Xiamen
7th. Jan. (2019)

Quark-Hadron Pasta in Neutron Stars:

A quick guide for EOS table

arXiv:1812.11889

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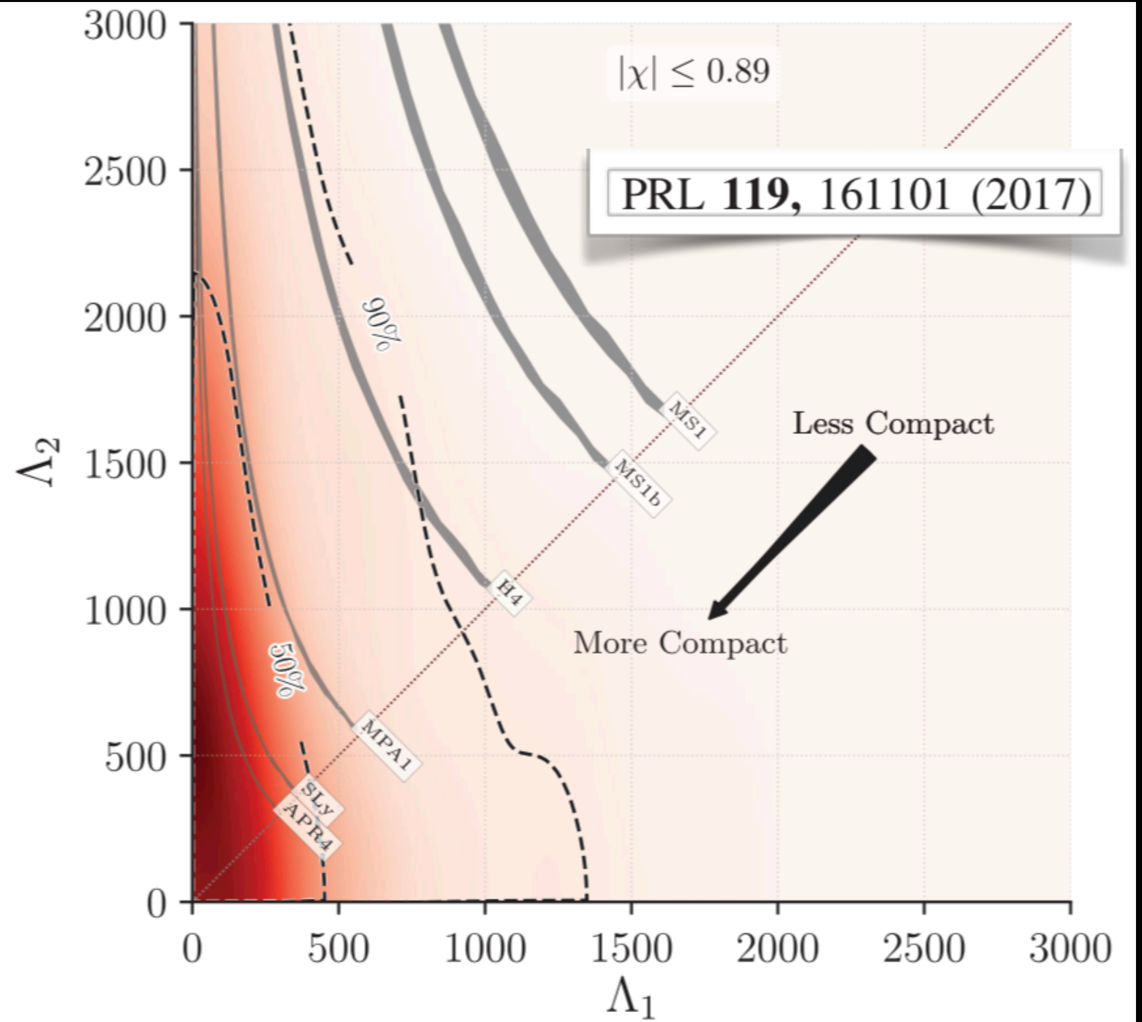
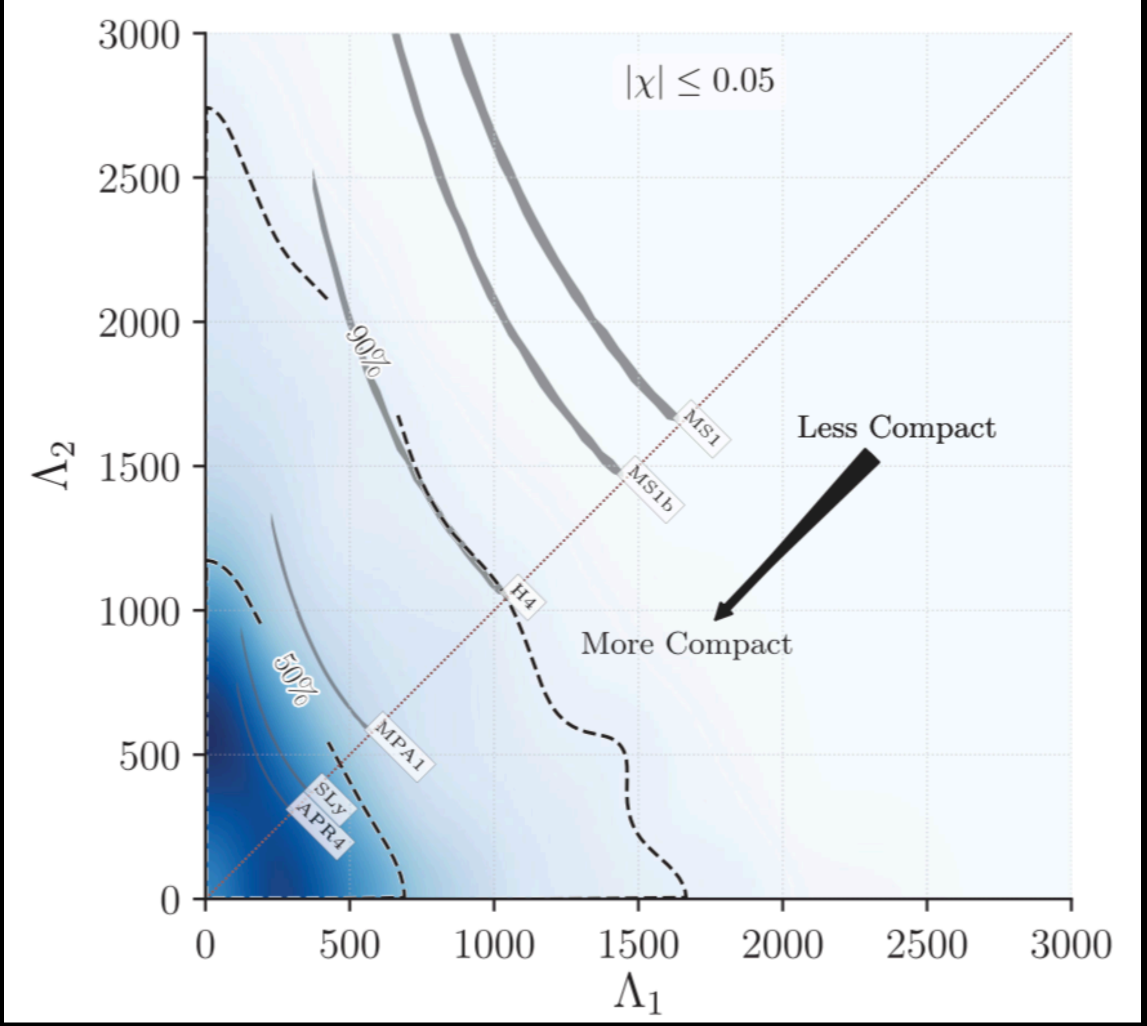
with

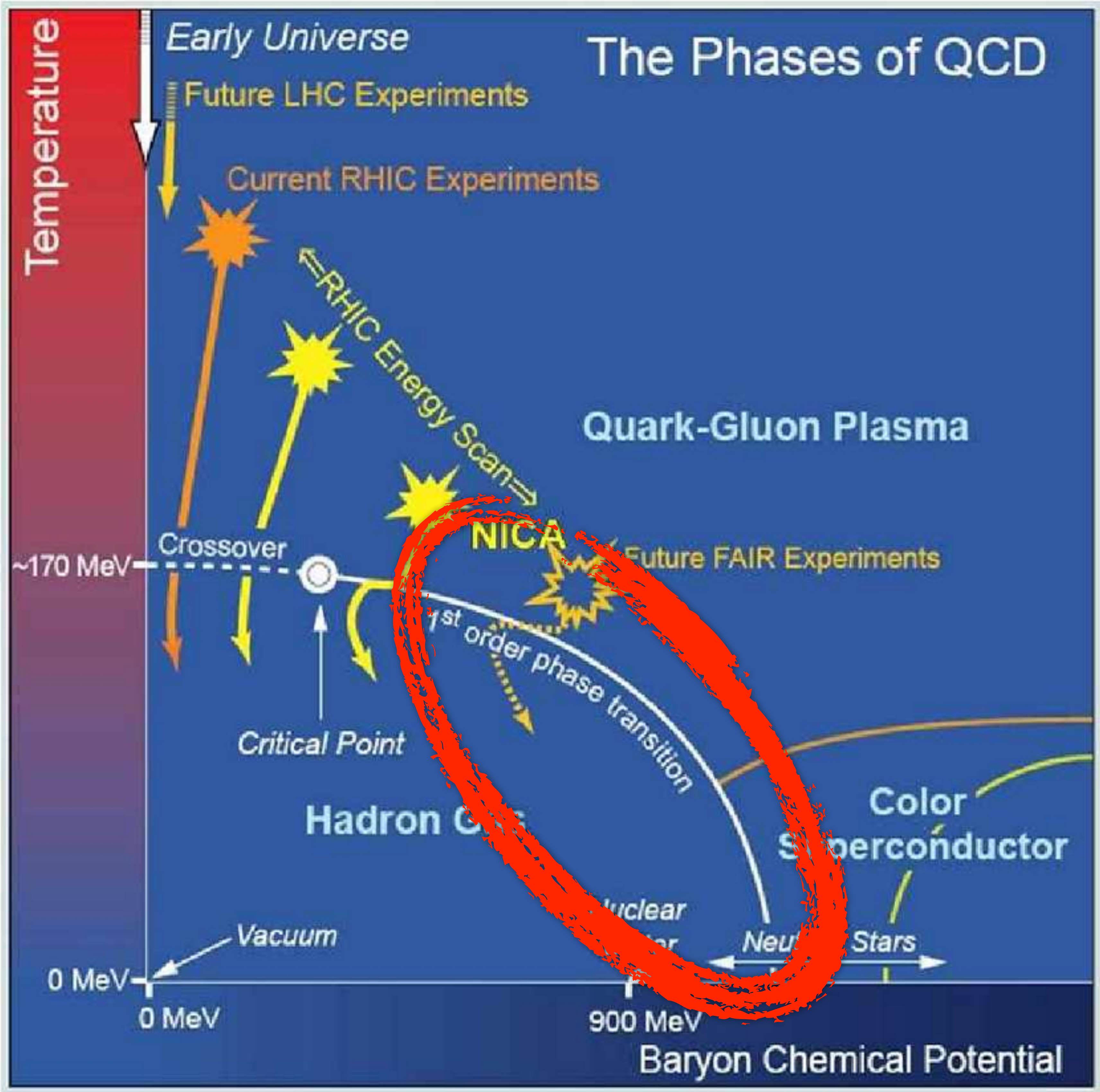
K. Maslov, A. Ayriyan, D. Blaschke, H. Grigorian,
T. Maruyama, T. Tatsumi, D. N. Voskresensky





