p,2p and p,pn reactions with rare isotopes

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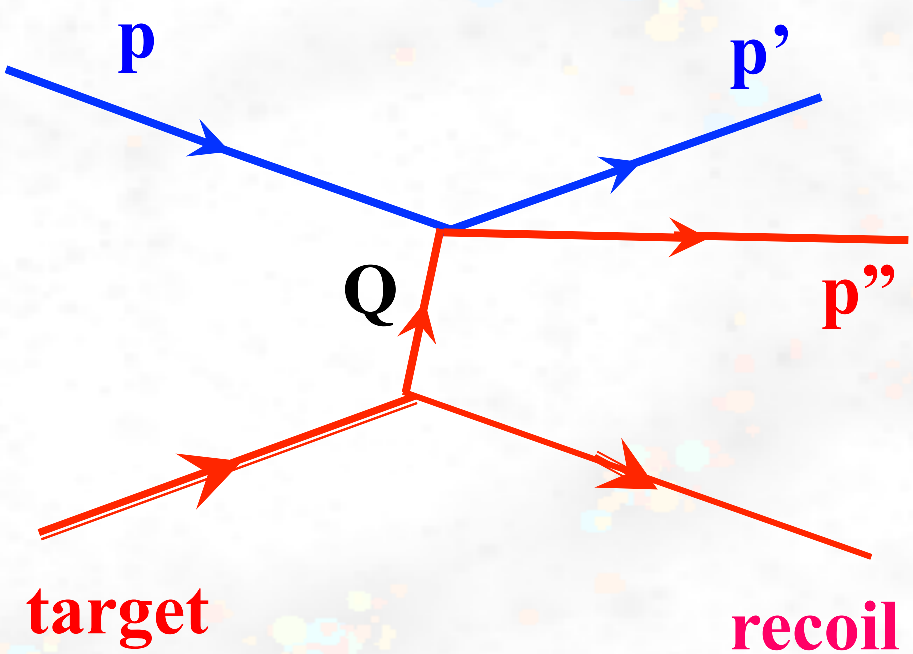
“Quasi-free (p,2p) and (p,pn) reactions with unstable nuclei”

T. Aumann, CB, J. Ryckebusch, PRC 88, 064610 (2013)

If (p,pN) was an IDEAL PROBE

- Impulse approximation: single hit + wave distortion (DWIA)
- For E_p ~ 200 MeV or higher, "Glauber" may apply
- Multiple scattering only responsible for distortion + absorption
- No off-shell effects, no correlations, etc.



Spectroscopic factor

Kinematic factor

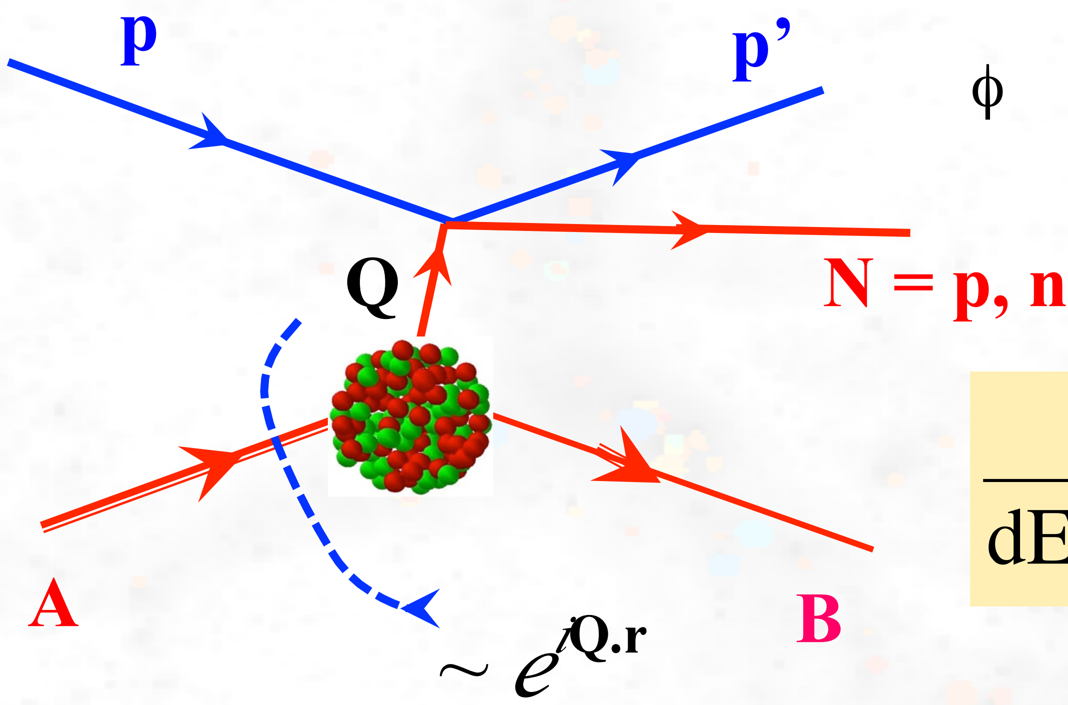
$$\sigma_{KO} = C^2 S \, K_F \, \sigma_{pp} \, |\phi(Q)|^2$$

½-shell pp cross section

Momentum distribution

Differential cross sections

Plane-Wave Impulse Approximation



ϕ = bound state w.f.

Kinematic factor

$$\frac{d\sigma}{dE_p d\Omega_p d\Omega_B} = K_F \frac{d\sigma_{pN}}{d\Omega_p} \left|\phi_N(\mathbf{Q})\right|^2$$

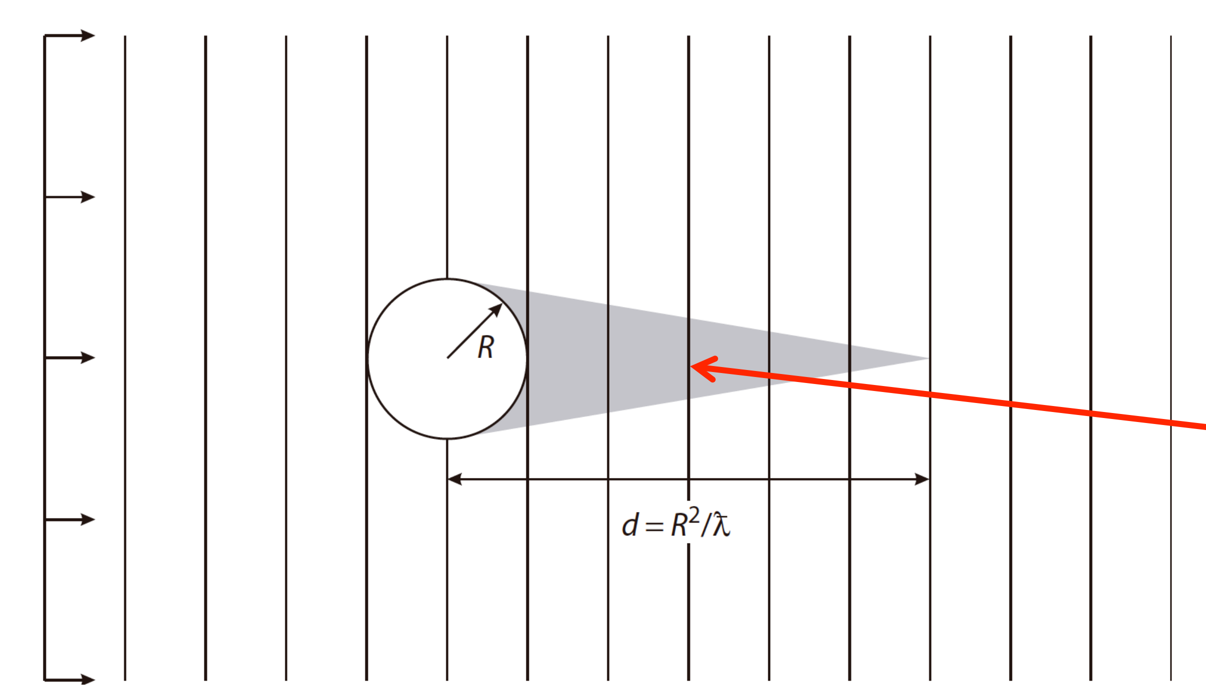
off-shell pN cross section

Momentum distribution of N within A

$$\frac{d\sigma_{pN}}{d\Omega_p} \sim \left(\frac{d\sigma_{pN}}{d\Omega_p}\right)_{free}$$

probably good if pp and pn cross sections slowly dependent on E_p

Estimates: p + ^{12}C



black disk theory →

$$\sigma_{\text{reaction}} \sim \pi R^2 \sim 250\ \text{mb}$$

+ optical theorem →

$$\sigma_{\text{elastic}} \sim \pi R^2 \sim 250\ \text{mb}$$

experiment (E_p > 200 MeV) →

$$\sigma_{\text{knockout}}(\text{p,2p}) \sim 30\ \text{mb}$$

but one expects:

$$\sigma_{\text{knockout}}(\text{p,2p}) \sim \frac{A}{2}\sigma_{\text{pp}}^{\text{free}} \sim 250\,\text{mb}$$

in fact, for nuclear excitation below threshold:

$$\sigma_{\text{inelastic}}(\text{p,p}') \sim 10 - 20\,\text{mb}$$

Conclusion: much of the knockout cross section goes to absorption

Absorption = whatever leads to other channels than (p,2p) or (p,pn)

➡ **one has to go beyond PWIA →** $d\sigma_{pN}/d\Omega_p$ **not really elastic**

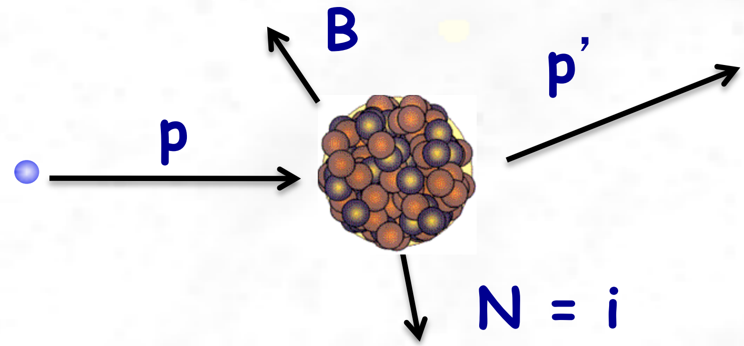
and/or $\phi_N(\mathbf{Q})$ **not really a simple Fourier transform of ϕ_N**

Distorted Wave Born Approximation

$$\frac{d\sigma}{dE_p d\Omega_p d\Omega_B} = K_F \sum_i \left| t_i(p,p',i) \right|^2$$

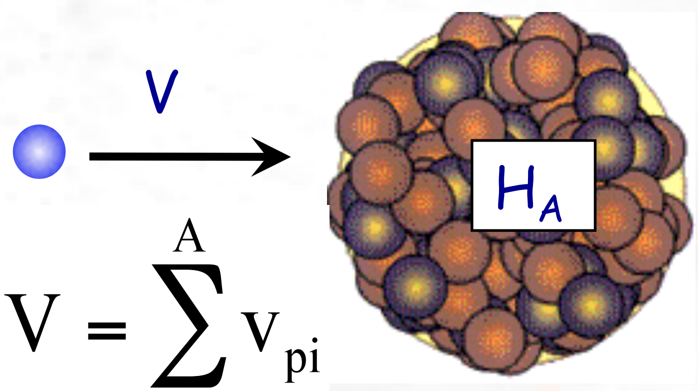
Kinematic factor
(phase-space factors)

sum over all p + i interactions



incoming and outgoing waves of p, p' and i are distorted by all p + i interactions

How to get t_i ?



$$V = \sum_{i=1}^{A} V_{pi}$$

Sum of binary interactions

$$H = -\frac{\hbar^2}{2m}\nabla^2 + V + H_A$$

intrinsic

T-matrix

$$T = V + V\left[E - (H - V) + i\varepsilon\right]^{-1} T$$

"Ab initio" p-A Scattering Theory

Factorization

$$T = \sum_i \tau_{pi}(E)\,\eta_i(E)$$

$$\tau_{pi} = v_{pi} + v_{pi}\left[E - (H - V) + i\varepsilon\right]^{-1}\tau_{pi}$$

τ_{pi} is an (A+1)-body operator

$$\eta_i(E) = 1 + \left[E - (H - V) + i\varepsilon\right]^{-1}\sum_{j\neq i}\tau_{pj}(E)\,\eta_j(E)$$

Too complicated!

Need to simplify

$$T = \sum_i \tau_{pi}(E) + \sum_{i,\, j\neq i}\tau_{pi}(E)\left[E - (H - V) + i\varepsilon\right]^{-1}\tau_{pj}(E) + \cdots$$

Effective theory expansion for T

Effective Theory for p-A Scattering

1) Project onto ground state $\quad P_0 = |\Psi_0\rangle\langle\Psi_0|$

2) Introduce p-A **Optical Potential**

$$T = U + U\left\{|\Psi_0\rangle\langle\Psi_0|\left[E + \hbar^2\nabla^2/2m - K_A + i\varepsilon\right]\right\}^{-1} T$$

K_A = p-A CM kinetic energy

3) τ_{pi} (A+1)-body → t_{pi} free NN t-matrix

4) Effective theory expansion for p - A scattering

$$U = \sum_{i}^{A} t_{pi}(E) + \sum_{i\neq j}^{A} t_{pi}(E) \frac{1 - |\Psi_0\rangle\langle\Psi_0|}{E + \hbar^2\nabla^2/2m - K_A + i\varepsilon} t_{pj}(E) + \cdots$$

$$= U^{(LO)} + U^{(NLO)} + \cdots$$

$$U = U^{(LO)} + U^{(NLO)} + \cdots$$

$U^{(LO)}$ = sum of binary (free) interactions

$U^{(NLO)}$ = sum of sequential binary interactions with account of **(virtual) intermediate states**

$U^{(NNLO)}$ = ...

