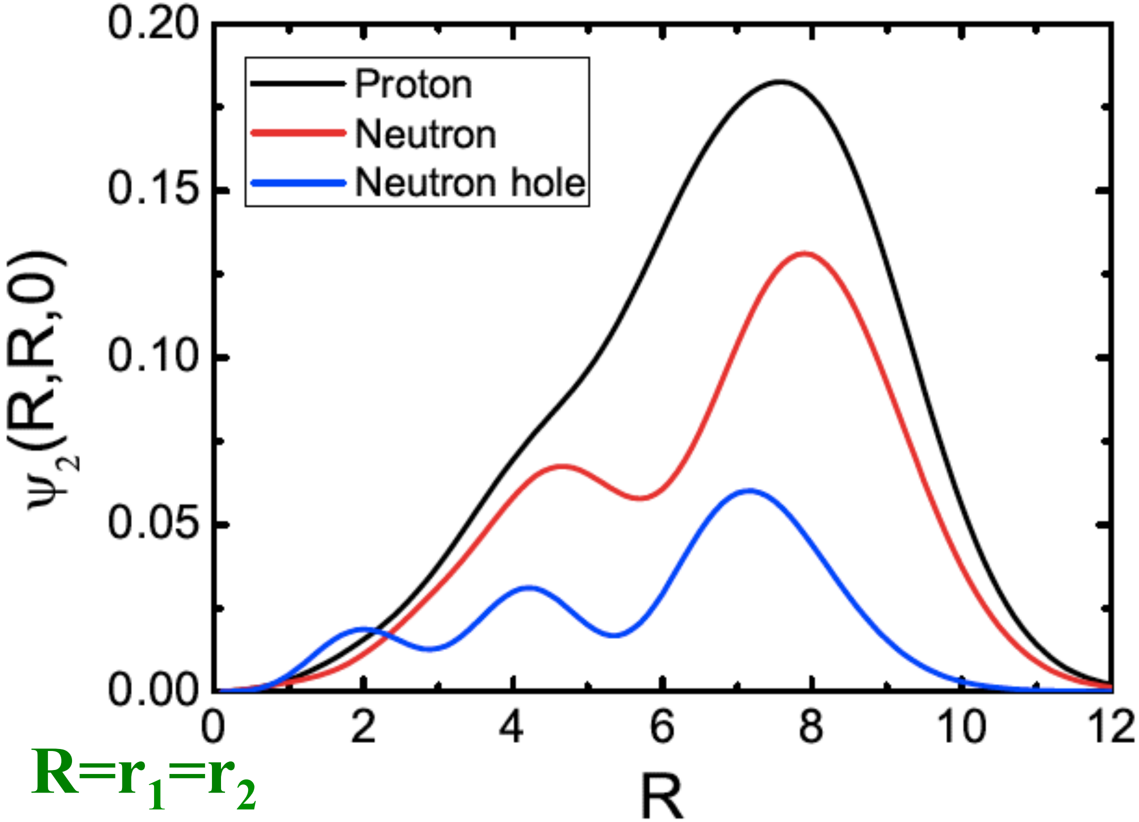
$$\mathscr{F}_\alpha(R;^{210}\mathrm{Po(gs)}) = \int d\mathbf{R}d\xi_\alpha\phi_\alpha(\xi_\alpha)\Psi(\mathbf{r_1r_2};^{206}\mathrm{Pb(gs)})\Psi(\mathbf{r_3r_4};^{210}\mathrm{Po(gs)}).$$