2017 GW from NS+NS merger !!

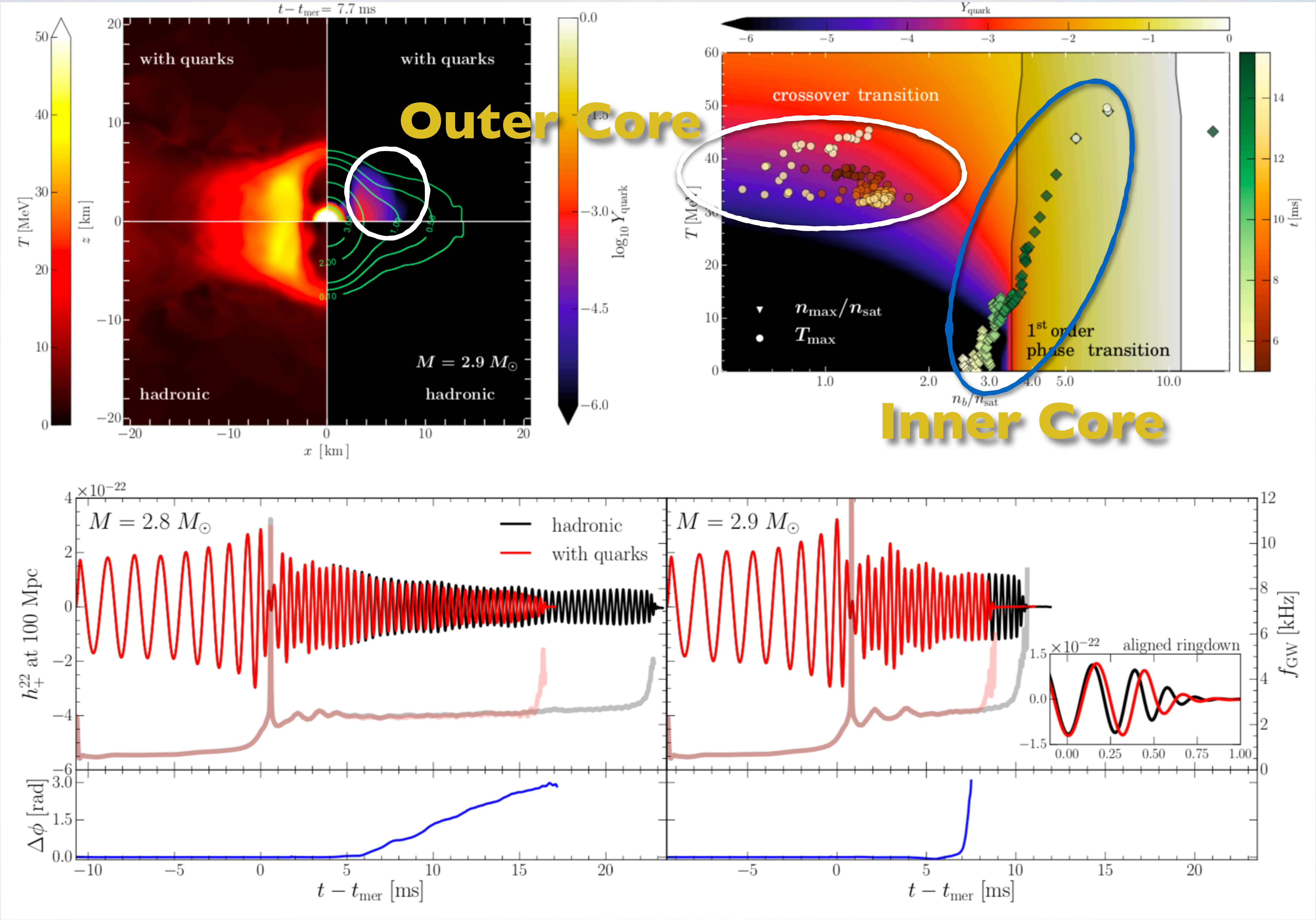
Progress of the NICA project (2009)



If we want to know the "real" phase diagram, we should know the non-uniform phase transition.

How to know ?

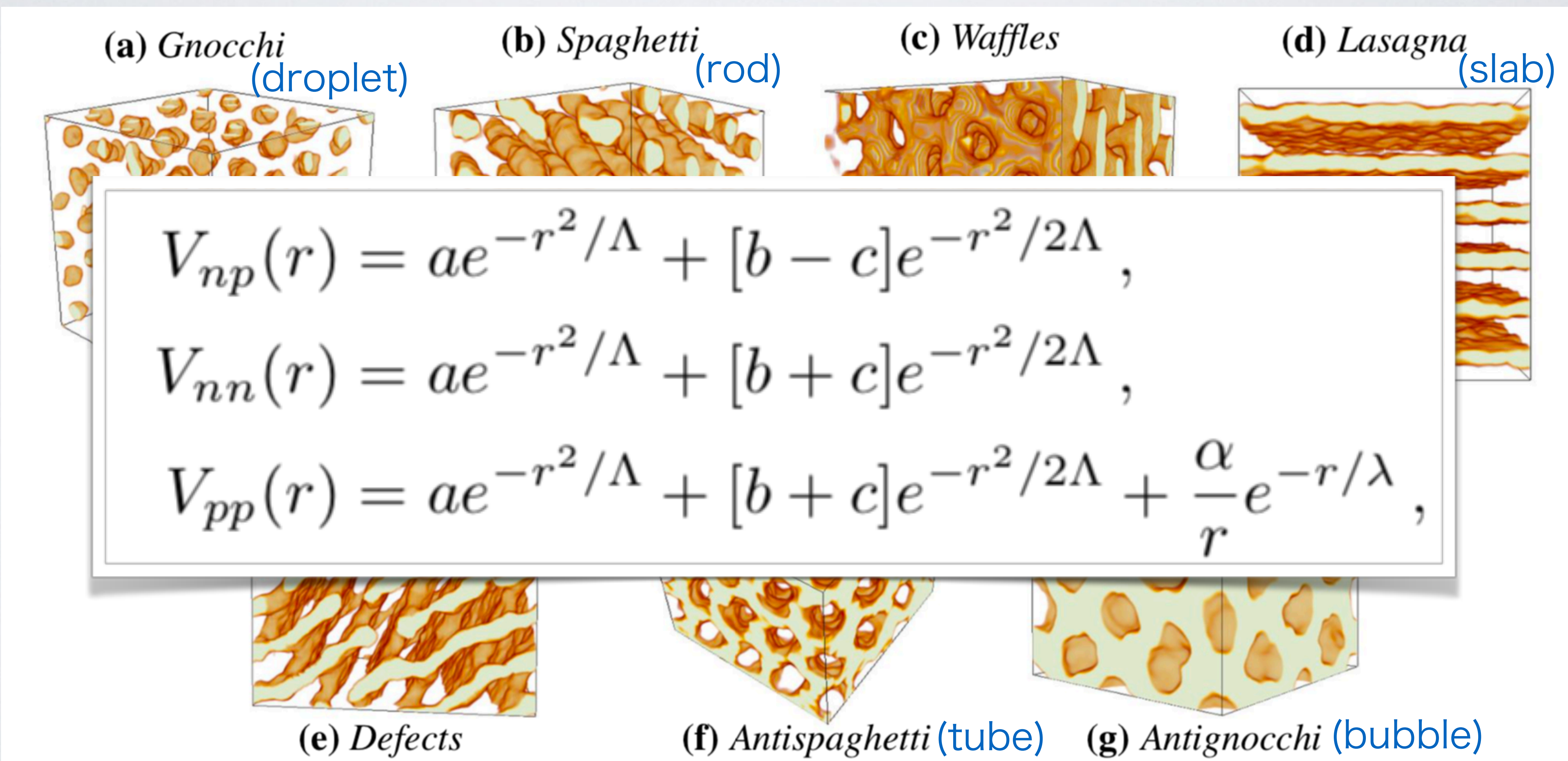
Most,Papenfort, Dexheimer, Hanauske, Schramm, Sto¨cker, Rezzolla (2018) arXiv1807.03684



What is pasta structure ?

It appears in the 1st order phase transition of multi-component system, such as the liquid-gas phase transition.