$U^{(....LO)}$ = ...

Optical Potential

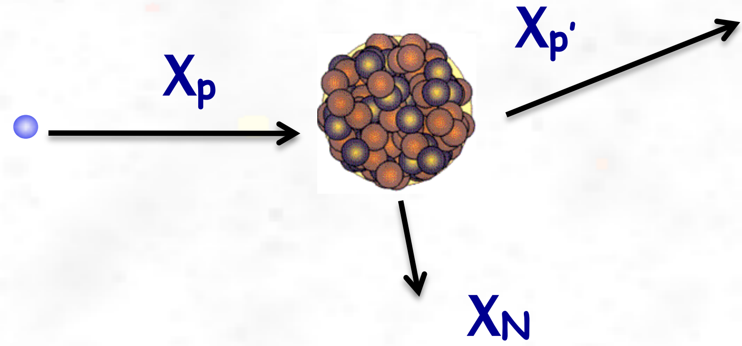
Nobody does this, really.

$$U(r) = f(a, b, ..., r) + ig(a', b', ..., r)$$

Woods-Saxon, etc.

DWIA

$$\frac{d\sigma}{dE_p\,d\Omega_{p'}\,d\Omega_N} = (\text{phase space}) \left| \left\langle \chi_p^- \chi_N^- \middle| U \middle| \overbrace{\chi_{p'}^+ \varphi_N}^{\chi_{N+B}} \right\rangle \right|^2$$



χ = distorted waves

ϕ = bound state w.f., includes spectroscopic amplitudes

First order, one encounter (impulse) for p - N, $U \sim U^{(LO)} = t_{pN}(E_p)$

$$\frac{d\sigma}{dE_p\,d\Omega_{p'}\,d\Omega_N} = (\text{phase space}) \left| \left\langle \chi_p^- \chi_N^- \middle| t_{pN}(E_p) \middle| \chi_{p'}^+ \varphi_N \right\rangle \right|^2$$

DWIA

Off-shell effects neglected

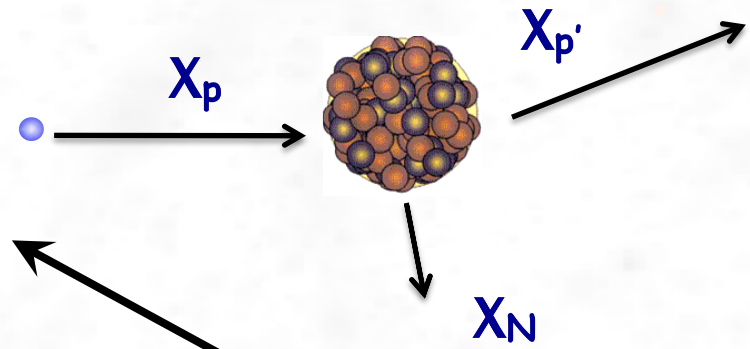
$$\frac{d\sigma}{d\Omega_{pN'}} = (\text{phase space}) \left| t_{pN}(E_p) \right|^2 \sum_{\Lambda} \left| \mathcal{T}^{(p,pN)} \right|^2$$

Quasi-free

$$\boxed{\frac{d\sigma}{dE_p\,d\Omega_{p'}\,d\Omega_N} = K_F \frac{d\sigma_{pN}}{d\Omega} \sum_{\Lambda} \left| \mathcal{T}^{(p,pN)} \right|^2}$$

DWIA

$$\frac{d\sigma}{dE_p\, d\Omega_{p'}\, d\Omega_N} = K_F \frac{d\sigma_{pN}}{d\Omega} \sum_{\Lambda} \left| \mathcal{T}^{(p,pN)} \right|^2$$



χ = distorted waves

ϕ = bound state w.f., includes spectroscopic amplitudes

DWIA

$$\mathcal{T}^{(p,pN)} = \int d^3r\, \chi_p^{(-)*}\left(\frac{A-1}{A}\mathbf{r}\right) \chi_{p'}^{(-)*}(\mathbf{r})\, \overbrace{\chi_N^{(+)*}(\mathbf{r})\varphi(\mathbf{r})}^{\chi_{N+B}}$$

if all χ = plane waves

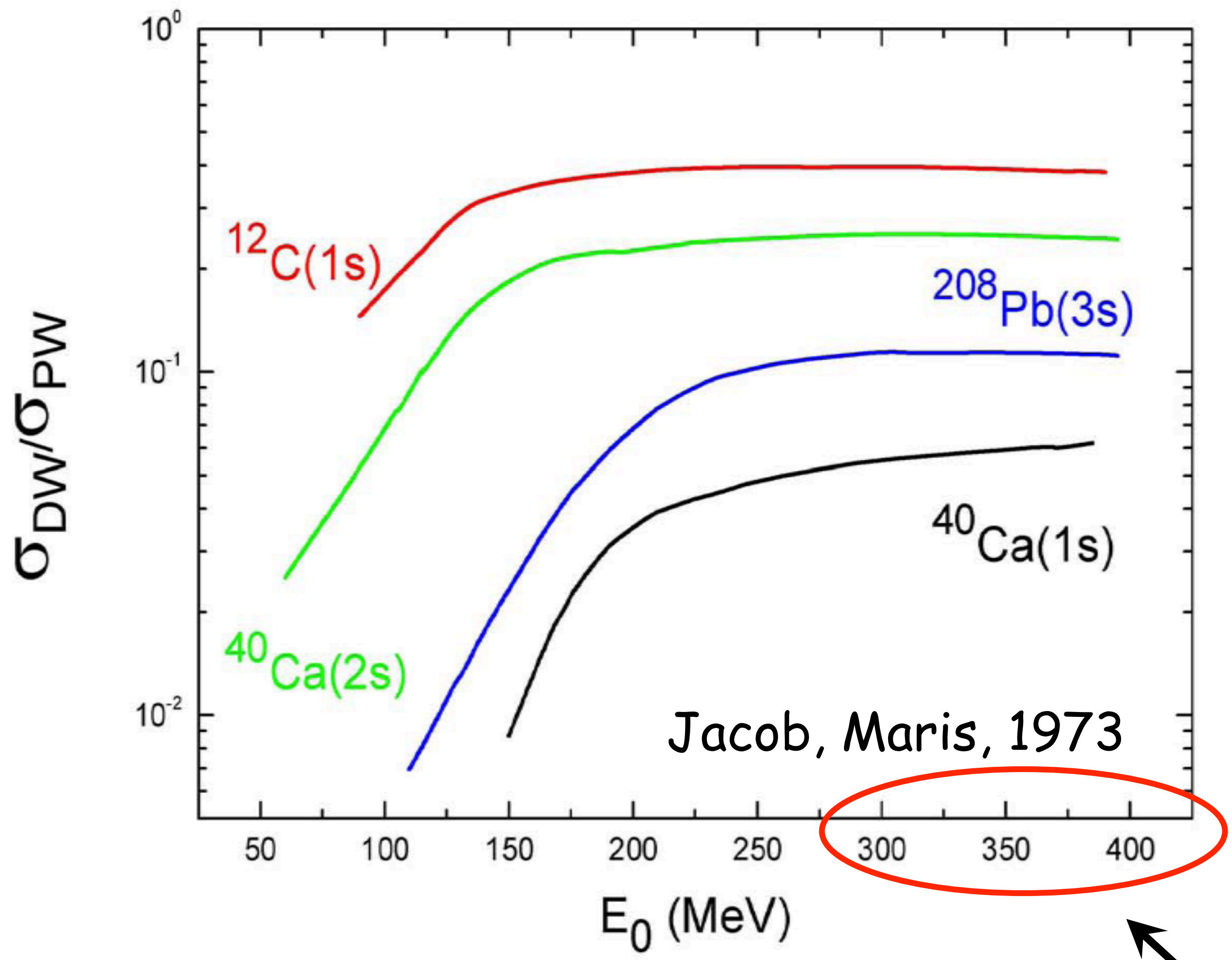
$$\frac{d\sigma}{dE_p\, d\Omega_p\, d\Omega_B} \sim \frac{d\sigma_{pN}}{d\Omega_p} \left| \phi_N(\mathbf{Q}) \right|^2$$

PWIA

$$\mathbf{Q} = \mathbf{k}_{p_1} + \mathbf{k}_{p_2} + \frac{A-1}{A}\mathbf{k}_p$$

DWIA x PWIA

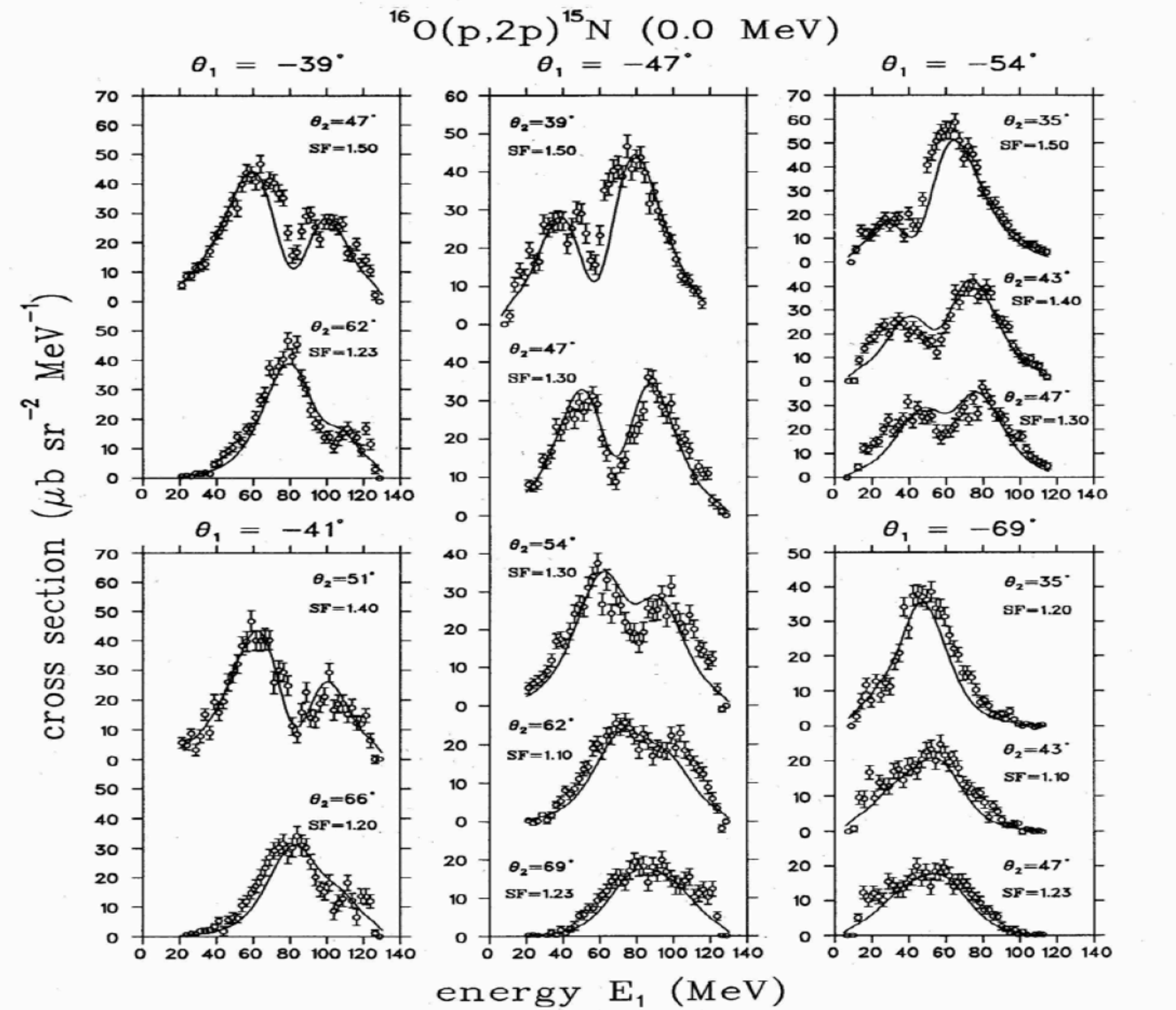
To calculate χ we need U_{pA} , $U_{p'A}$ and U_{NA}