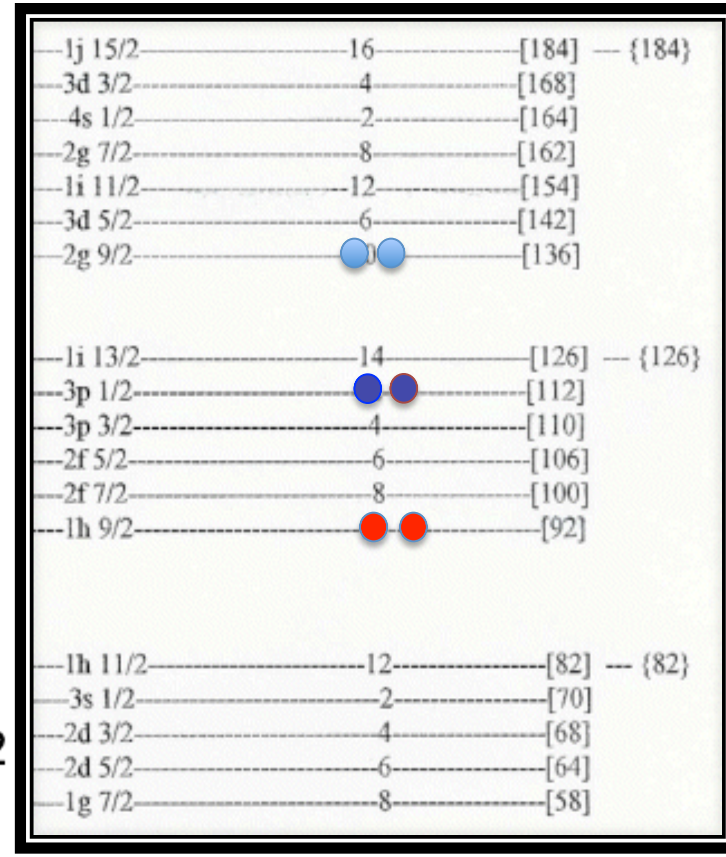
Two-body clustering

Two-body clustering

$$\Psi_2(\mathbf{r_1},\mathbf{r_2}) = (\chi_1\chi_2)_0\Phi_2(r_1,r_2,\theta_{12}) = (\chi_1\chi_2)_0\frac{1}{4\pi}\sum_p\sqrt{\frac{2j_p+1}{2}}X_p\phi_p(r_1)\phi_p(r_2)P_{l_p}(\cos\theta_{12}),$$