Depended on "density" and temperature", each charged particle clusterizes automatically by r-dependent interactions such as "**meson interactions**", and/or "**Coulomb interactions** " balanced with "**surface tensions**" ; i.e. ***finite size effects.***



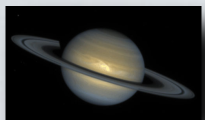
$$V_{np}(r) = ae^{-r^2/\Lambda} + [b - c]e^{-r^2/2\Lambda},$$

$$V_{nn}(r) = ae^{-r^2/\Lambda} + [b + c]e^{-r^2/2\Lambda},$$

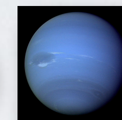
$$V_{pp}(r) = ae^{-r^2/\Lambda} + [b + c]e^{-r^2/2\Lambda} + \frac{\alpha}{r}e^{-r/\lambda},$$

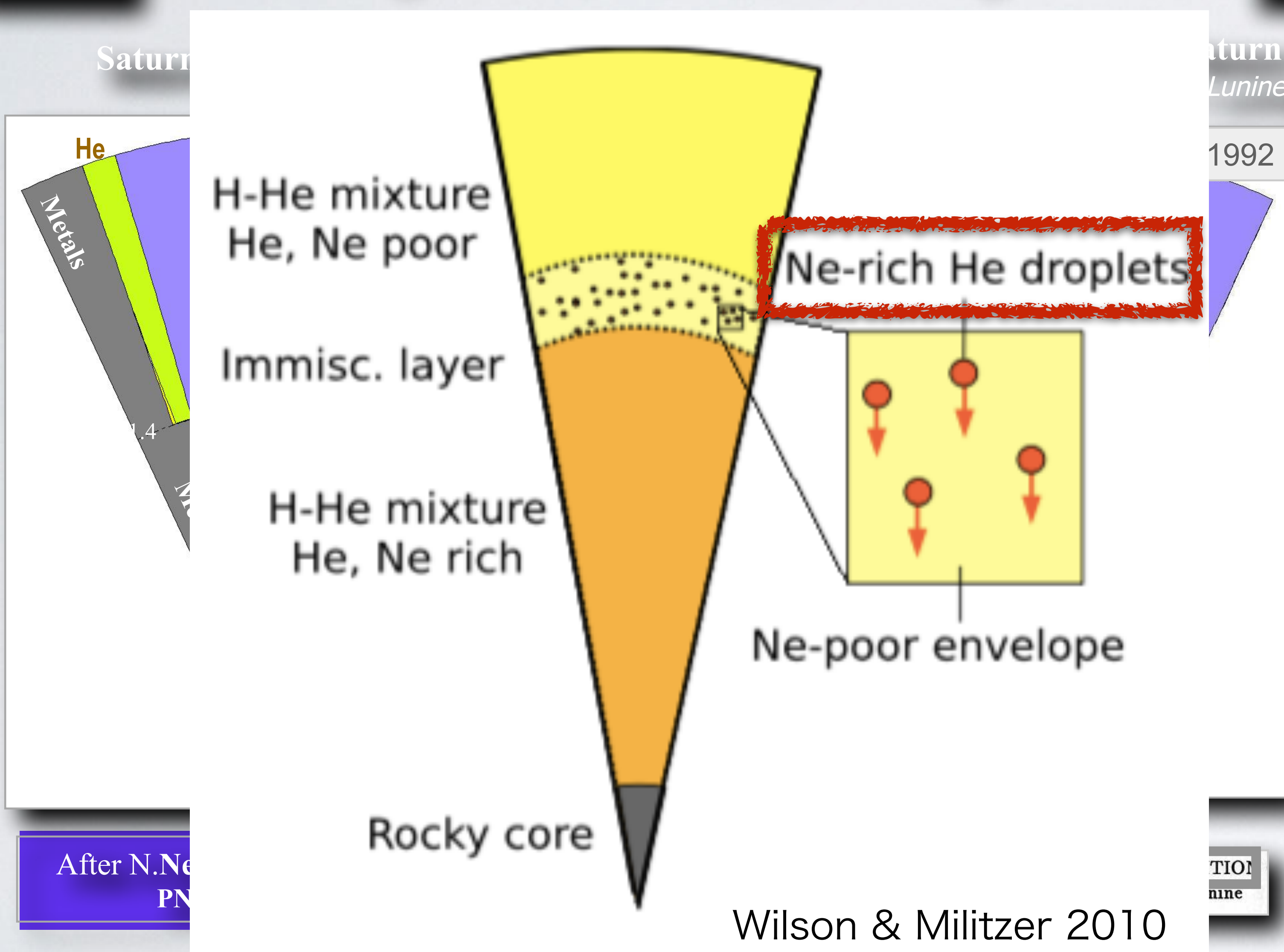
MD simulation by Horowitz et al., 2015; Schneider et al., 2014, 2013

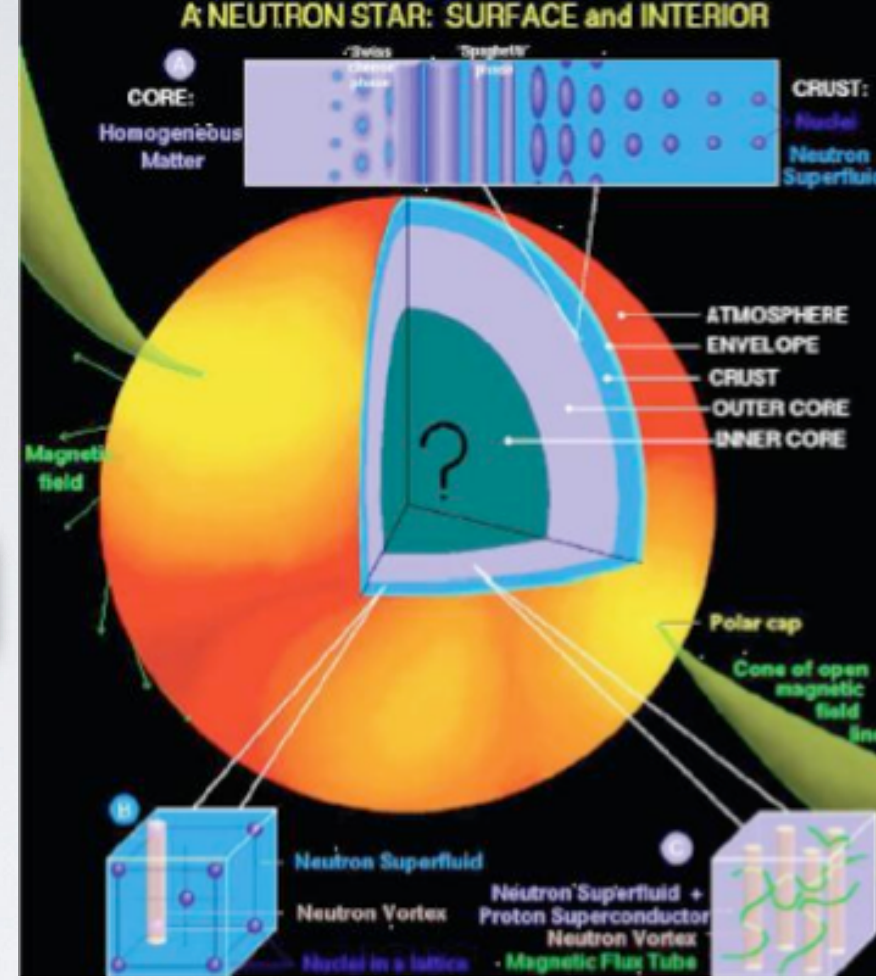
Examples of non-uniform matter



Giant planets interior composition

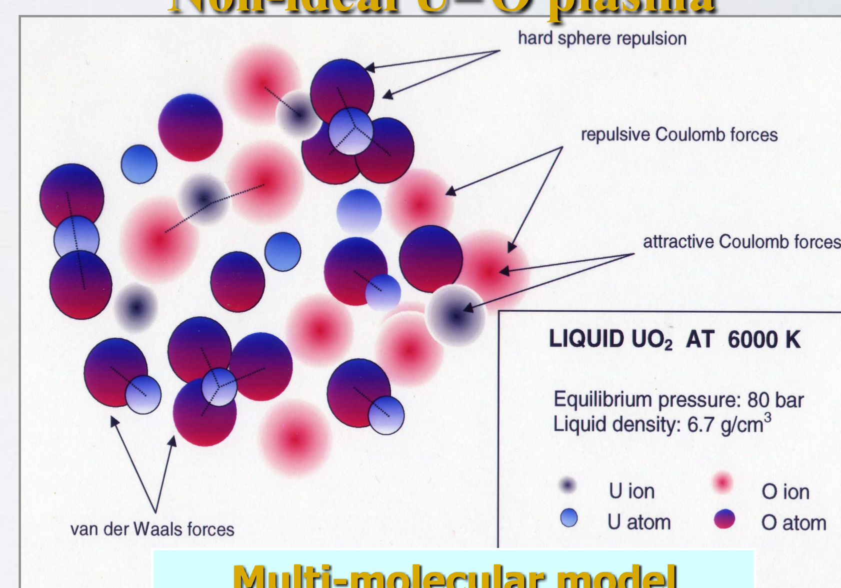






(c)D.Page

Non-ideal U–O plasma



Multi-molecular model
(*Liquid & Gas*)

$U + O + O_2 + UO + UO_2 + UO_3$

$U^+ + UO^+ + UO_2^+ + O^- + UO_3^- + e^-$

$U + 2O \Leftrightarrow UO_2$
$2O \Leftrightarrow O_2$
$U^+ + e \Leftrightarrow U$
$UO_3 + e \Leftrightarrow UO_3^-$
......