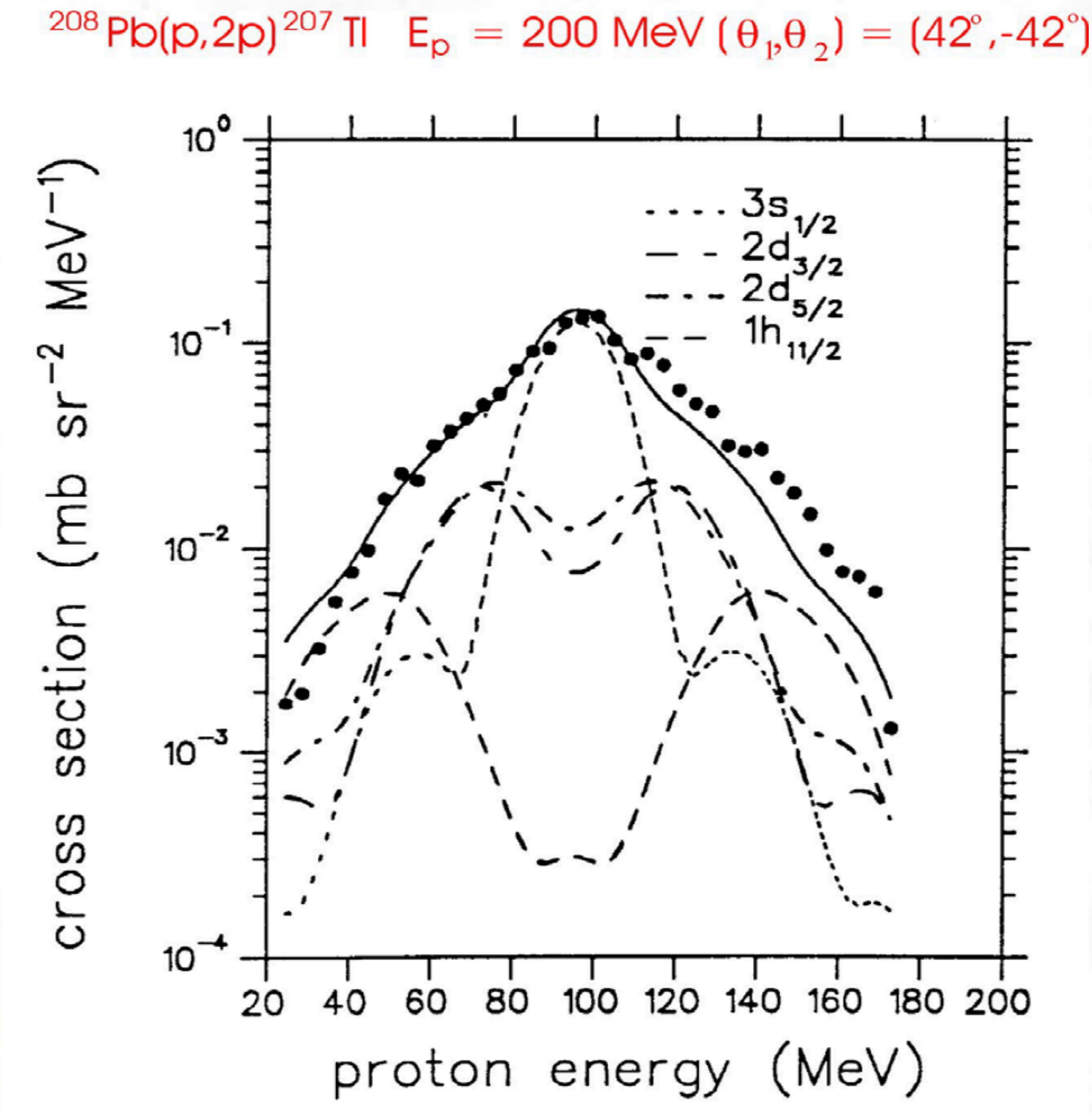
Distortions important

recent energy with inverse kinematics with radioactive beams

Lessons from the past: DWIA works well

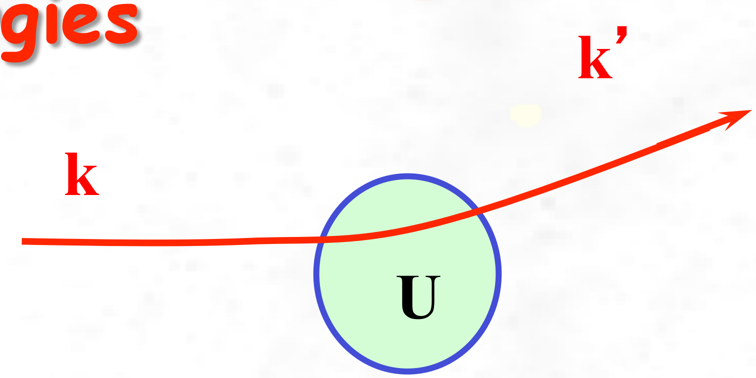


Lessons from the past: DWIA works well



Distorted Waves at Low Energies

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + U\right]\Psi = E\Psi$$



$$\Psi = \sum_{lm} \frac{u_l(r)}{r} Y_{lm}(\hat{\mathbf{r}})$$ (partial waves)

Outgoing wave

$$u_l(r) \xrightarrow[r\to\infty]{} \frac{i}{2}\left\{H_l^{(-)}(kr) - S_l H_l^{(+)}(kr)\right\}$$

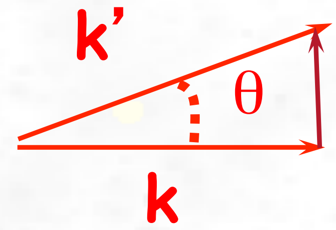
Incoming wave

"Survival" amplitude

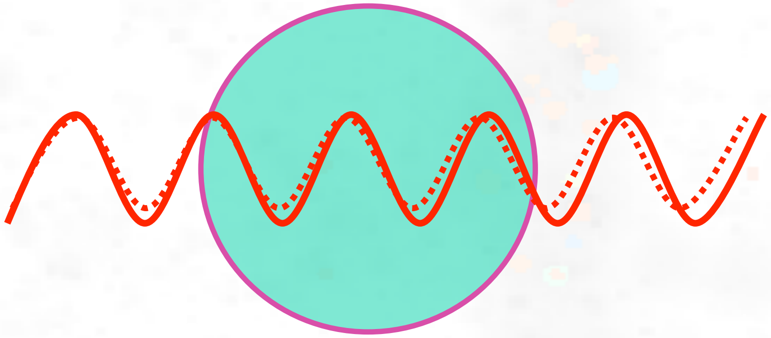
$$S_l = e^{2i\delta_l}$$ (δ_l = Phase shift)

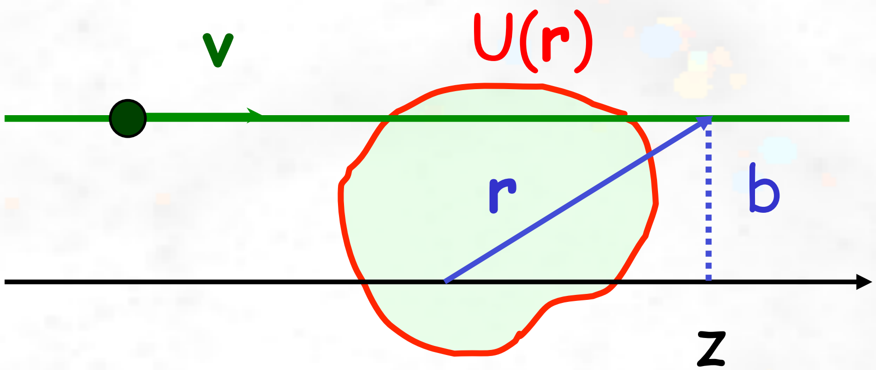
$|S_l|^2$ = "Survival" probability ≤ 1

Distorted Waves at High Energies



$\Delta E \ll E$, $\theta \ll 1$ radian , $|\Delta\psi/\psi|_{\Delta r=\lambda} \ll 1$





Assume $\Psi(\mathbf{r}) = S(\mathbf{b},z)\, e^{i\mathbf{k}\cdot\mathbf{r}}$

Insert in S.E. and neglect terms

$\partial_b^2\Psi$ compared to $\partial_b\Psi$

$\partial_z^2\Psi$ compared to $\partial_z\Psi$

eikonal S-matrix

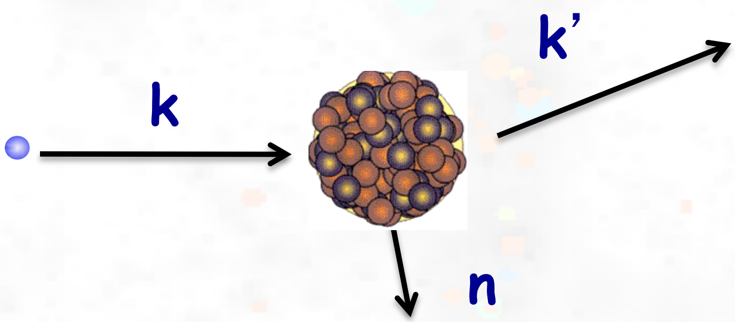
$$S(\mathbf{b},z) = \exp\left\{-\frac{i}{\hbar v}\int_{-\infty}^{z} U(\mathbf{r}')\,dz'\right\}$$

$\mathbf{r}' = (\mathbf{b},z')$

$$\Psi(\mathbf{r}) = S(\mathbf{b})\, e^{i\mathbf{k}\cdot\mathbf{r}} \qquad S(\mathbf{b}) = e^{i\chi(\mathbf{b})} = \exp\left\{-\frac{i}{\hbar v}\int_{-\infty}^{\infty} U(\mathbf{r}')\,dz'\right\}$$

$z \to \infty$ (after the collision)

Optical Potential at High Energies



$$U(E) = \langle \mathbf{k'}, \mathbf{n} | U | \mathbf{k}, \rangle$$
$$= (2\pi)^3 \delta(\mathbf{k'} + \mathbf{n} - \mathbf{k}) U(\mathbf{k}, \mathbf{k'}, E)$$

target nucleus density matrix

$$U^{(LO)}(\mathbf{k}', \mathbf{k}, E) = \int \frac{d^3\mathbf{p}_1}{(2\pi)^3} \rho(\mathbf{p}_1 + \mathbf{Q}; \mathbf{p}_1)\, t(\mathbf{k}_1, \mathbf{k}_1', E')$$

One-body density:

$$\rho(\mathbf{Q}) = \int \frac{d^3\mathbf{p}_1}{(2\pi)^3} \rho(\mathbf{p}_1 + \mathbf{Q}; \mathbf{p}_1)$$

Neglecting off-shell effects (**Q ~ 0**)

$$\Rightarrow \quad \boxed{U^{(1)}(\mathbf{k}', \mathbf{k}, E) = \rho(\mathbf{Q})\, t(\mathbf{k}', \mathbf{k}, E)}$$

(t-ρ approximation)

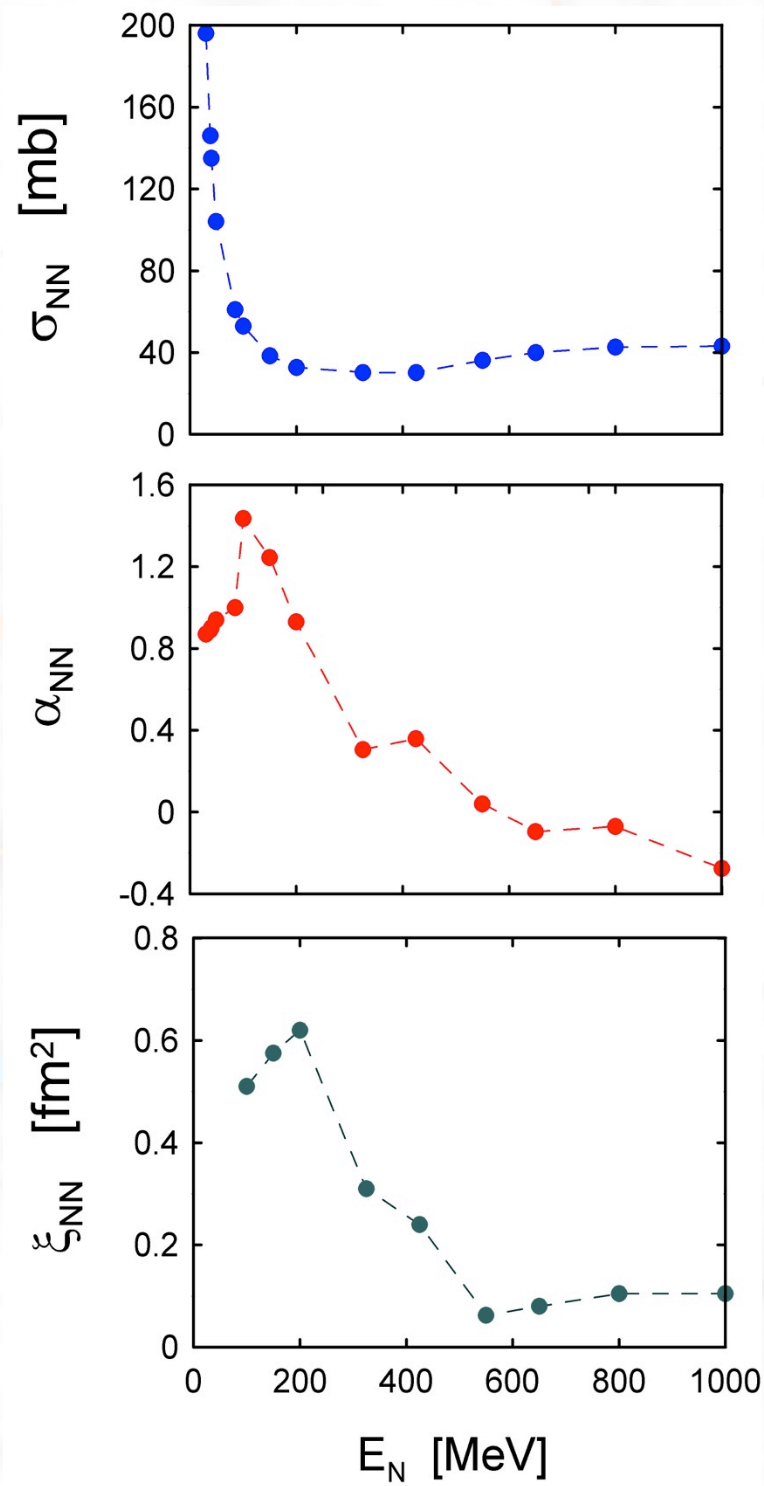
t-ρ Approximation

$$U^{(LO)}(E;r) = \int t_{NN}^{\text{forward}}(E;\mathbf{Q} = \mathbf{k'}-\mathbf{k})\,\rho_T(\mathbf{Q})\,d^3Q$$