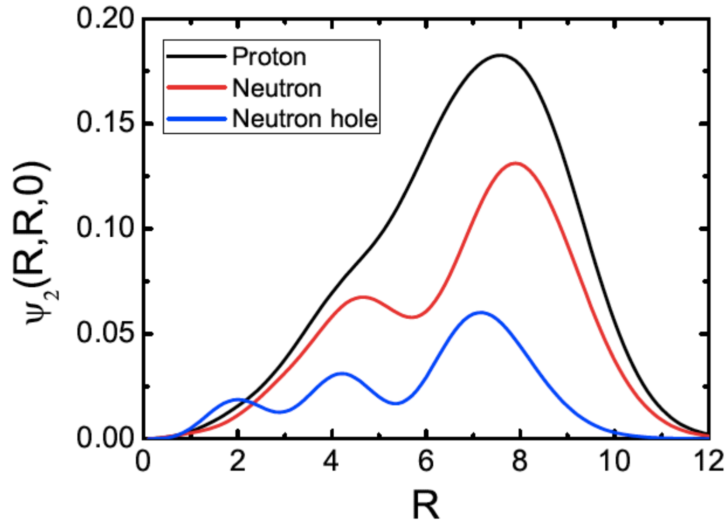
$R=r_1=r_2$

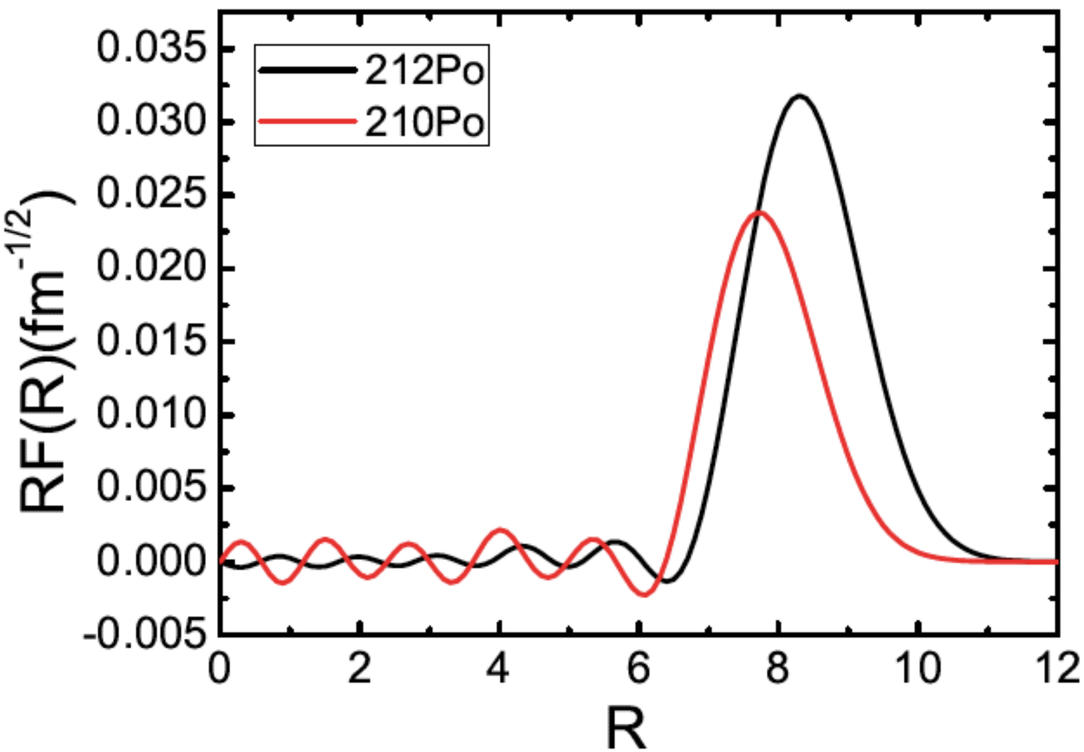


- The two-body wave functions are indeed strongly enhanced at the nuclear surface;
- The enhancement is much weaker in ^{206}Pb(gs) than that in ^{210}Pb(gs)
  - *Relatively small number of configurations in the hole-hole case;*
  - *$p_{1/2}$ dominance in ^{206}Pb(gs);*
  - *Radial wave functions of hole states less extended.*

Alpha formation amplitude

$$F_\alpha(R) = \sqrt{\frac{1}{4\pi}} \int dr_\alpha \phi_\alpha(r_\alpha) \Psi_{2\pi}(\mathbf{r_1}, \mathbf{r_2}) \Psi_{2\nu}(\mathbf{r_3}, \mathbf{r_4}),$$