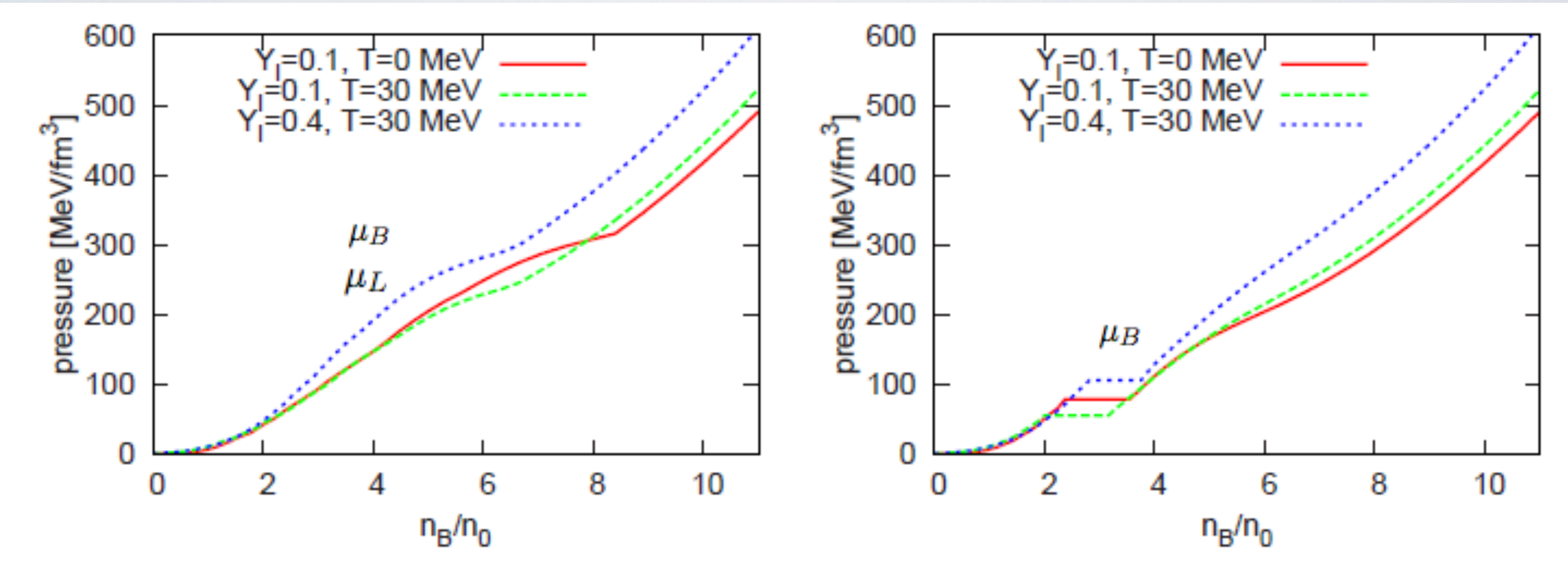
$\mu_U + 2\mu_U = \mu_{UO2}$
$2\mu_O = \mu_{O2}$
$\mu_{U+} + \mu_e = \mu_U$
$\mu_{UO3} + \mu_e = \mu_{UO3-}$
....

(c)Iosilevskiy et al.

If we want to know the real world (observations), we should know the non-uniform phase transition.

Uncertainty from the finite size effects in quark-hadron phase transition

Shen EOS + NJL model



the bulk Gibbs condition

Glendenning (1992) PRD

the Maxwell construction

↑

$\sigma_S = 0$ the finite size effects $\sigma_S > \sigma_C$

Voskresensky, Yasuhira, Tatsumi (2002) PLB, Maruyama et al., (2008) PRC…

Uncertainty of Q-H phase transition

Hempel et al., PRD 80, 125014 (2009)

TABLE III. As Table II, but now for the hadron-quark phase transition. $\mu_d = \mu_s$ is valid if strangeness is in equilibrium.

Case	Conserved densities/fractions Globally	Locally	Equilibrium conditions	Construction of mixed phase
0		n_B, (Y_p), (Y_L), n_C	-	Direct
Ia	n_B	Y_p, Y_L, n_C	$(1-Y_p)\mu_n + Y_p(\mu_p + \mu_e^H) + (Y_L - Y_p)\mu_\nu^H = (2-Y_p)\mu_d + (1+Y_p)\mu_u + Y_p\mu_e^Q + (Y_L - Y_p)\mu_\nu^Q$	Maxwell
Ib	n_B	Y_L, n_C	$\mu_n + Y_L\mu_\nu^H = 2\mu_d + \mu_u + Y_L\mu_\nu^Q$	Maxwell
Ic	n_B	Y_p, n_C	$(1-Y_p)\mu_n + Y_p(\mu_p + \mu_e^H) = (2-Y_p)\mu_d + (1+Y_p)\mu_u + Y_p\mu_e^Q$	Maxwell
Id	n_B	n_C	$\mu_n = 2\mu_d + \mu_u$	Maxwell
IIa	n_B, Y_L	Y_p, n_C	$(1-Y_p)\mu_n + Y_p(\mu_p + \mu_e^H) = (2-Y_p)\mu_d + (1+Y_p)\mu_u + Y_p\mu_e^Q$, $\mu_\nu^H = \mu_\nu^Q$	Maxwell/Gibbs
IIb	n_B, Y_L	n_C	$\mu_n = 2\mu_d + \mu_u$, $\mu_\nu^H = \mu_\nu^Q$	Gibbs
IIIa	n_B, Y_p	Y_L, n_C	$\mu_n + Y_L\mu_\nu^H = 2\mu_d + \mu_u + Y_L\mu_\nu^Q$, $\mu_p - \mu_n - \mu_\nu^H + \mu_e^H = \mu_u - \mu_d - \mu_\nu^Q + \mu_e^Q$	Gibbs
IIIb	n_B, Y_p	n_C	$\mu_n = 2\mu_d + \mu_u$, $\mu_p + \mu_e^H = 2\mu_u + \mu_d + \mu_e^Q$	Gibbs
IV	n_B, Y_L, Y_p	n_C	$\mu_n = 2\mu_d + \mu_u$, $\mu_\nu^H = \mu_\nu^Q$, $\mu_p + \mu_e^H = 2\mu_u + \mu_d + \mu_e^Q$	Gibbs
V	n_B, Y_L, Y_p, n_C		$\mu_n = 2\mu_d + \mu_u$, $\mu_\nu^H = \mu_\nu^Q$, $\mu_p = 2\mu_u + \mu_d$, $\mu_e^H = \mu_e^Q$	Gibbs

1st. order phase transition with inhomogeneous structures

Voskresensky, Yasuhira, Tatsumi (2002) PLB, Maruyama et al., (2008) PRC…

Chemical equilibrium for quarks, hadrons, and leptons

$$\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_C^Q, \quad \mu_d = \mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_C^Q,$$

$$\mu_n = \mu_\Lambda = \mu_B, \quad \mu_p = \mu_B + \mu_{C,H}, \quad \mu_{\Sigma^-} + \mu_p = 2\mu_B,$$

$$\mu_L^{H(Q)} = \mu_{\nu_e}^{H(Q)}, \quad \mu_C^{H(Q)} = \mu_L^{H(Q)} - \mu_e^{H(Q)},$$

Charge screening effects

$$\mu* = \frac{\delta F}{\delta \rho_i^\alpha}, \quad \alpha = \{\mathrm{I}, \mathrm{II}\}$$

$$= \frac{\partial \epsilon_{\mathrm{kin+str}}^\alpha}{\partial \rho_i^\alpha} - N_i^{\mathrm{ch},\alpha}(V^\alpha - V^0), \quad N_i^{\mathrm{ch},\alpha} = Q_i^\alpha / e,$$

Free energy for quarks, hadrons, and leptons

$$F = \int_{V_H} dr^3 \mathcal{F}_H[n_i] + \int_{V_Q} dr^3 \mathcal{F}_Q[n_q] + F_e + F_{\nu_e} + \underline{E_C} + \underline{E_S}$$

$$E_C = \frac{e^2}{2}\int_{V_W} d^3\boldsymbol{r} d^3\boldsymbol{r}' \, \frac{n_{\mathrm{ch}}(\boldsymbol{r}) n_{\mathrm{ch}}(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|}$$

$$\int_{\partial D} dS \epsilon_S$$

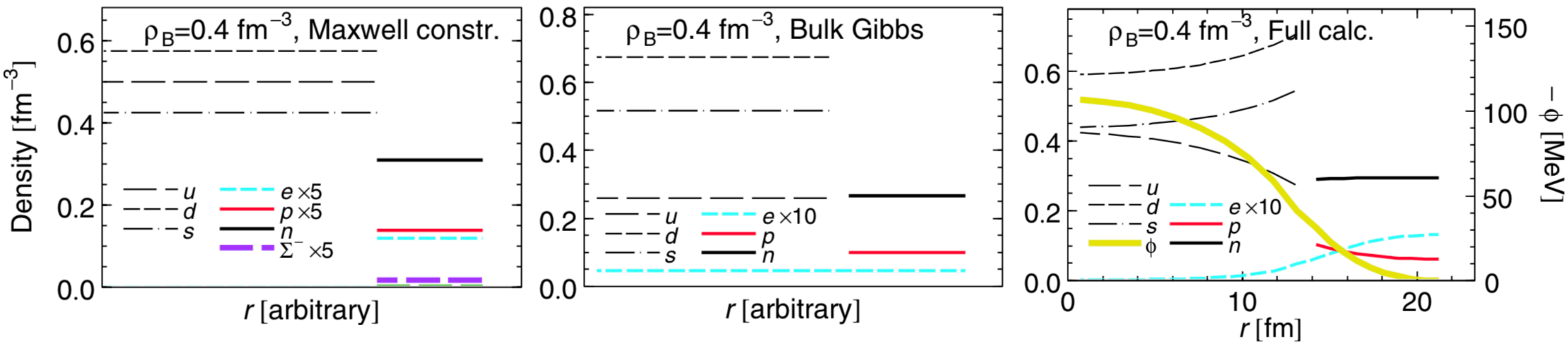
Charge screening effects

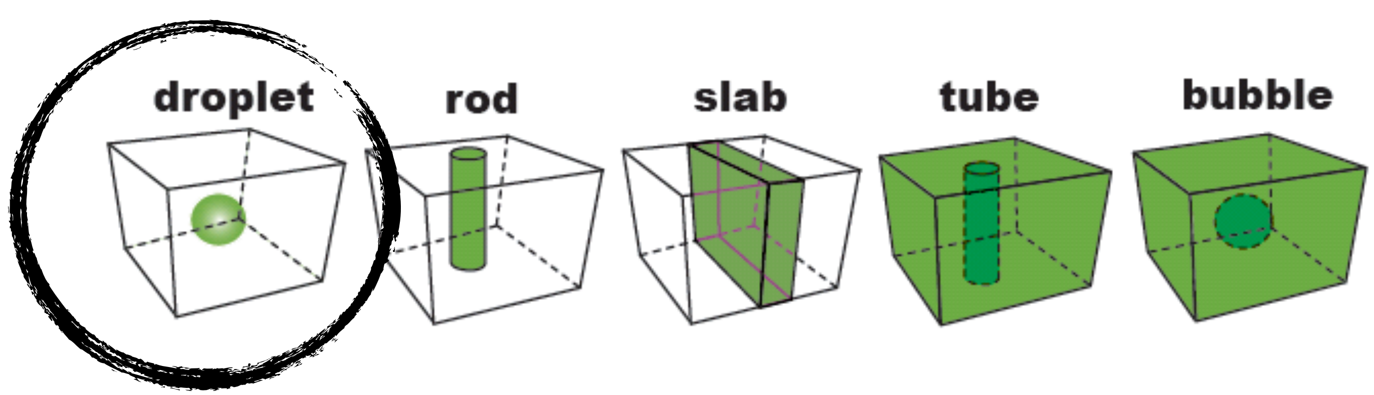
Voskresensky, Yasuhira, Tatsumi (2002) PLB,   Maruyama et al., (2008) PRC…

$$\mu_{B,H} = \mu_{B,Q}$$

$$\mu_{B,H} = \mu_{B,Q} \quad \mu_{C,H} = \mu_{C,Q}$$

$$\mu^*_{B,H} = \mu^*_{B,Q} \quad \mu^*_{C,H} = \mu^*_{C,Q}$$





$$P^{\mathrm{I}} = P^{\mathrm{II}} + \sigma \frac{dS}{dv^{\mathrm{I}}},$$

Formalism

NY, et al. (2013)*Recent Advances in Quarks Research*, Nova, Chap.4, pp.63, ISBN 9781622579709, arXiv:1208.0427[astro-ph].