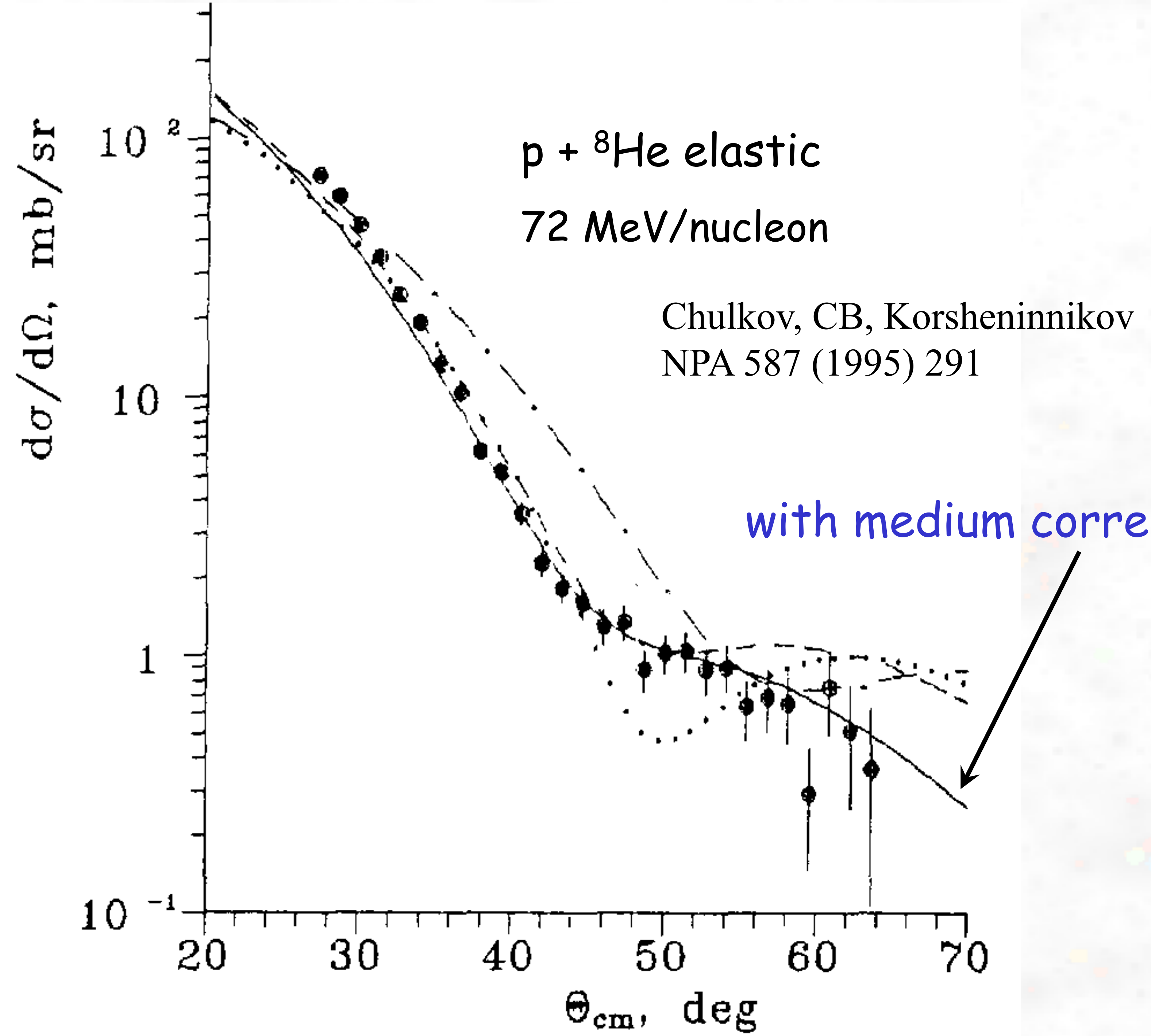
$$t_{NN}^{\text{forward}}(E;Q) = -\frac{\hbar^2}{4\pi^2\mu} f_{NN}(Q \sim 0)$$

$$f_{NN}(Q \sim 0) = \frac{k_{NN}}{4\pi}\sigma_{NN}\left(i + \alpha_{NN}\right)e^{-\xi_{NN} Q^2}$$

needs medium correction

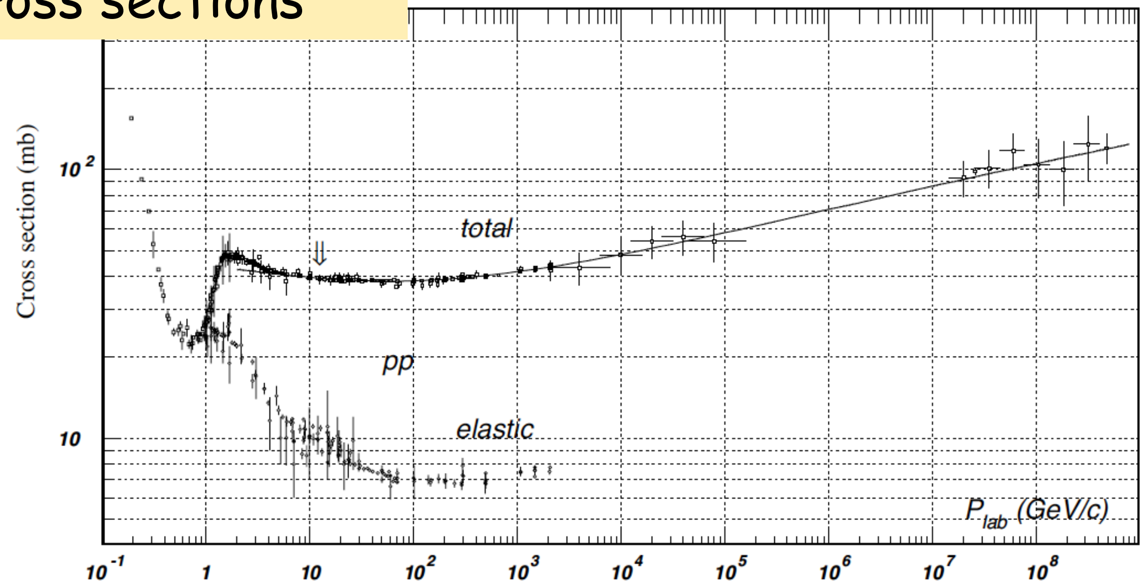


t-ρ Approximation - Example

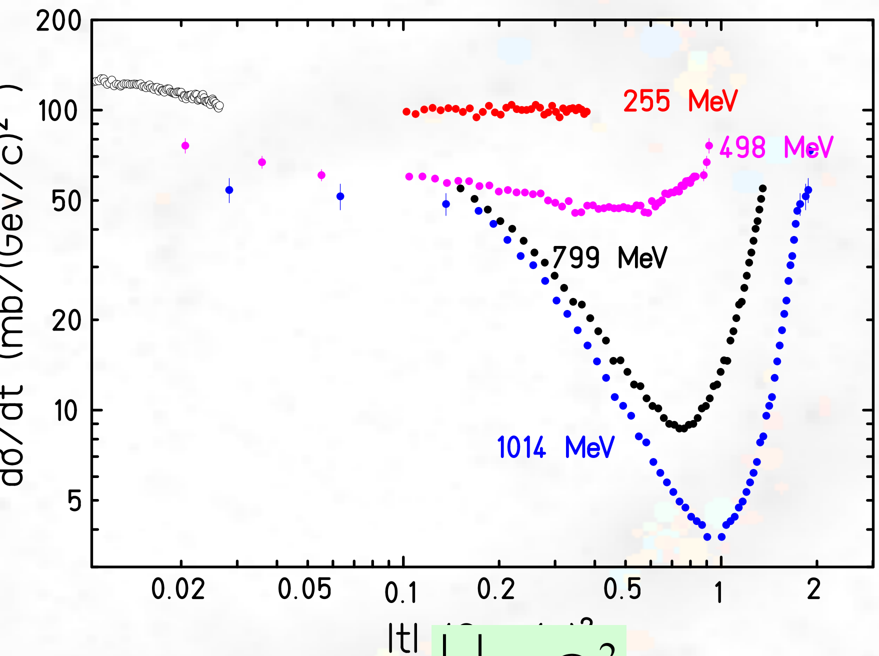


NN cross sections

Total x elastic pp cross sections

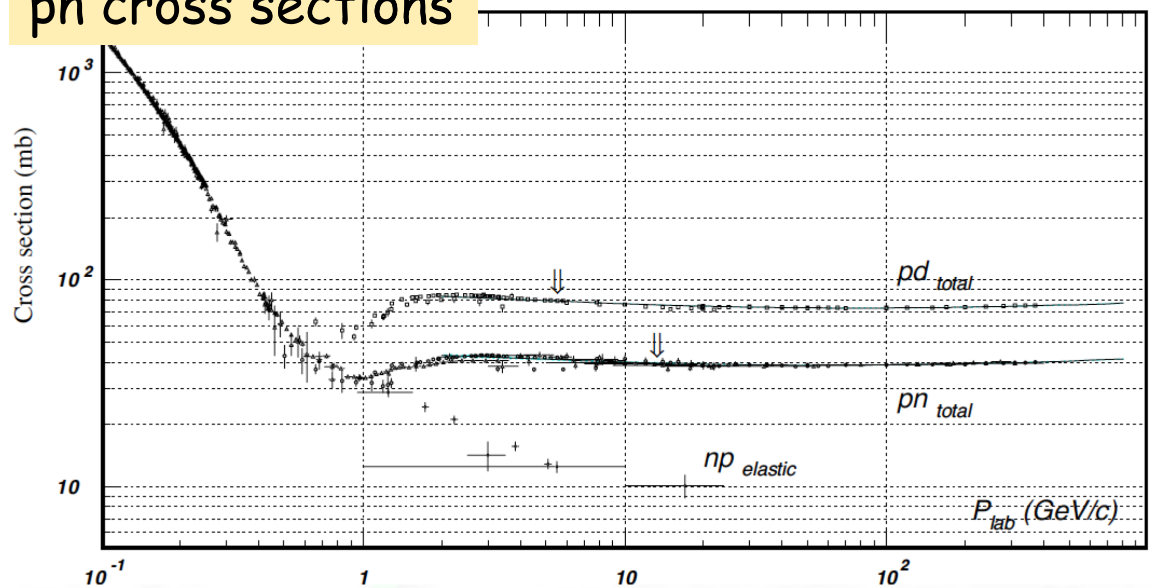


Differential pp cross section



$$|t| = Q^2$$

Total x elastic pn cross sections



Medium corrections of σ_{NN}

Lippmann-Schwinger + Pauli ⇨ Bethe-Goldstone

$$\langle \mathbf{k}|G|\mathbf{k}_0\rangle = \langle \mathbf{k}|v_{NN}|\mathbf{k}_0\rangle - \int \frac{d^3k'}{(2\pi)^3} \frac{\langle \mathbf{k}|v_{NN}|\mathbf{k'}\rangle Q(\mathbf{k'})\langle \mathbf{k'}|G|\mathbf{k}_0\rangle}{E(\mathbf{k'}) - E_0 - i\varepsilon}$$

$$E(\mathbf{P},\mathbf{k}) = e(\mathbf{P}+\mathbf{k}) + e(\mathbf{P}-\mathbf{k})$$

e = single-particle energies

E_0 = E on-shell

$$Q(\mathbf{P},\mathbf{k}) = 1, \quad \text{if} \quad k_{1,2} > k_F \qquad \mathbf{k}_{1,2} = \mathbf{P} \pm \mathbf{k}$$
$$= 0, \quad \text{otherwise}$$

(Brueckner theory)

Solve self-consistently for v_{NN} (or σ_{NN})

$$v(p) = \langle p|v|p\rangle = \mathrm{Re} \sum_{q \le k_F} \langle pq|G|pq - qp\rangle$$

- e depends on v
- v depends on G
- G depends on v

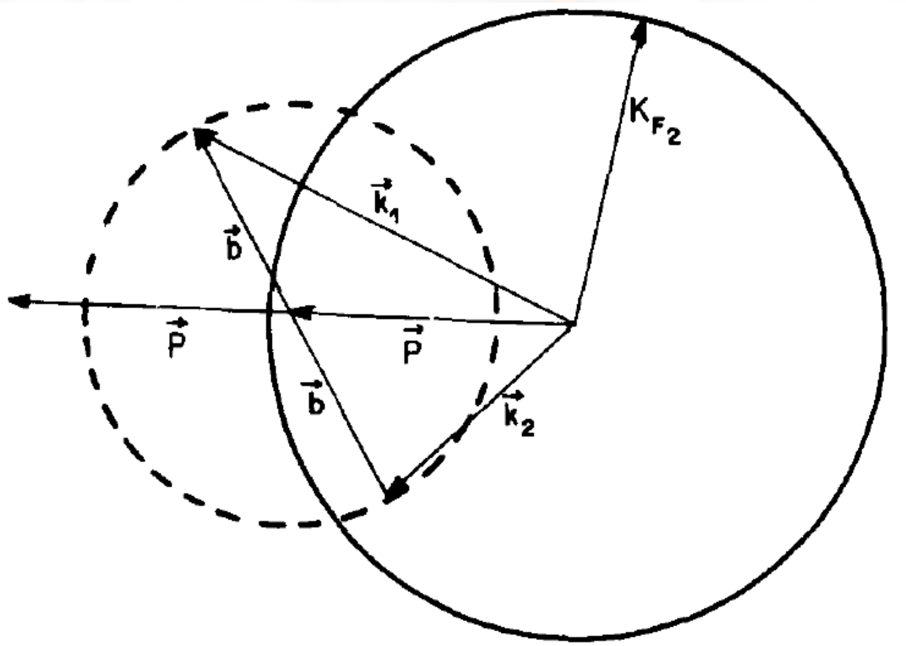
⇨ Complicated and often needs simplifications

Simplified Medium Corrections of σ_{NN}

Local Density Approximation

$$K_F^2 = \left[\frac{3\pi^2}{4}\rho(r)\right]^{2/3} + \frac{5}{2}\xi\left(\frac{\nabla\rho}{\rho}\right)^2$$

surface correction



Pauli Blocking:

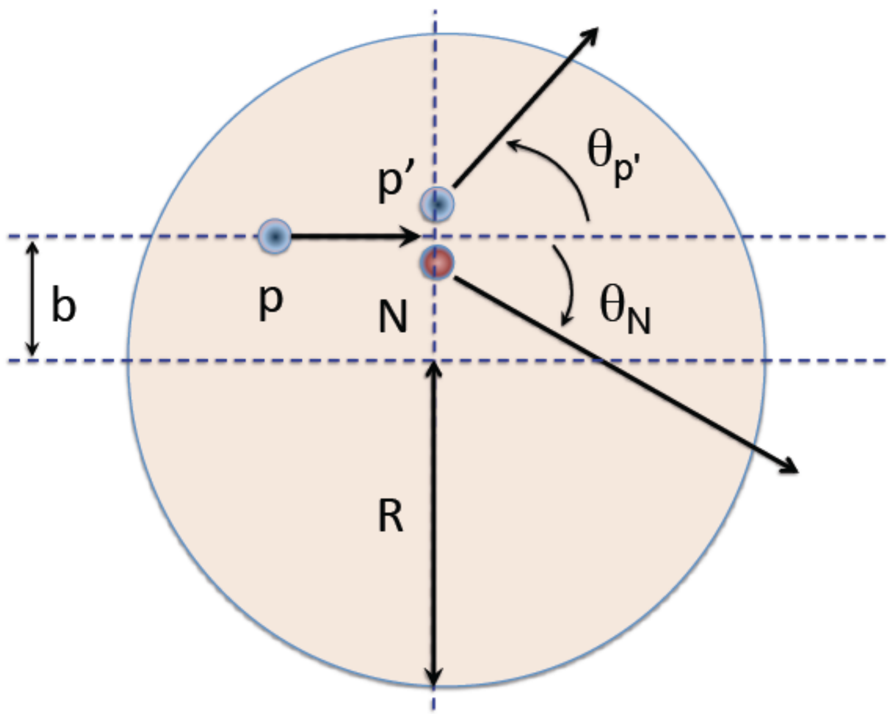
scattering phase space limited

$$\sigma_{NN}(E,\rho) = \sigma_{NN}^{(free)}(E)P(E,\rho)$$

$$P(E,\rho) = P(x), \qquad x = \frac{E_F}{E}$$

$$P(x) = \begin{cases} 1 - 7x/5, & x \le 1/2 \\ 1 - 7x/5 + (2x/5)(2 - 1/x)^{5/2}, & x \ge 1/2 \end{cases}$$

Summary: waves + potentials for (p,pN)



$$\chi_p^{(in)}(\mathbf{r}) = S_p(b_p,z)\exp\left[i\frac{A-1}{A}\mathbf{k}_p\cdot\mathbf{r}\right]$$

$$S_p^{in}(b_p,z) = -\frac{i}{\hbar v}\int_{-\infty}^{z} U(r')dz'$$

$$\chi_{p'}^{(out)}(\mathbf{r}) = S_{p'}(b_{p'},z)\exp\left[i\mathbf{k}_{p'}\cdot\mathbf{r}\right]$$

$$S_{p'}^{(out)}(b_{p'},z) = -\frac{i}{\hbar v}\int_{z}^{\infty} U(r')dz'$$

and similarly for χ_N

$$U = U_{Real} + iU_{Imag}$$

⟵ Constructed from t-rho (absorption)

$$U_{Imag} = -\frac{\hbar v\overline{\sigma}_{NN}}{8\pi}\int dq\, q\, \rho_A(q)\rho_p(q)J_0(qb)$$

Constructed from M3Y folding model

$$\rho_p(q) = \frac{1}{1+q^2/a^2}, \qquad a^2 = 0.71\,\mathrm{fm}^{-2}$$

Momentum distributions of recoiled residual nuclei B

Instead of $\frac{d\sigma}{dE_p d\Omega_{p'} d\Omega_N} = K_F \frac{d\sigma_{pN}}{d\Omega} \sum_\Lambda \left|\mathcal{T}^{(p,pN)}\right|^2$ one can get a much simpler expression for $d\sigma / d^3Q$ by using the eikonal formalism described above:

$$\mathcal{T}^{(p,pN)} = \int d^3r \, \chi_p^{(-)*}\left(\frac{A-1}{A}\mathbf{r}\right) \chi_{p'}^{(-)*}(\mathbf{r}) \, \chi_N^{(+)*}(\mathbf{r}) \varphi(\mathbf{r}) \qquad \mathbf{Q} = \mathbf{k}_p^{'} + \mathbf{k}_N - \frac{A-1}{A}\mathbf{k}_p$$

$$\Rightarrow \quad \frac{d\sigma}{d^3Q} = \frac{1}{(2\pi)^2}\frac{1}{2j+1}\sum_m \left\langle \frac{d\sigma_{pN}}{d\Omega} \right\rangle_Q \left| \int d^3r \, e^{-i\mathbf{Q}.\mathbf{r}} \langle S(b,\theta)\rangle_Q \varphi_{jlm}(\mathbf{r}) \right|^2$$

$$S(b,\theta) = S_{pA}(E_p;b) S_{p'B}(E_{p'};\theta_{p'};b) S_{NB}(E_N;\theta_N;b)$$

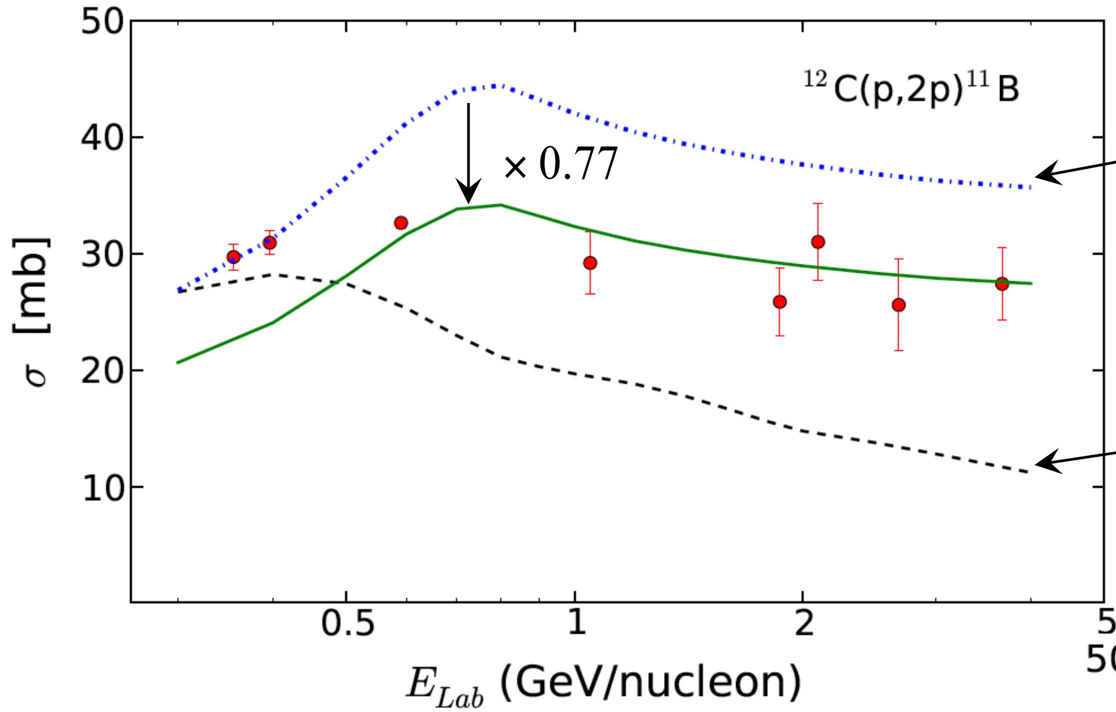
Averages need to be done, as there are several energies and scattering angle combinations leading to the same Q

$$\left\langle \frac{d\sigma_{pN}}{d\Omega} \right\rangle_Q = \frac{\int_{Q=\mathrm{const}} d^3p' d^3p_N \frac{d\sigma}{d\Omega_{pN}}}{\int_{Q=\mathrm{const}} d^3p' d^3p_N}$$

and similarly for $\langle S(b,\theta)\rangle_Q$

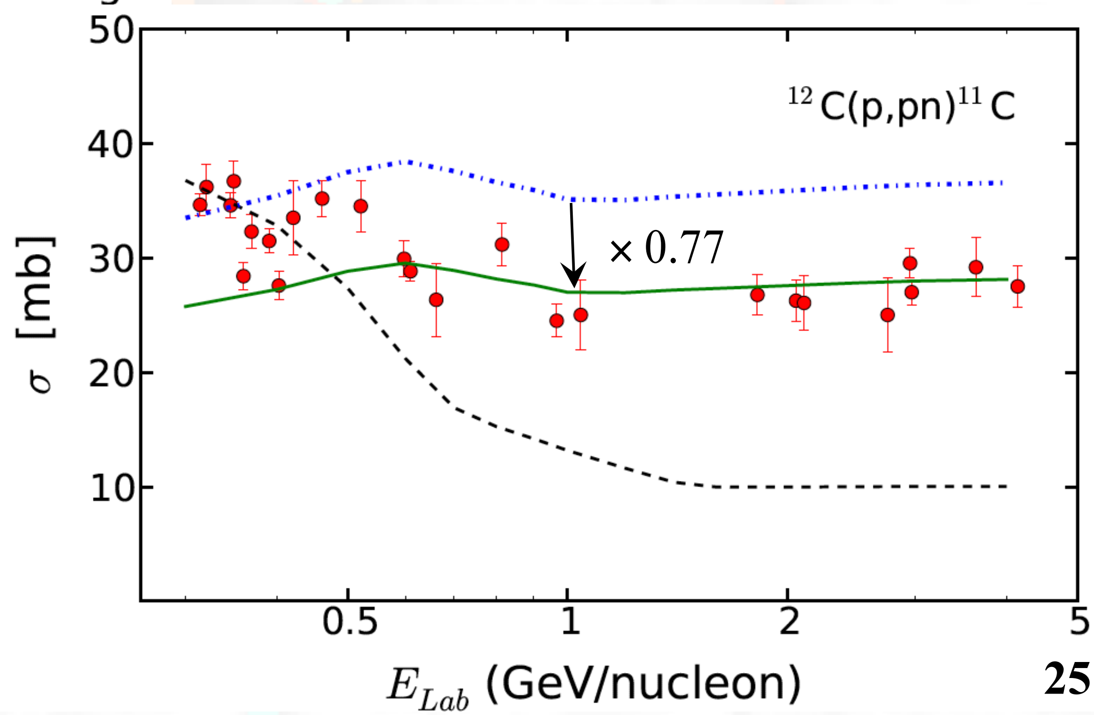
Applications: $^{12}C(p,pN)$

Consider removal from p-states in ^{12}C with separation energy of 15.9 (18.7) MeV for proton (neutron).

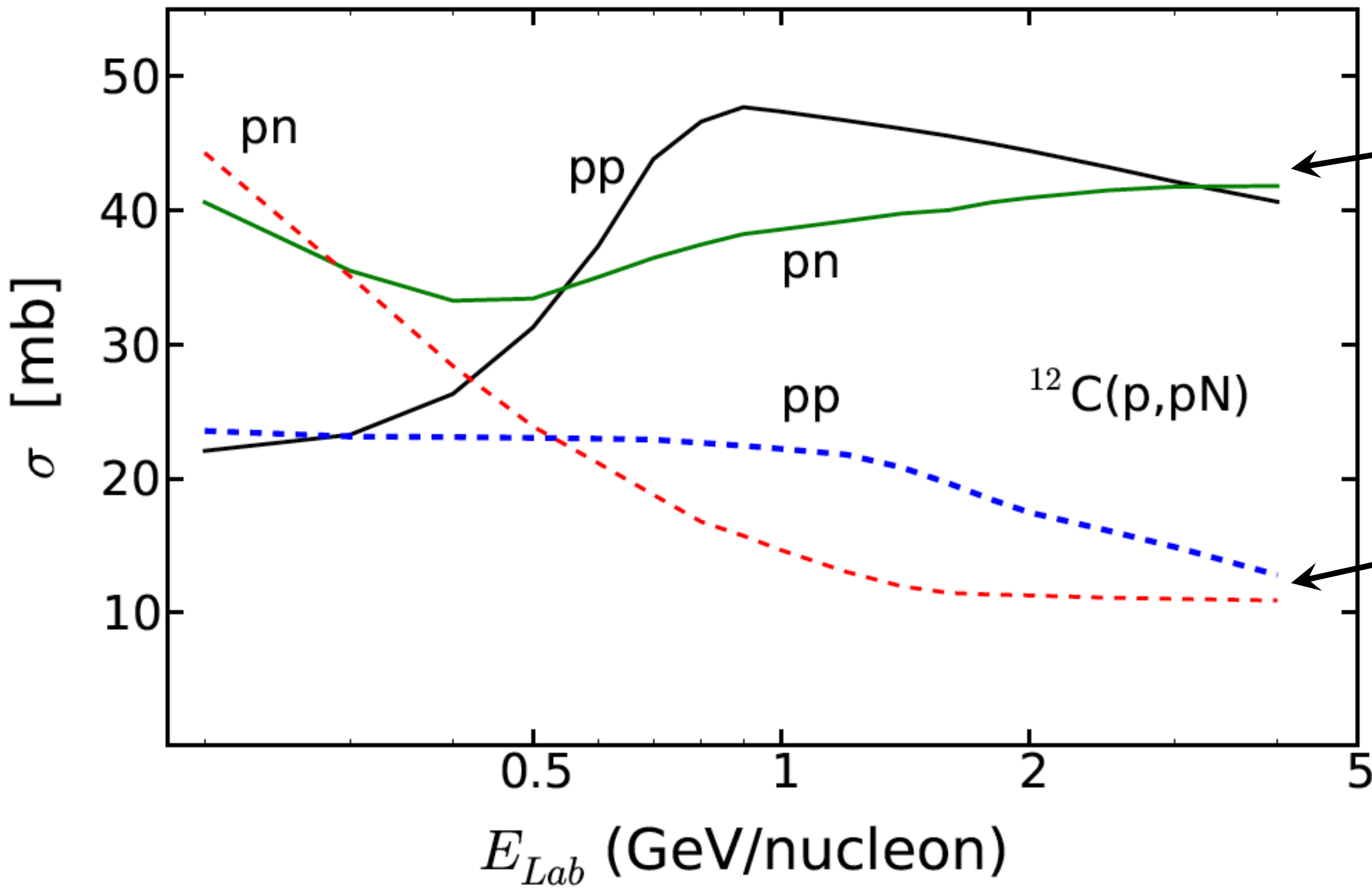


$$\left\langle \frac{d\sigma^{total}_{pN}}{d\Omega} \right\rangle_{o.s.} = \frac{\sigma^{total}_{pN}}{4\pi}$$ assuming isotropic

$$\left\langle \frac{d\sigma^{elast}_{pN}}{d\Omega} \right\rangle_{o.s.}$$ and accounting for anisotropy