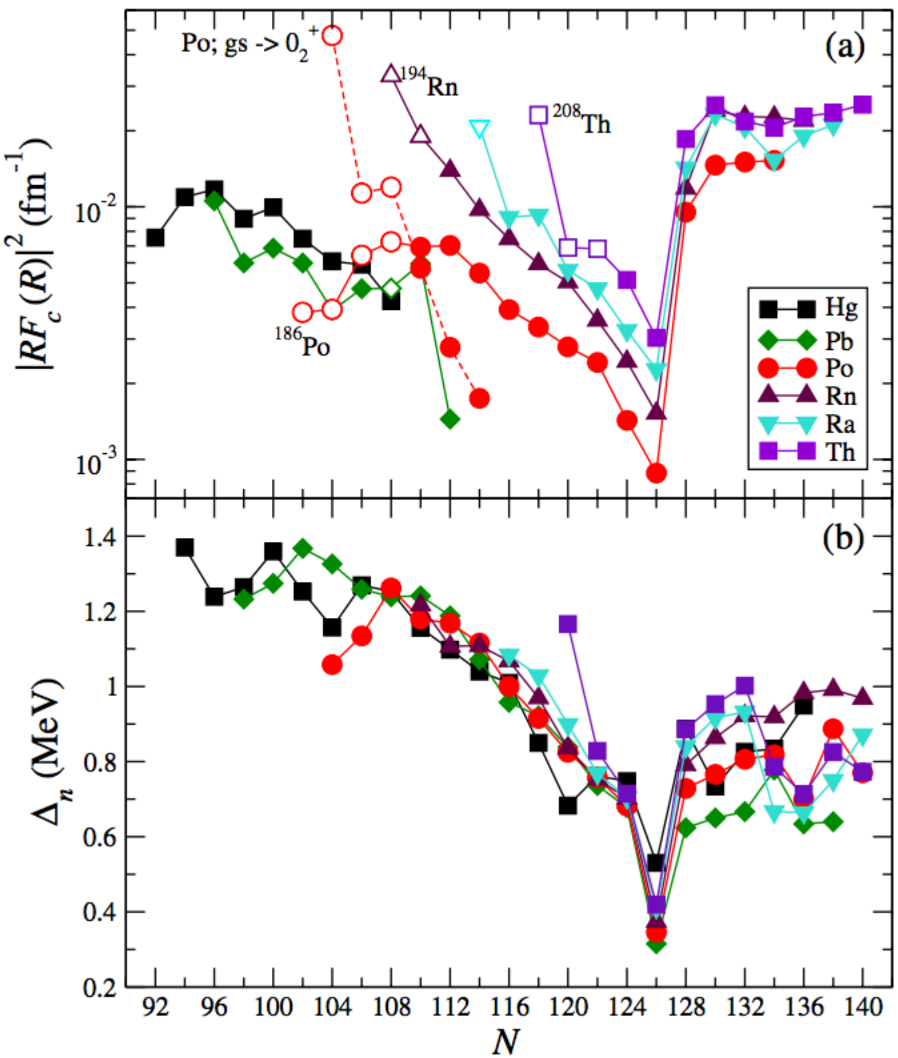




- ➢**Alpha particle is formed on the nuclear surface;**
- ➢**The clustering induced by the pairing mode is inhibited if the configuration space does not allow a proper manifestation of the pairing collectivity.**

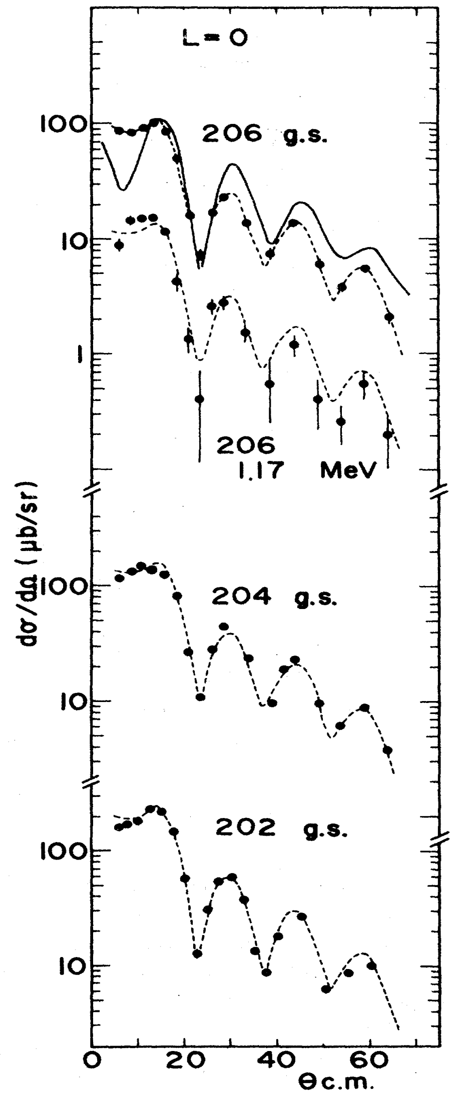
CQ et al., Phys. Rev. C 81, 064319 (2010).