Hadron matter

- **Brueckner-Hartree-Fock model** (Baldo et al. 1998, Schulze et al. 1995, Yamamoto et al. 2013)
- **Relativistic Mean Field theory** (**KVOR**, **MKVOR**, …) **etc…**

↕ phase transition (non-uniform)

Quark matter

- **Dyson-Schwinger method** (Huan et al. 2012, etc.)
- **Non-local (P)NJL model** (Blaschke et al. 2012, Benic et al. 2014 etc.),
- **String flip model** (Bastian, Kaltenborn, Blaschke, 2017 etc.) **etc…**

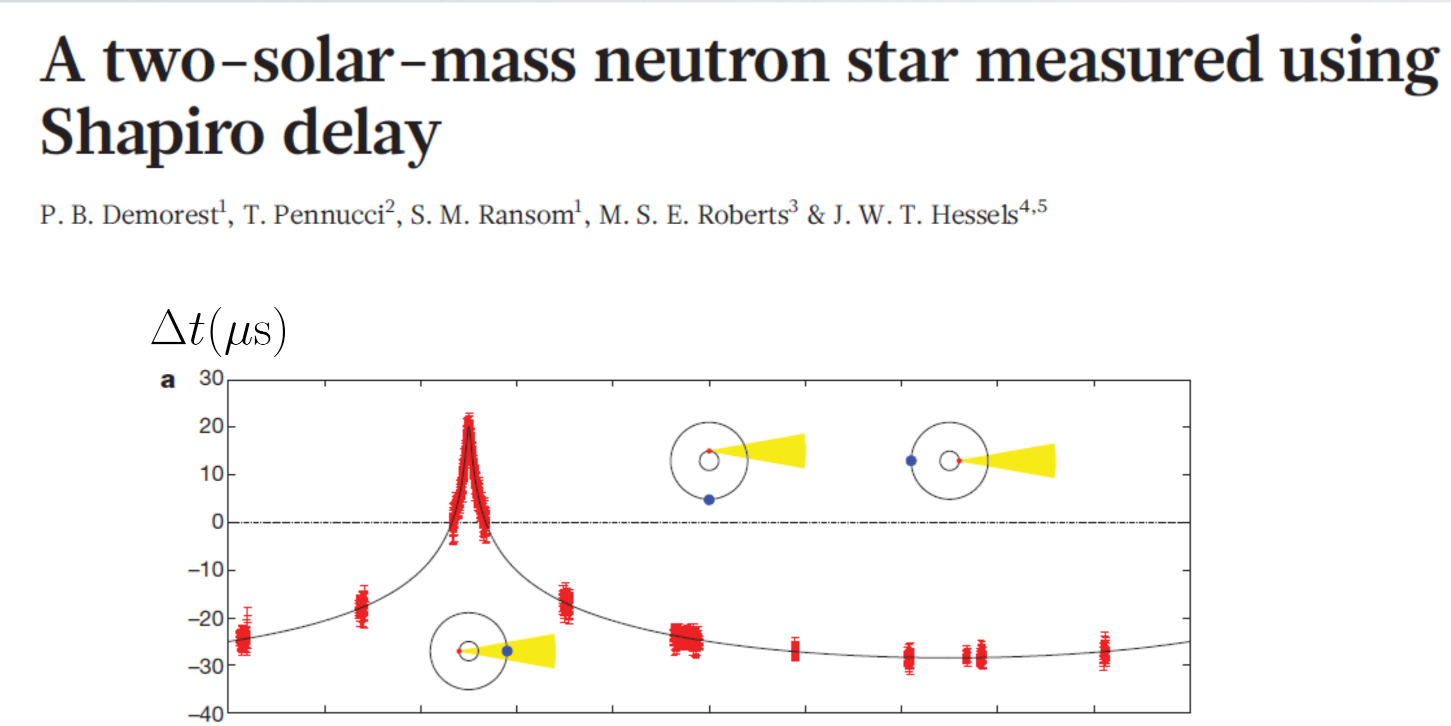
We assume the non-uniform structures of the mixed phase as droplet, rod, slab, tube, and bubble under Wigner-Seitz cell approximation.

In calculations of mixed phase, we consider

- charge neutrality
- chemical equilibrium
- baryon number conservation
- balance between "surface tension" and "Coulomb interaction"

Changing all of them, we search the minimum free energy.

Constraints on EOS ①


A two-solar-mass neutron star measured using Shapiro delay

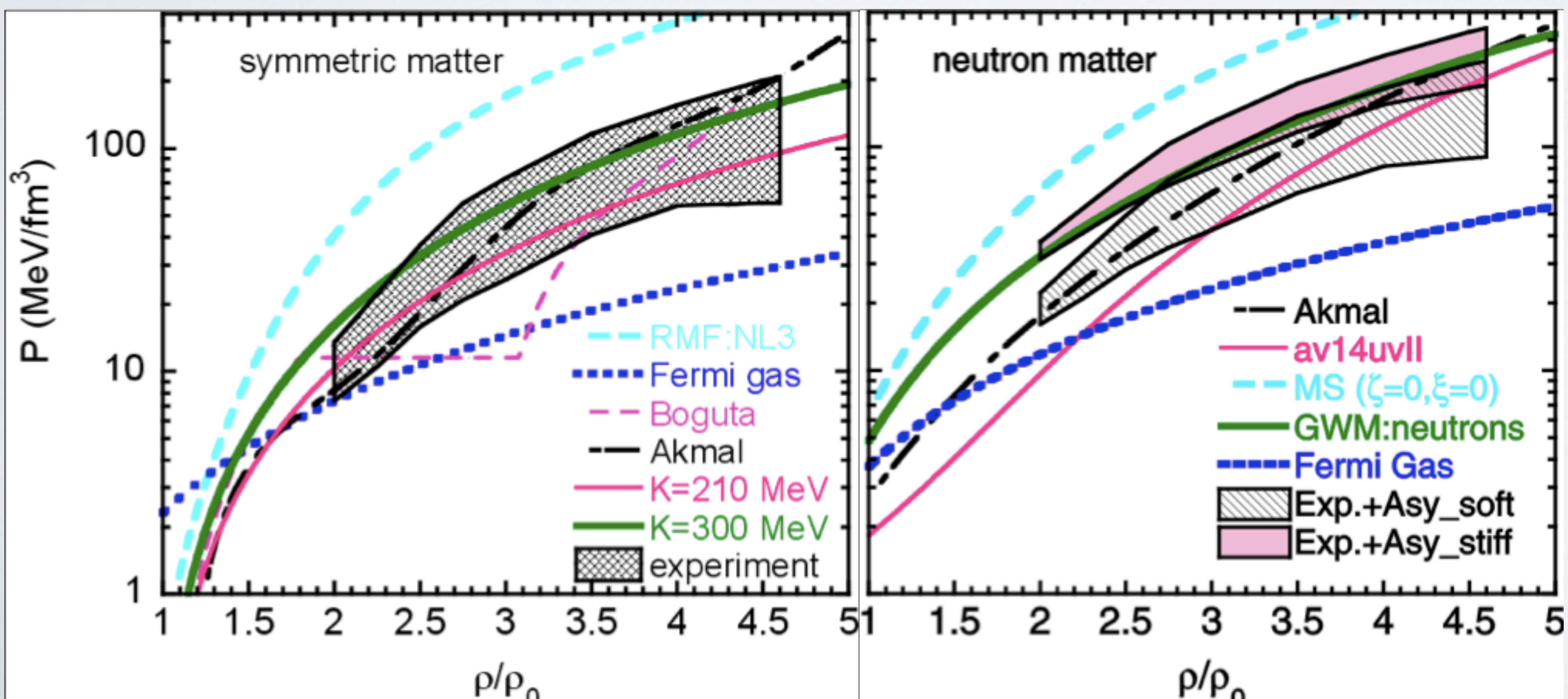
P. B. Demorest[1], T. Pennucci[2], S. M. Ransom[1], M. S. E. Roberts[3] & J. W. T. Hessels[4,5]

Demorest et al. 2010 nature

M~1.97 Ms

"Shapiro delay"

Radar signals passing near a massive object take slightly longer to travel to a target and longer to return than they would if the mass of the object were not present.