Applications: $^{12}C(p,pN)$



free σ^{total}_{pN}

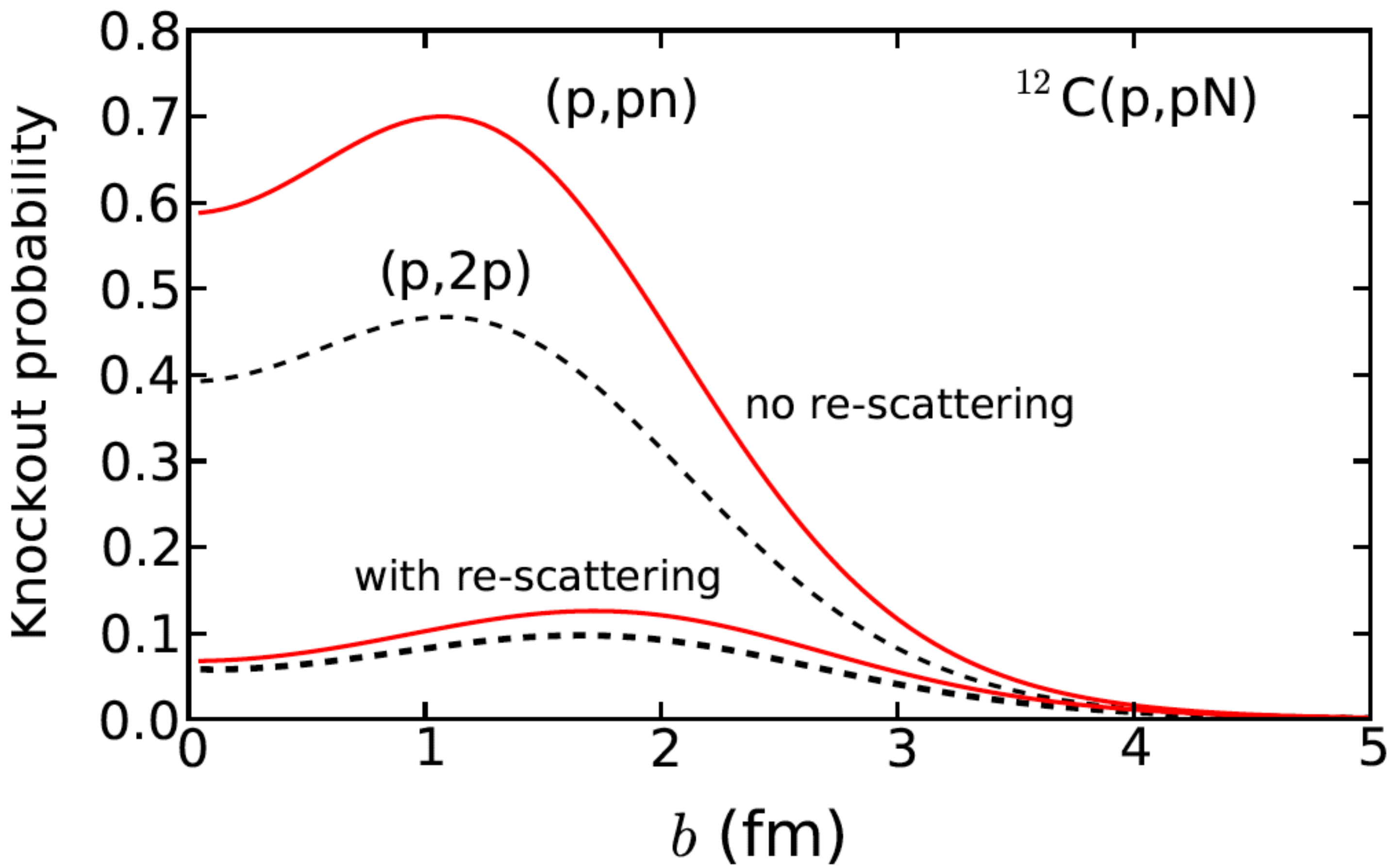
p', N, energies, angles, averaged

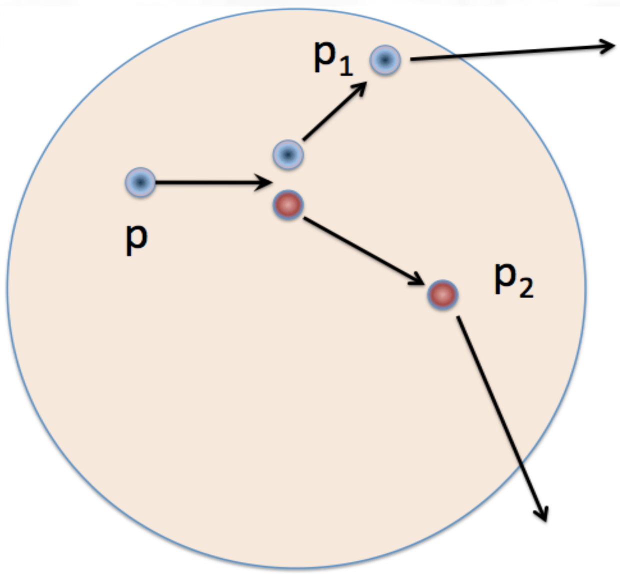
$$\left\langle \sigma^{elastic}_{pN} \right\rangle$$

Knockout Probabilities

$$P(b) = \frac{1}{2j+1} \sum_m \left\langle \frac{d\sigma_{pN}}{d\Omega} \right\rangle_{o.s.} \left|C_{lm}\right|^2 \left|\langle S(b,\theta)\rangle_{o.s.}\right|^2 \int_{-\infty}^{\infty} dz \left| \frac{u_{lj}(r)}{r} P_{lm}(b,z) \right|^2$$

includes re-scattering



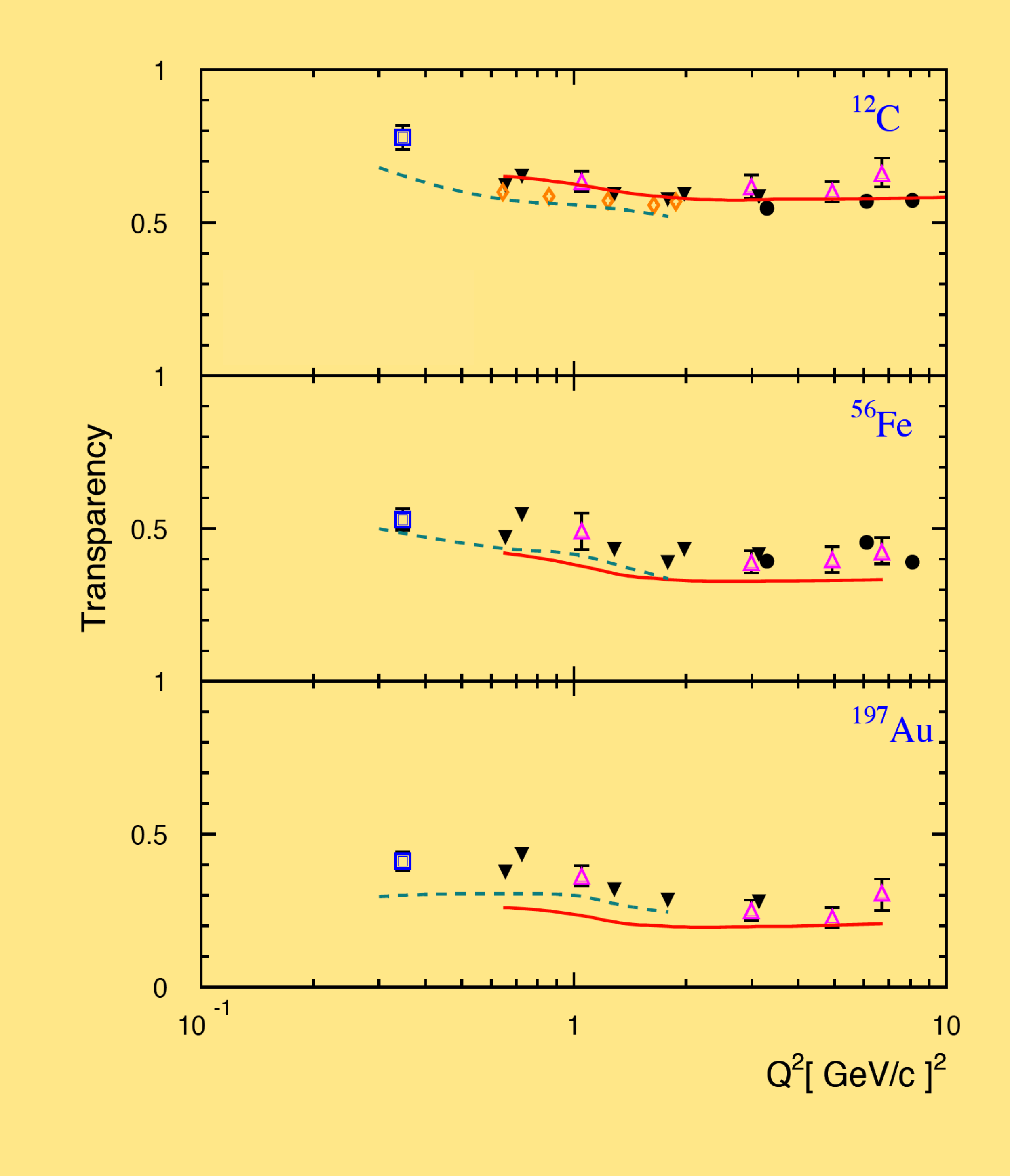


S = 1
no re-scattering

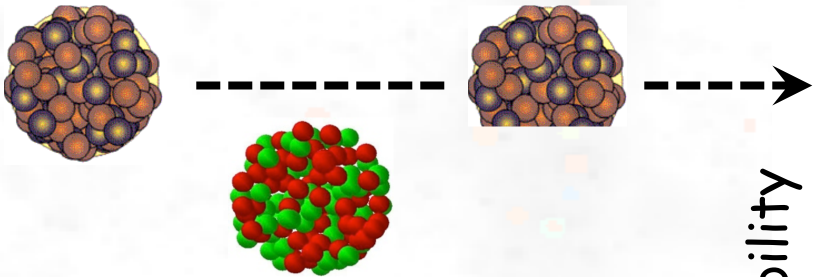
Comparison with Absortpion in (e,e'p)

Ryckebusch

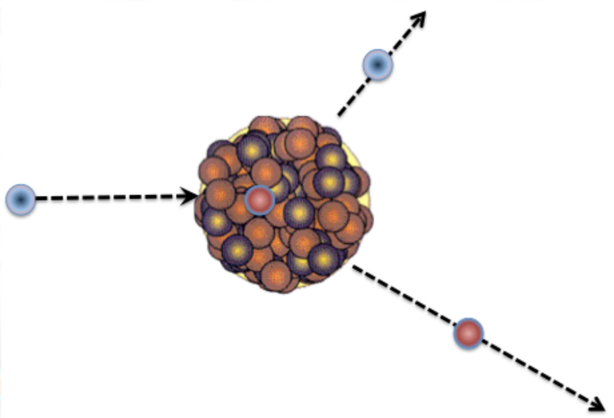
A(e,e'p)
$100 \leq E_p \leq 4$ GeV



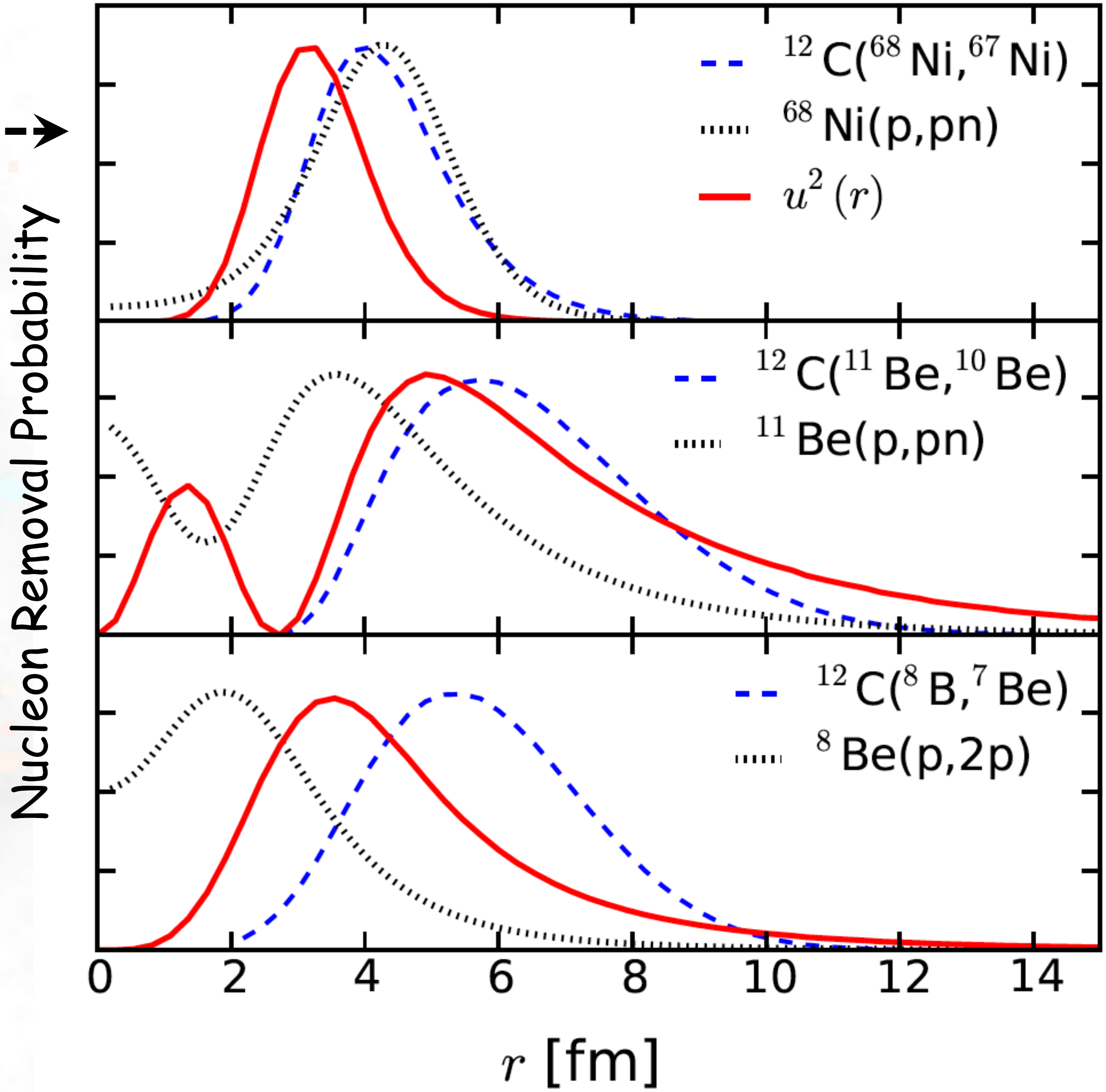
Comparison with heavy ion knockout reactions