Pairing gap and the alpha formation



A.N. Andreyev CQ et al., PhysRevLett.110.242502 (2013).

Cross sections of (p,t) reactions on Pb isotopes



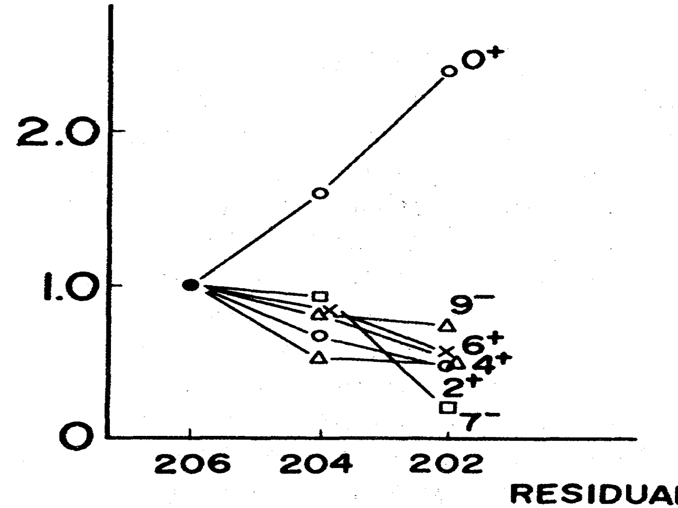
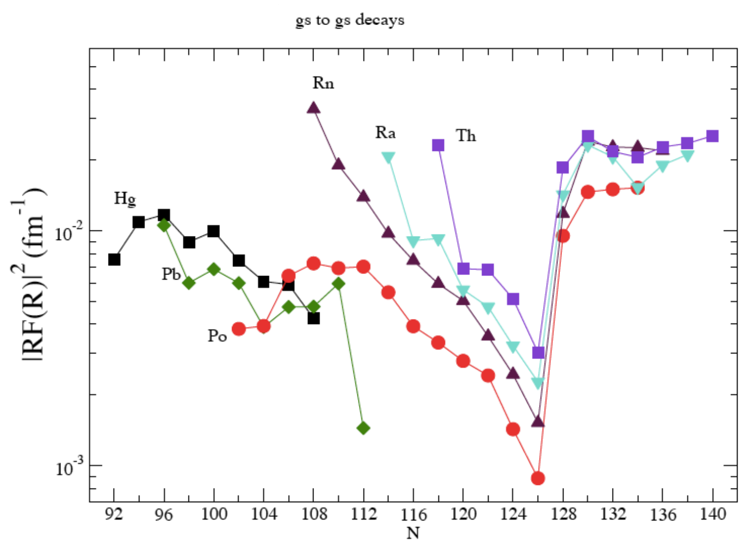


FIG. 13. Isotope dependence of triton strengths leading
› the lowest state of $J^{\pi}=0^{+}, 2^{+}, 4^{+}, 6^{+}, 7^{-}$, and 9^{-}.