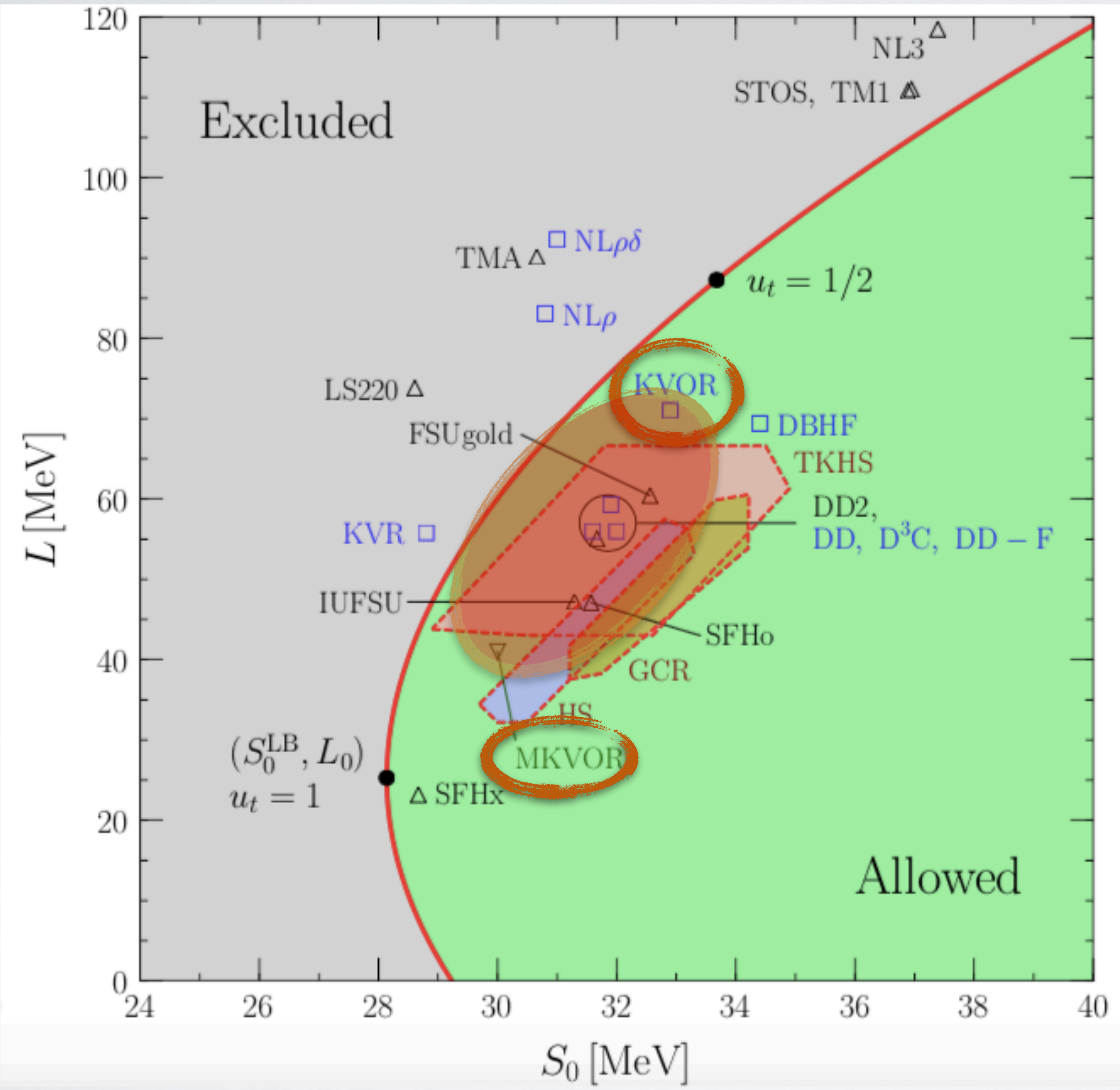


Danielewiez et al. 2002 science

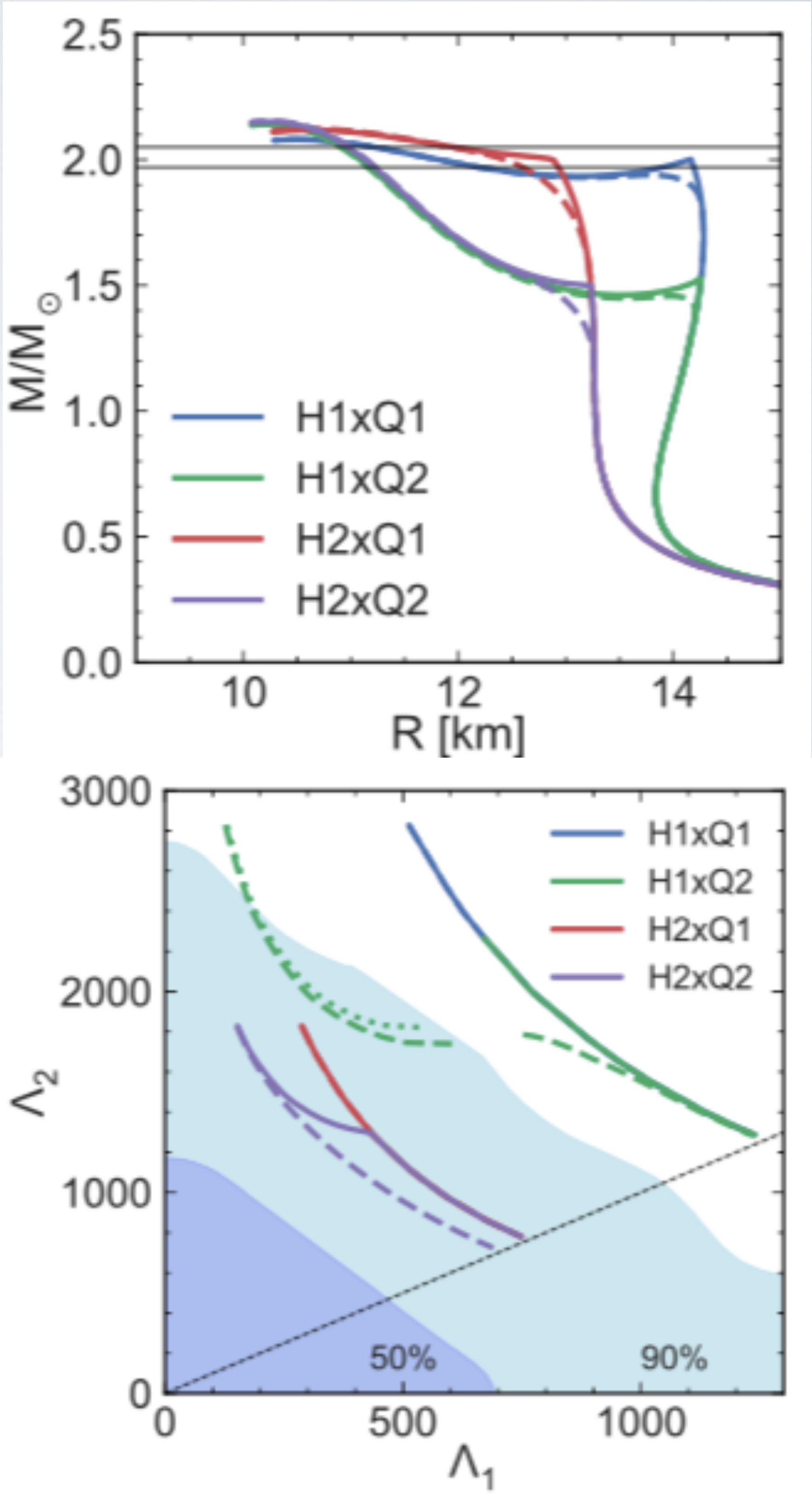
"Constraint by Experiment"

There is the upper limit of hard EOS.

Constraints on EOS ②



Tews, Lattimer, Ohnishi, Kolomeitsev 2017



Maslov, **NY**, Ayriyan, Grigorian, Blaschke, Voskresensky, Maruyama, Tatsumi, arXiv: 1812.11889, and the next one coming soon.

Generalized relativistic mean-field model

E. E. Kolomeitsev, D.N. Voskresensky, NPA 759 (2005)
K. A. M, E. E. K. and D. N. V., Phys. Lett. B 748 (2015),
E. E. K., K. A. M. and D. N. V., arXiv:1610.09746, to be published in NPA 961 (2017)

$$\mathcal{L} = \mathcal{L}_{\rm bar} + \mathcal{L}_{\rm mes} + \mathcal{L}_l,$$

$$\mathcal{L}_{\rm bar} = \sum_{i=b\cup r} (\bar{\Psi}_i \left(iD_\mu^{(i)}\gamma^\mu - m_i\Phi_i(\sigma)\right)\Psi_i,$$

$$D_\mu^{(i)} = \partial_\mu + ig_{\omega i}\chi_{\omega i}(\sigma)\omega_\mu + ig_{\rho i}\chi_{\rho i}(\sigma)\vec{t}\vec{\rho}_\mu + ig_{\phi i}\chi_{\phi i}(\sigma)\phi_\mu,$$

$$\{b\} = (N, \Lambda, \Sigma^{\pm,0}, \Xi^{-,0}, \Delta^-, \Delta^0, \Delta^+, \Delta^{++})$$

$$\mathcal{L}_{\rm mes} = \frac{\partial_\mu\sigma\partial^\mu\sigma}{2} - \frac{m_\sigma^2\Phi_\sigma^2(\sigma)\sigma^2}{2} - U(\sigma) +$$

$$+\frac{m_\omega^2\Phi_\omega^2(\sigma)\omega_\mu\omega^\mu}{2} - \frac{\omega_{\mu\nu}\omega^{\mu\nu}}{4} + \frac{m_\rho^2\Phi_\rho^2(\sigma)\vec{\rho}_\mu\vec{\rho}^\mu}{2} - \frac{\rho_{\mu\nu}\rho^{\mu\nu}}{4} +$$

$$+\frac{m_\phi^2\Phi_\phi^2(\sigma)\phi_\mu\phi^\mu}{2} - \frac{\phi_{\mu\nu}\phi^{\mu\nu}}{4},$$