knockout



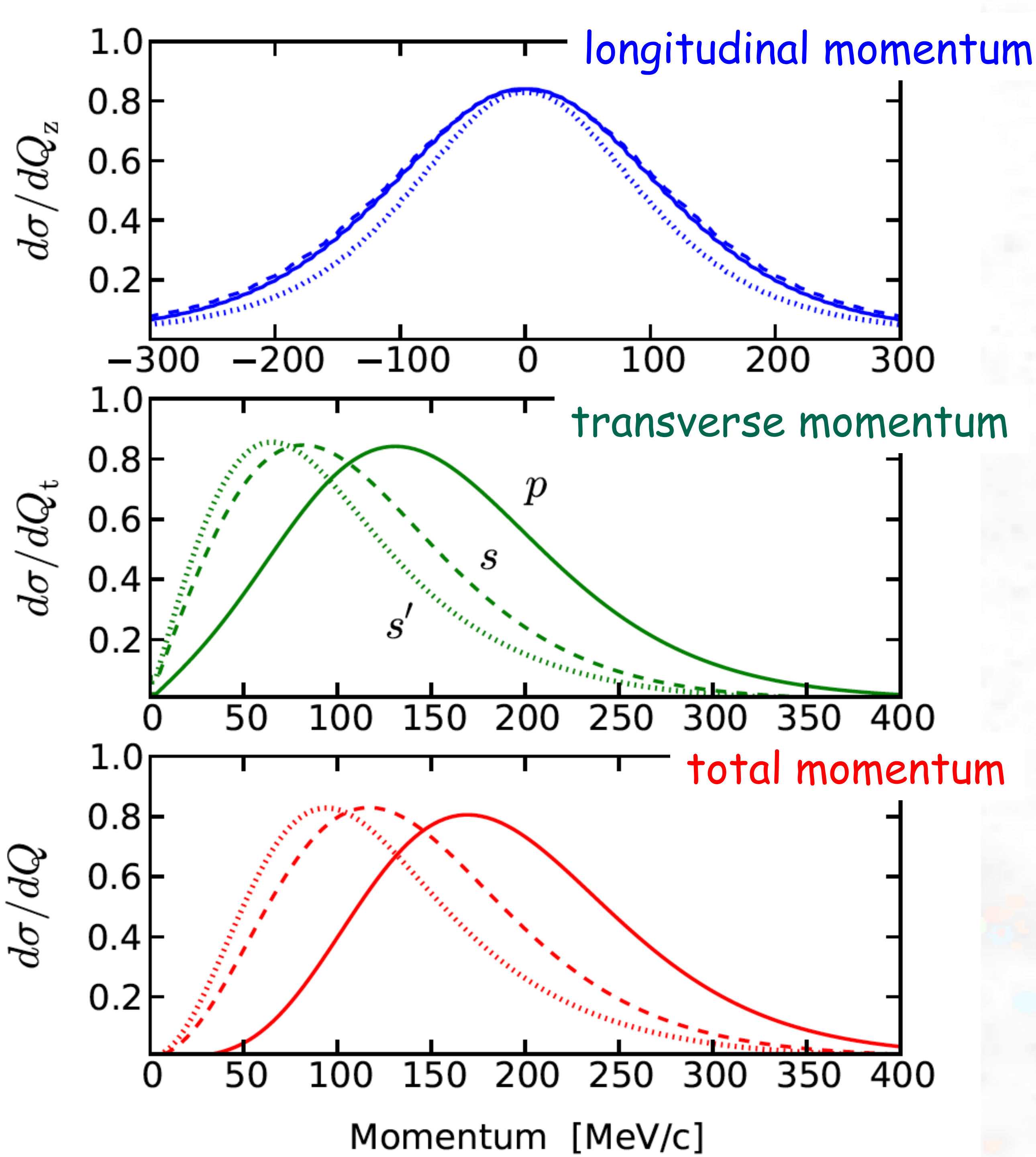
p,2p



Dependence on Angular Momentum and Energy

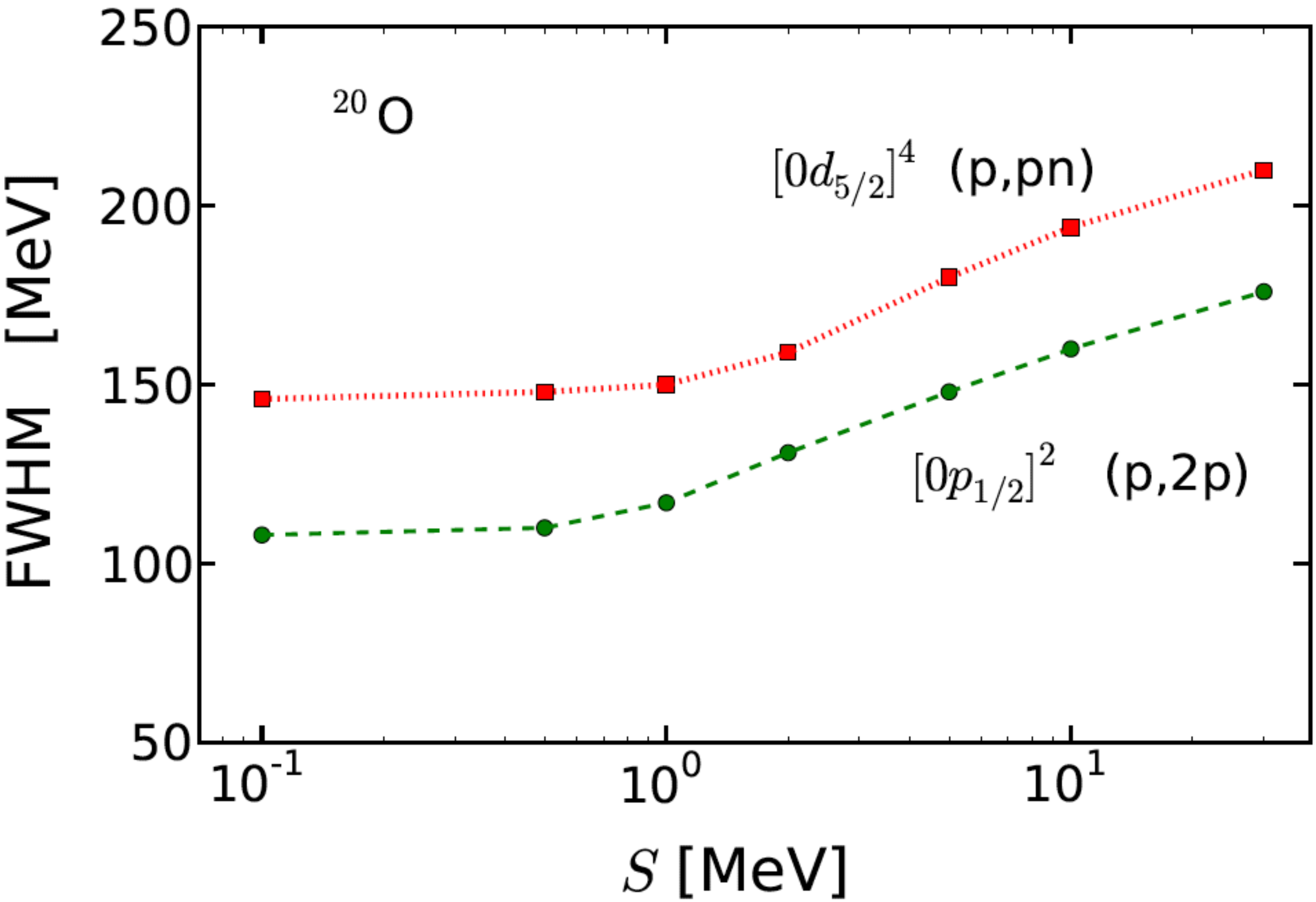
Energy of state s′ shifted to the same as p-state