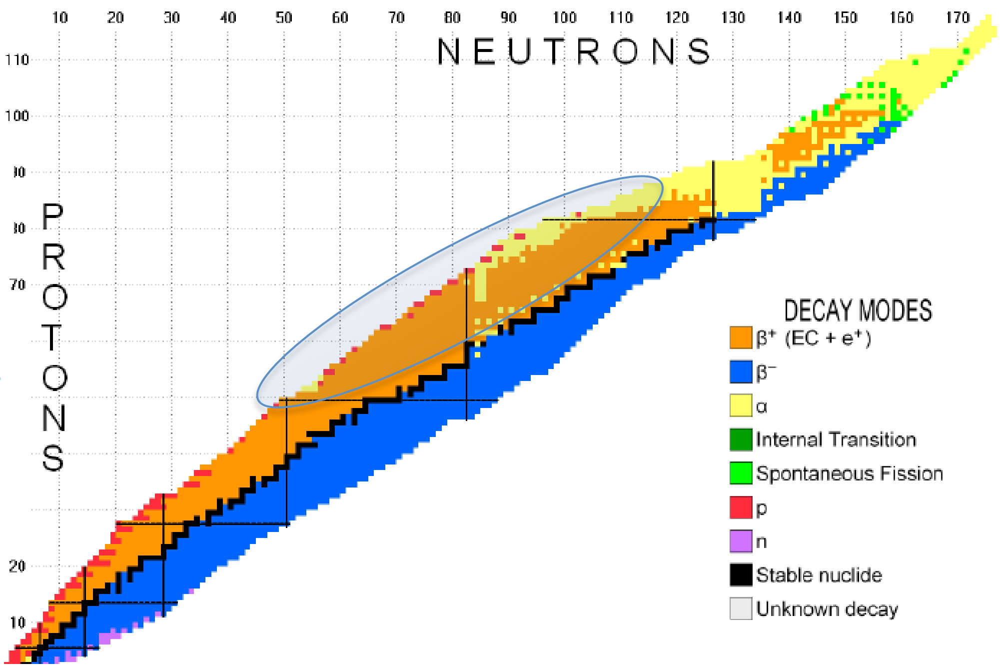
M. Takahashi, PRC27,1454(1983)

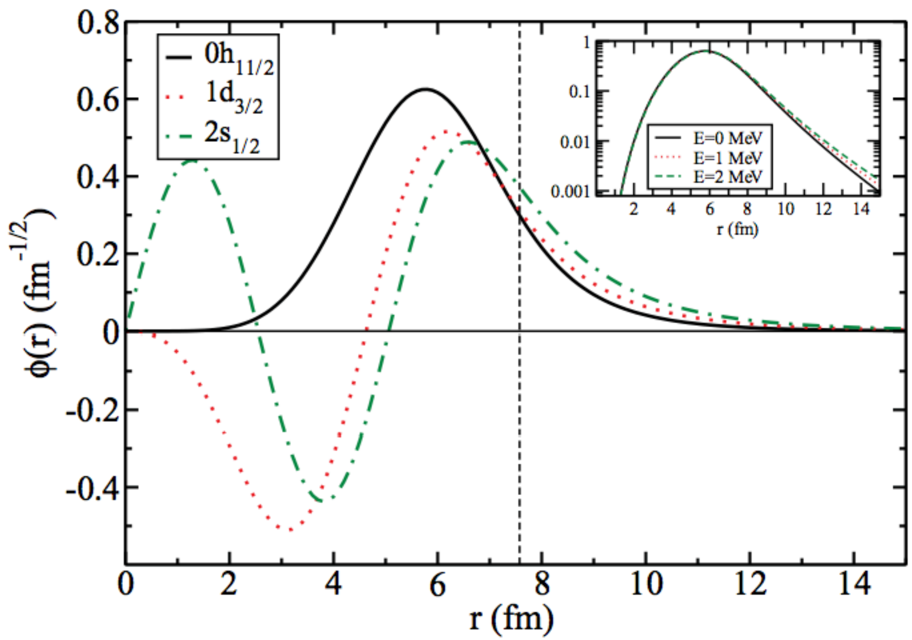
Proton decay

$$\mathcal{F}_l(R) = \int d\mathbf{R}\, d\xi_d [\Psi(\xi_d)\xi_p Y_l(\mathbf{R})]^*_{J_m M_m} \Psi_m(\xi_d, \xi_p, \mathbf{R}),$$

that $\mathcal{F}_l(R)$ would indeed be the wave function of the outgoing particle $\psi_p(R)$ if the mother nucleus would behave at the point R as

$$\Psi_m(\xi_d, \xi_p, \mathbf{R}) = [\Psi(\xi_d)\xi_p \psi_p(R) Y_l(\mathbf{R})]_{J_m M_m}. \quad (3)$$