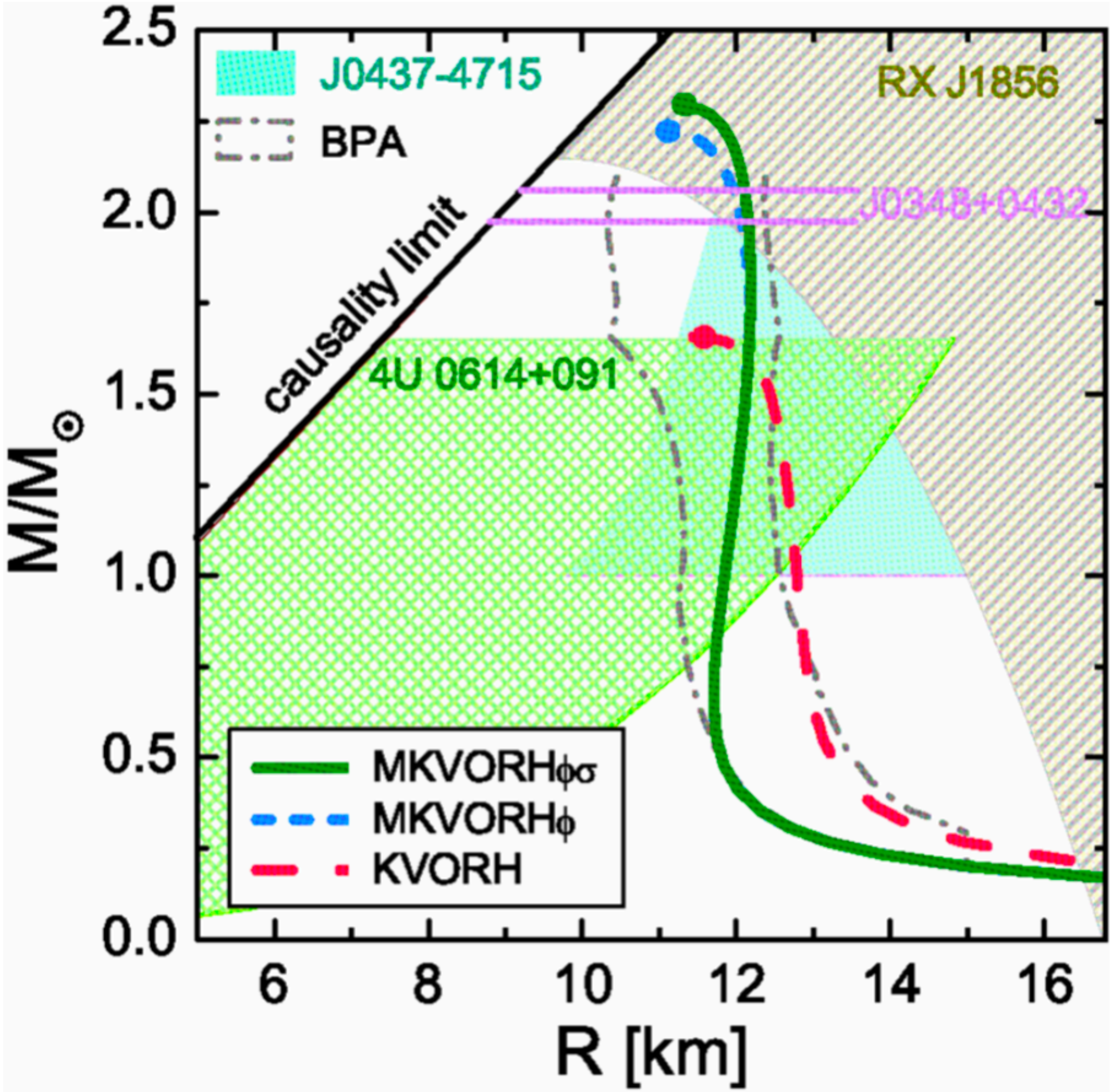
$$\omega_{\mu\nu} = \partial_\nu\omega_\mu - \partial_\mu\omega_\nu, \quad \vec{\rho}_{\mu\nu} = \partial_\nu\vec{\rho}_\mu - \partial_\mu\vec{\rho}_\nu,$$

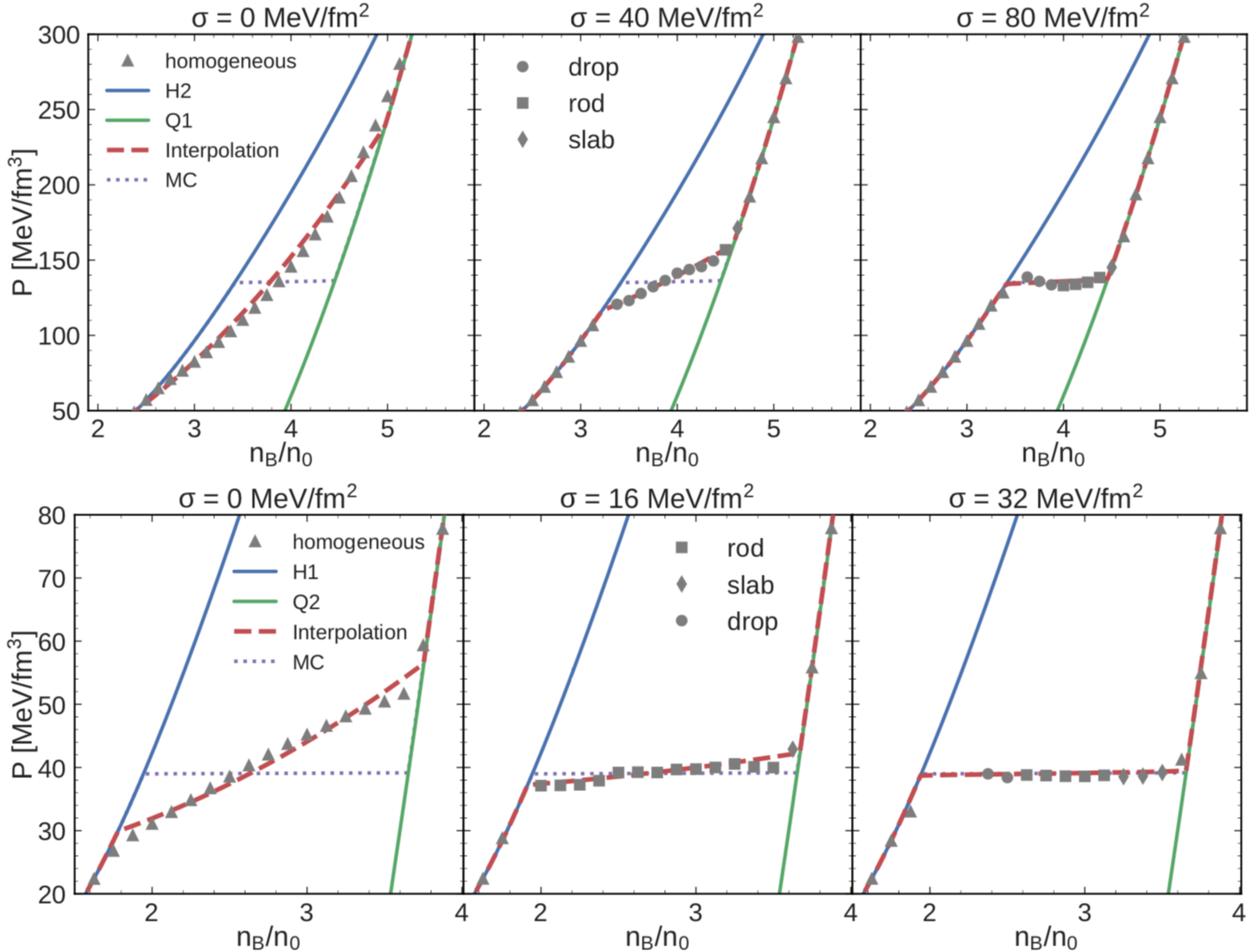
$$\phi_{\mu\nu} = \partial_\nu\phi_\mu - \partial_\mu\phi_\nu,$$

$$\mathcal{L}_l = \sum_l \bar{\psi}_l(i\partial_\mu\gamma^\mu - m_l)\psi_l, \quad \{l\} = (e,\ \mu).$$

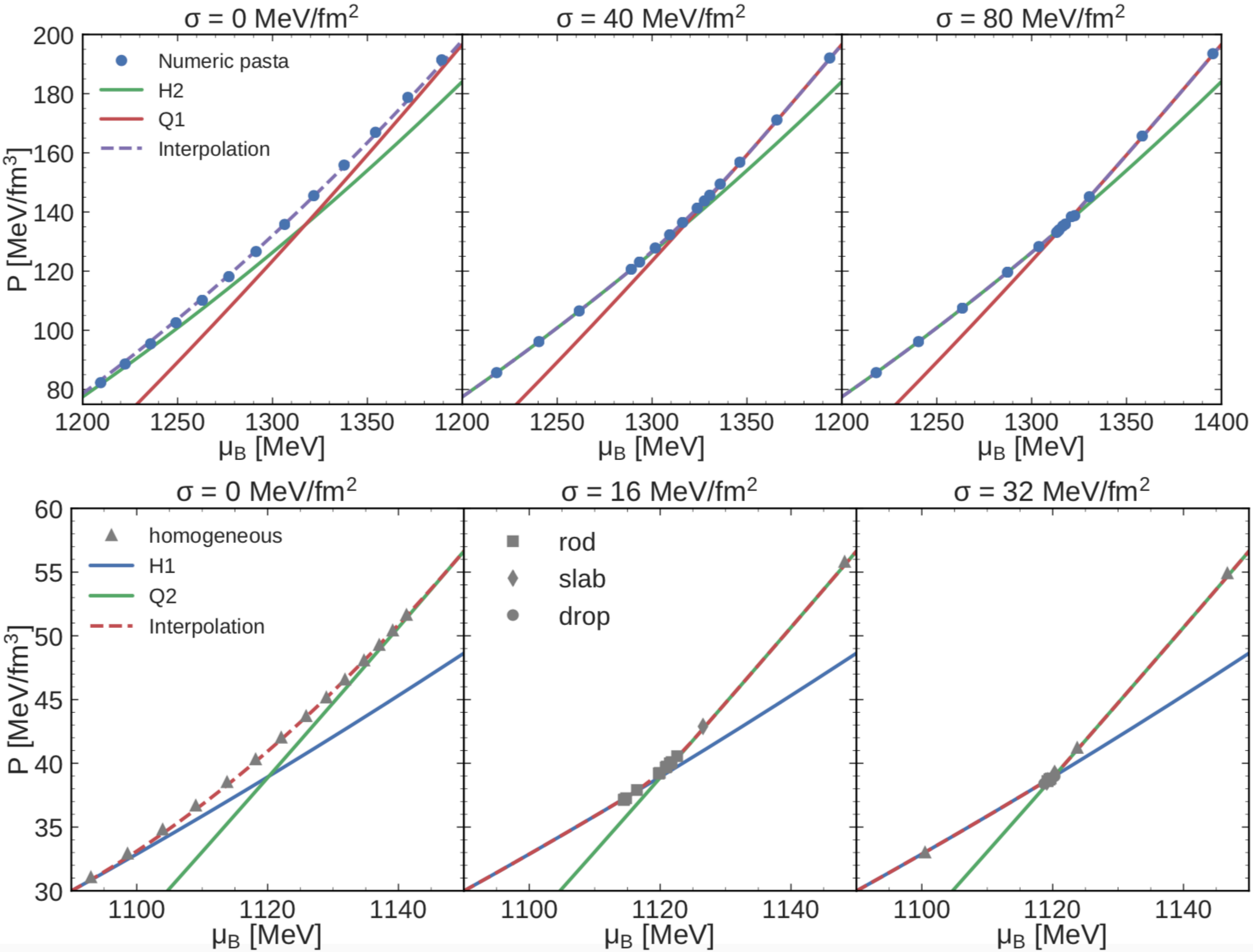
Maximum mass constraint

- The largest precisely measured NS mass $M[PSRJ0348 + 0432] = 2.01 \pm 0.04 M_{\odot}$ (Antoniadis et al., 2012).
- 4U 0614+091: QPO; RX J1856: isolated NS thermal radiation