$^{12}C(p,2p)$

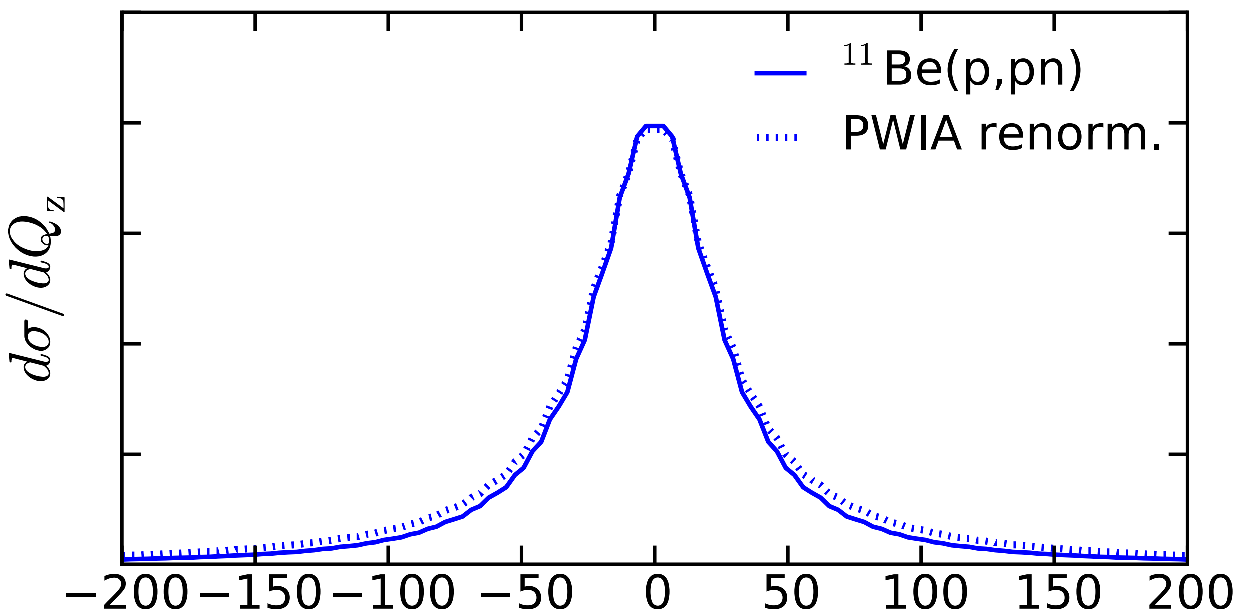


Dependence on Separation Energies

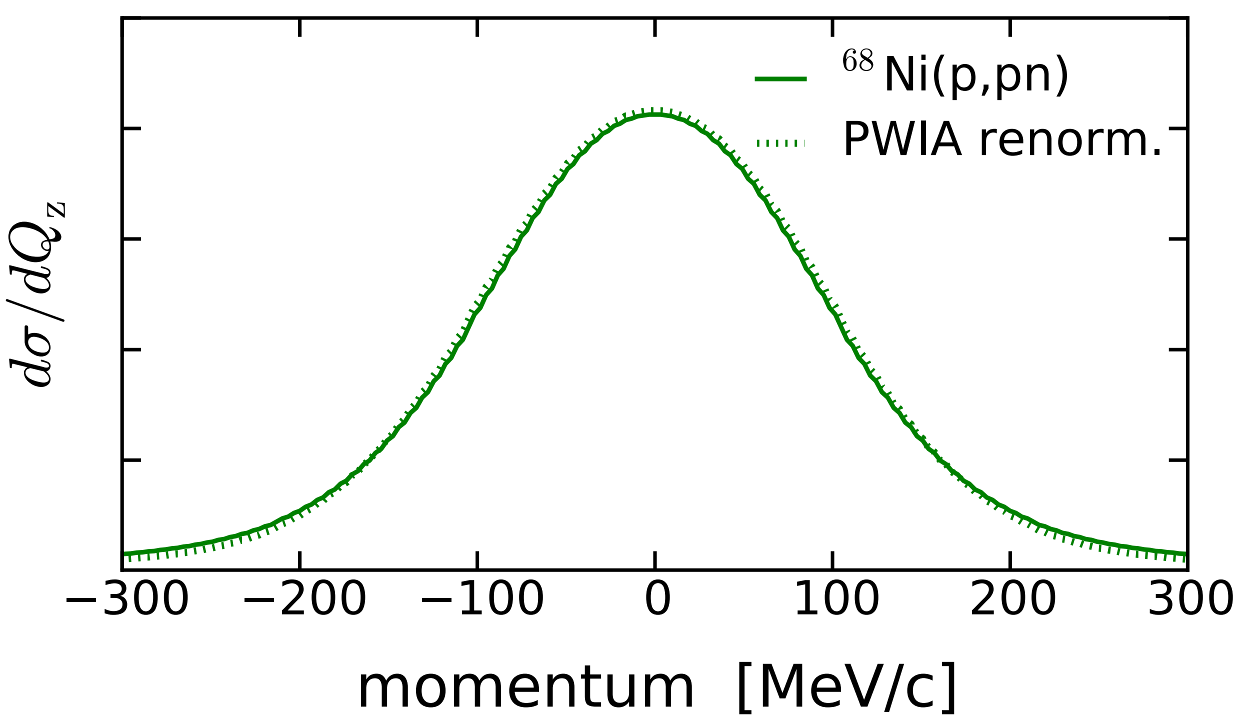
^{20}O at 500 MeV/nucleon



DWIA x PWIA



$S_n(^{11}Be - 1s_{1/2}) = 0.502$ MeV

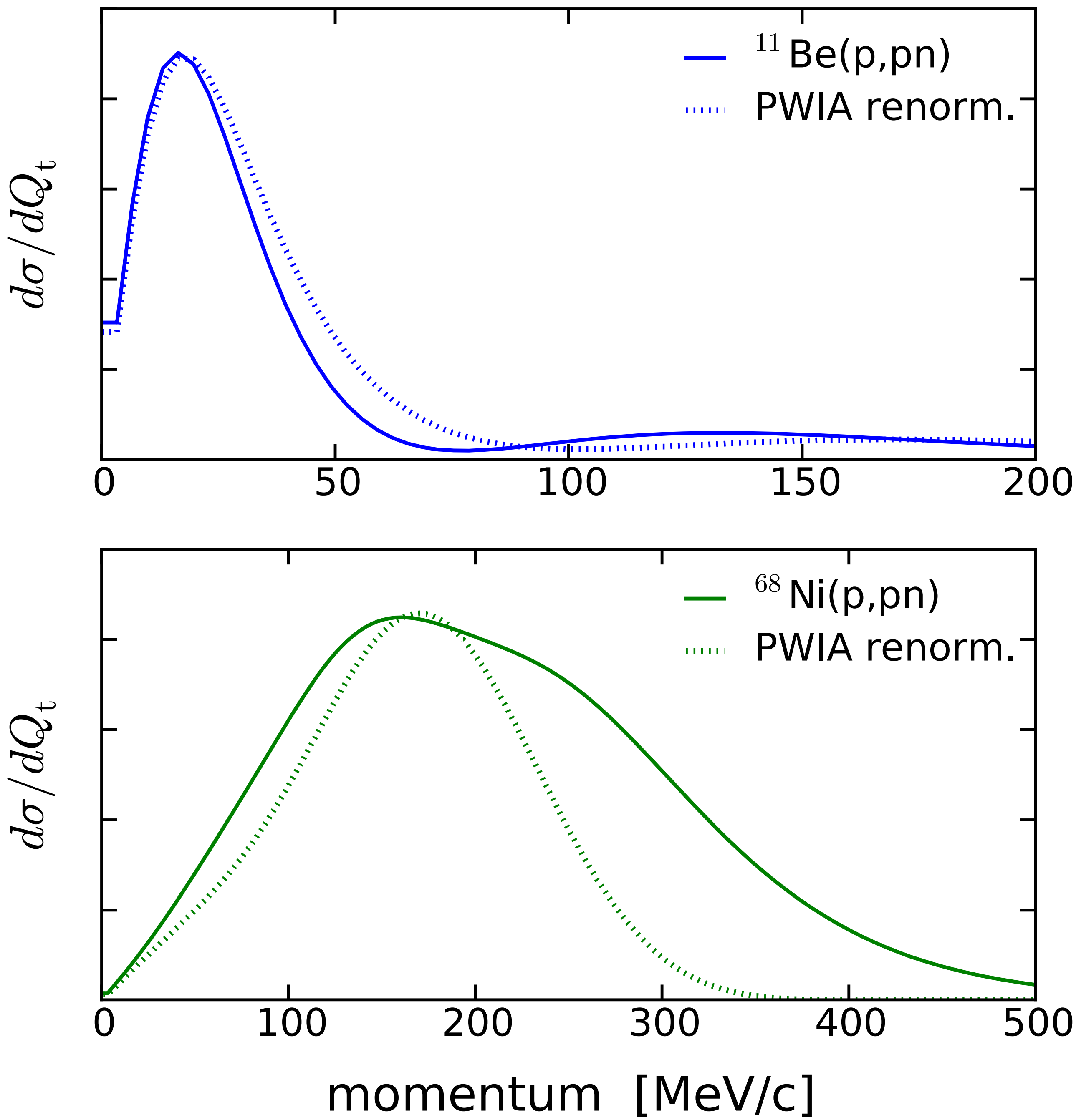


$S_n(^{68}Ni - 0f_{3/2}) = 15.7$ MeV

$$\left(\frac{d\sigma}{dQ}\right)_{DWIA} \sim \left| \int d^3r e^{i\mathbf{Q}\cdot\mathbf{r}} \mathcal{F}(\mathbf{r}) \varphi(\mathbf{r}) \right|^2$$

$$\left(\frac{d\sigma}{dQ}\right)_{PWIA} \sim \left| \int d^3r e^{i\mathbf{Q}\cdot\mathbf{r}} \varphi(\mathbf{r}) \right|^2$$

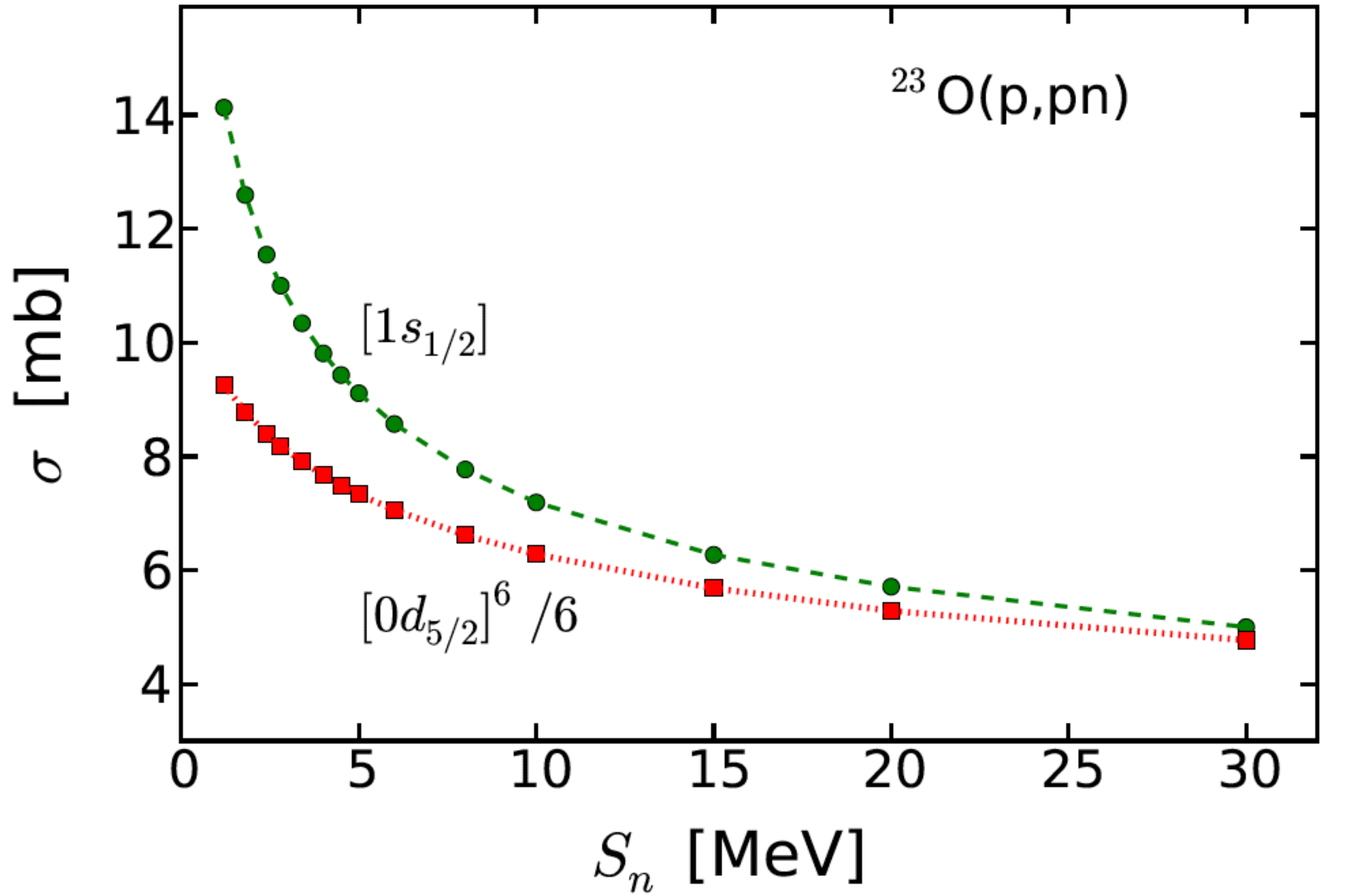
DWIA x PWIA



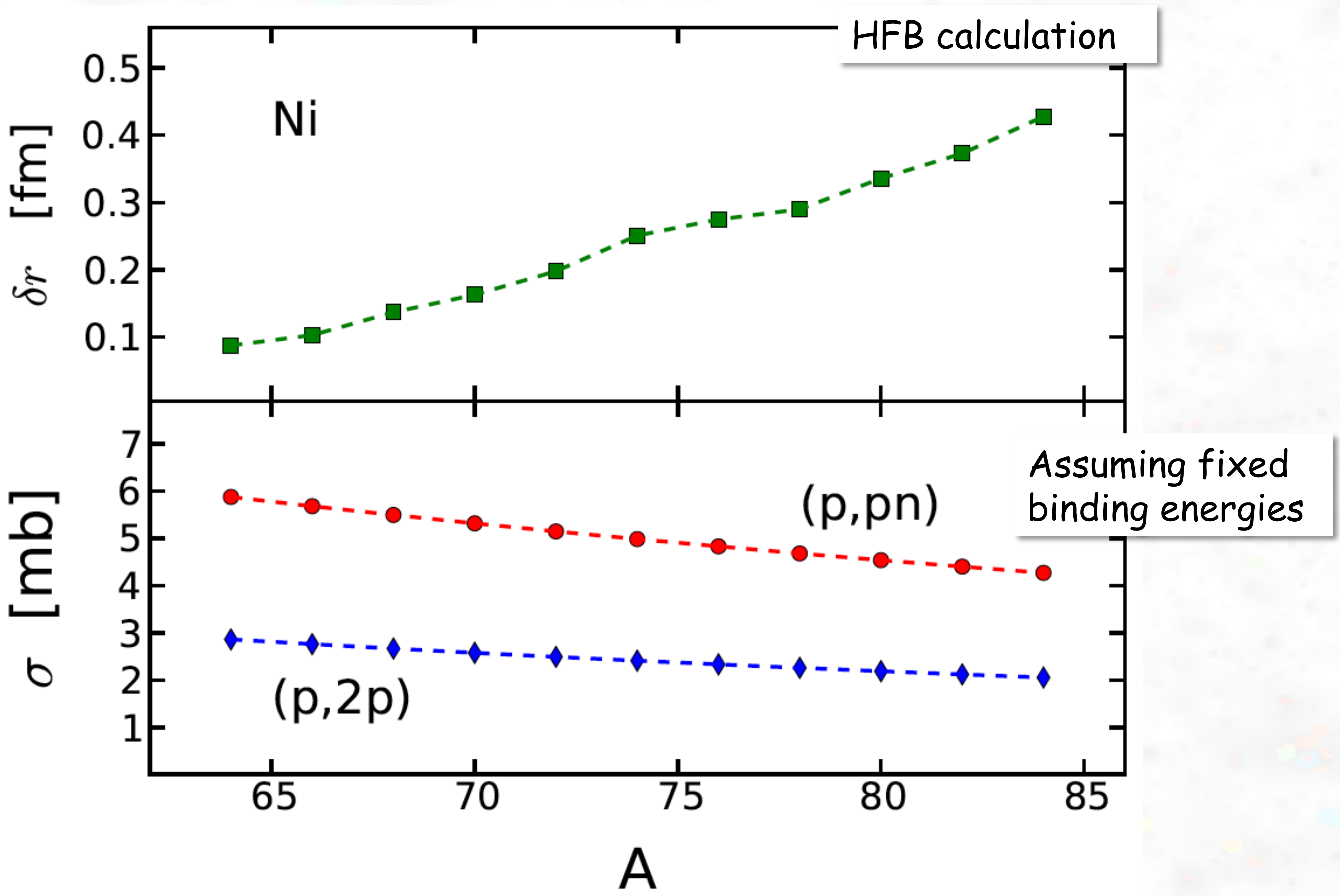
$S_n(^{11}Be) = 0.502$ MeV

$S_n(^{68}Ni) = 15.7$ MeV

Dependence on Separation Energy and Ang. Mom.



Dependence on # of nucleons



Conclusions

• p,2p and p,pn reactions very useful spectroscopic tools for studies of nuclei far from the stability.

• Very much used in the 60's and 70's – most people working with it are either dead or retired.

• High beam energies might simplify theory (Glauber, kinematics, etc.)

• First experiments being carried out in Europe and Asia (Japan, China)

• New theories and codes need development for consistent experimental analysis.