C. Qi, D.S. Delion, R.J. Liotta, and R. Wyss, 85, 011303(R) (2012)

Formation vs ‘u’

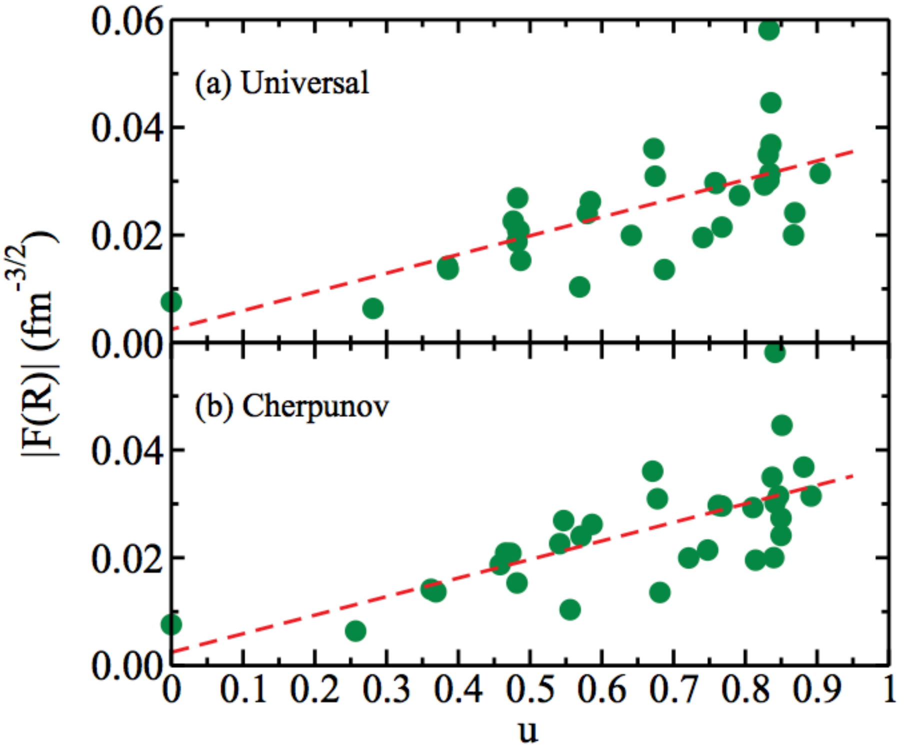


FIG. 4. (Color online) The formation amplitudes $|F_l(R)|$ extracted from experimental data for proton decays of nuclei $N \geqslant 75$ ($Z > 67$) as a function of u calculated from BCS calculations using for the Woods-Saxon mean field the universal parameters [23] (upper) and the Cherpunov parameters [24] (lower).

Summary

- OES and pairing gaps
- Seniority in many shells
- OES as an indication of the pairing collectivity
- Alpha clustering in heavy nuclei and pairing correlation
- Inclusion of continuum configurations
- Application in studying the pair correlation effects in decay and reaction processes

Thank you!

Microscopic description of alpha decay

R.G. Thomas, Prog. Theor. Phys. **12**, 253 (1954).

$$T_{1/2} = \frac{\hbar \ln 2}{\Gamma_c} = \frac{\ln 2}{\nu} \left| \frac{H_l^+(\chi, \rho)}{RF_c(R)} \right|^2,$$

ν is the outgoing velocity of the emitted particle
$F_c(R)$ is the formation amplitude
H_l^+ is the Coulomb-Hankel function The penetrability is proportional to $|H_l^+(\chi, \rho)|^{-2}$.
R is the distance between the center of mass of the cluster and daughter nucleus which divides the decay process into an internal region and complementary external region.

$$m \rightarrow d + \alpha$$

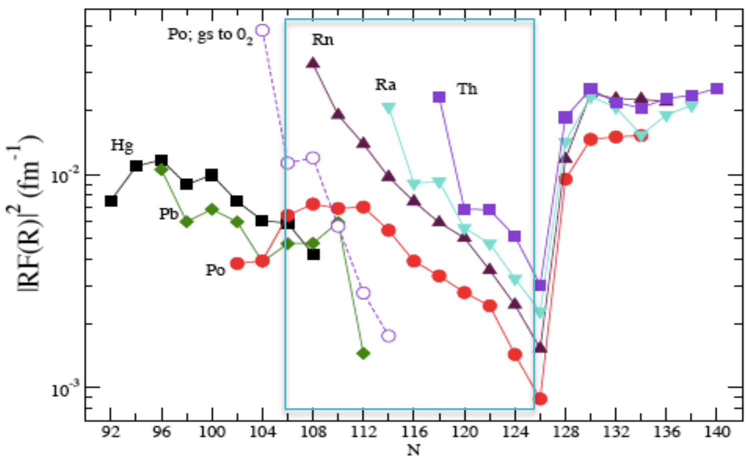
$$\mathcal{F}_l(R) = \int d\mathbf{R} d\xi_d d\xi_\alpha [\Psi(\xi_d)\phi(\xi_\alpha)Y_l(\mathbf{R})]^*_{J_m M_m} \Psi_m(\xi_d, \xi_\alpha, \mathbf{R}),$$

Formation amplitude:

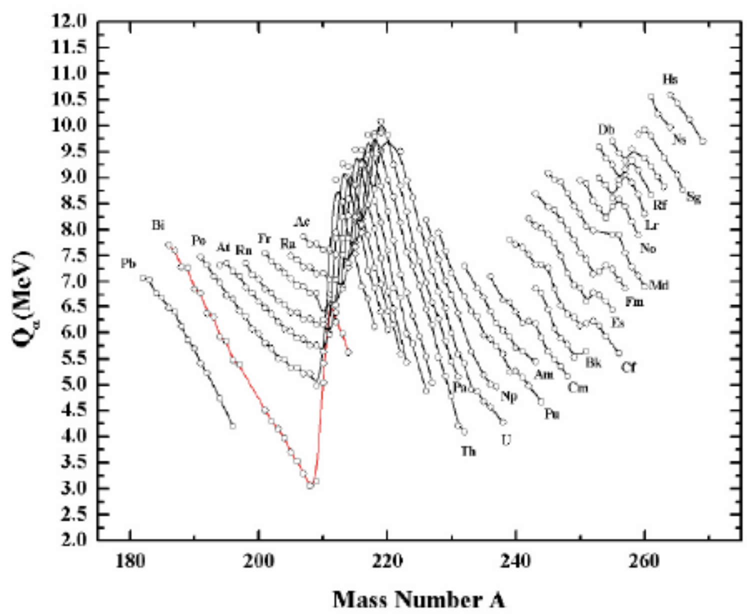
- Can be extracted from experimental data in a model-independent way;
- Can be calculated microscopically in a direct way;

Shell Model
H.J. Mang, PR 119,1069 (1960); I. Tonozuka, A. Arima, NPA 323, 45 (1979).
BCS approach
HJ Mang and JO Rasmussen, Mat. Fys. Medd. Dan. Vid. Selsk. (1962)
DS Delion, A. Insolia and RJ Liotta, PRC46, 884(1992).

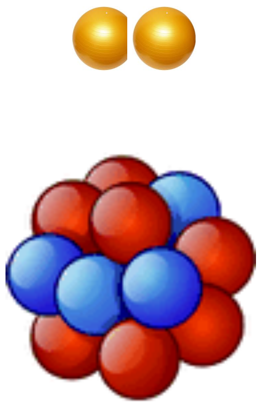
GN law works if the formation probability is a constant or proportional to $Q^{-1/2}$



$\log|RF(R)|^2 \propto Q^{-1/2}$



2p decay?



Strong pairing