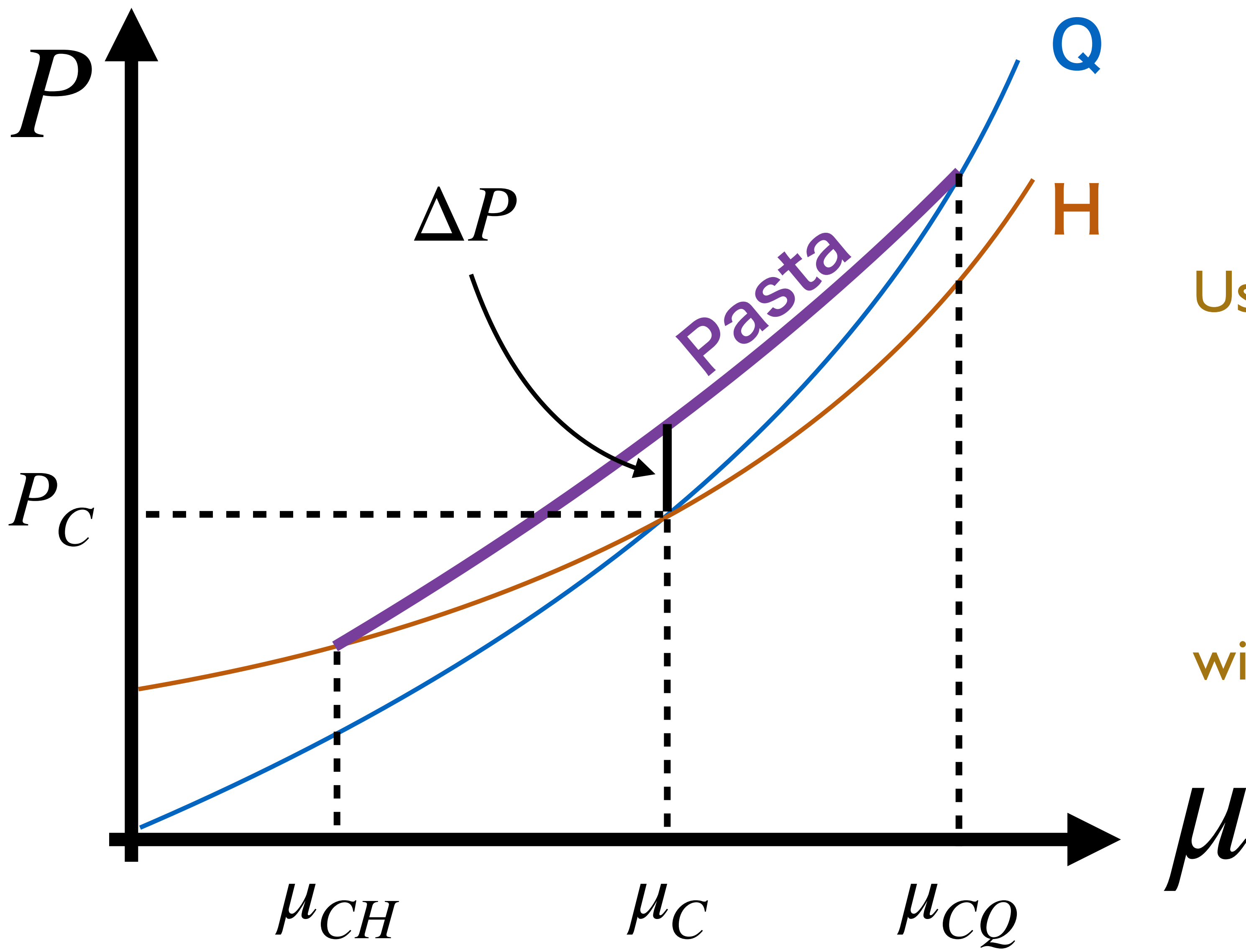
σ = 0 MeV/fm²
σ = 40 MeV/fm²
σ = 80 MeV/fm²
homogeneous
H2
Q1
Interpolation
MC
drop
rod
slab
P [MeV/fm³]
n_B/n_0
σ = 0 MeV/fm²
σ = 16 MeV/fm²
σ = 32 MeV/fm²
homogeneous
H1
Q2
Interpolation
MC
rod
slab
drop
P [MeV/fm³]
n_B/n_0



σ = 0 MeV/fm²
σ = 40 MeV/fm²
σ = 80 MeV/fm²
Numeric pasta
H2
Q1
Interpolation
P [MeV/fm³]
μB [MeV]
σ = 0 MeV/fm²
σ = 16 MeV/fm²
σ = 32 MeV/fm²
homogeneous
H1
Q2
Interpolation
rod
slab
drop
P [MeV/fm³]
μB [MeV]

Empirical formula of EOS with pasta

$$P(\mu) = \begin{cases} P^{(H)}(\mu), & \mu < \mu_{cH}\,, \\ a(\mu-\mu_c)^2 + b(\mu-\mu_c) & \\ +P_c + \Delta P, & \mu_{cH} < \mu < \mu_{cQ}\,, \\ P^{(Q)}(\mu), & \mu_{cQ} < \mu, \end{cases}$$



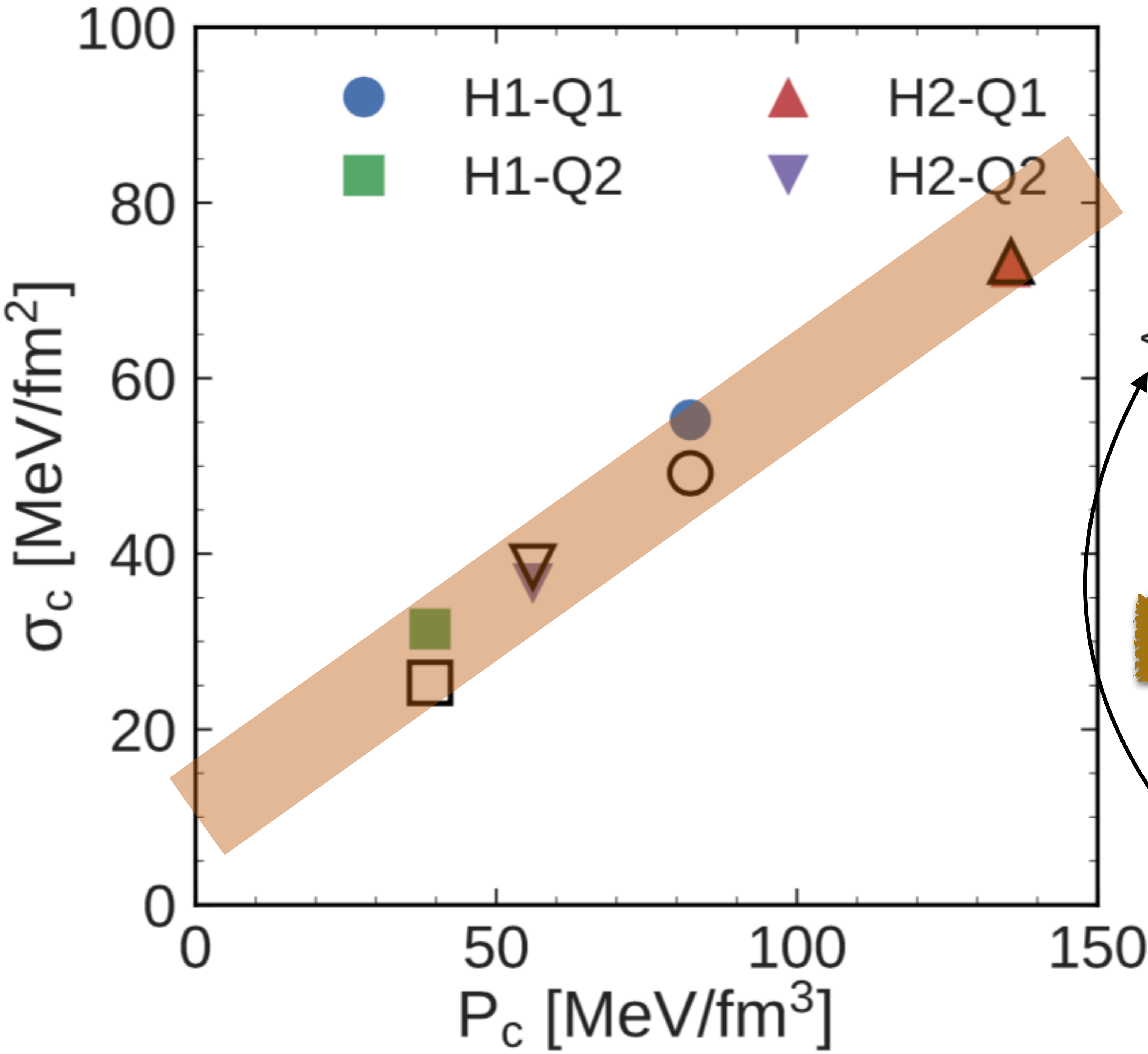
Using this formula, you can enjoy

- MR-relations,
- NS cooling,
- magnetar evolution, etc.

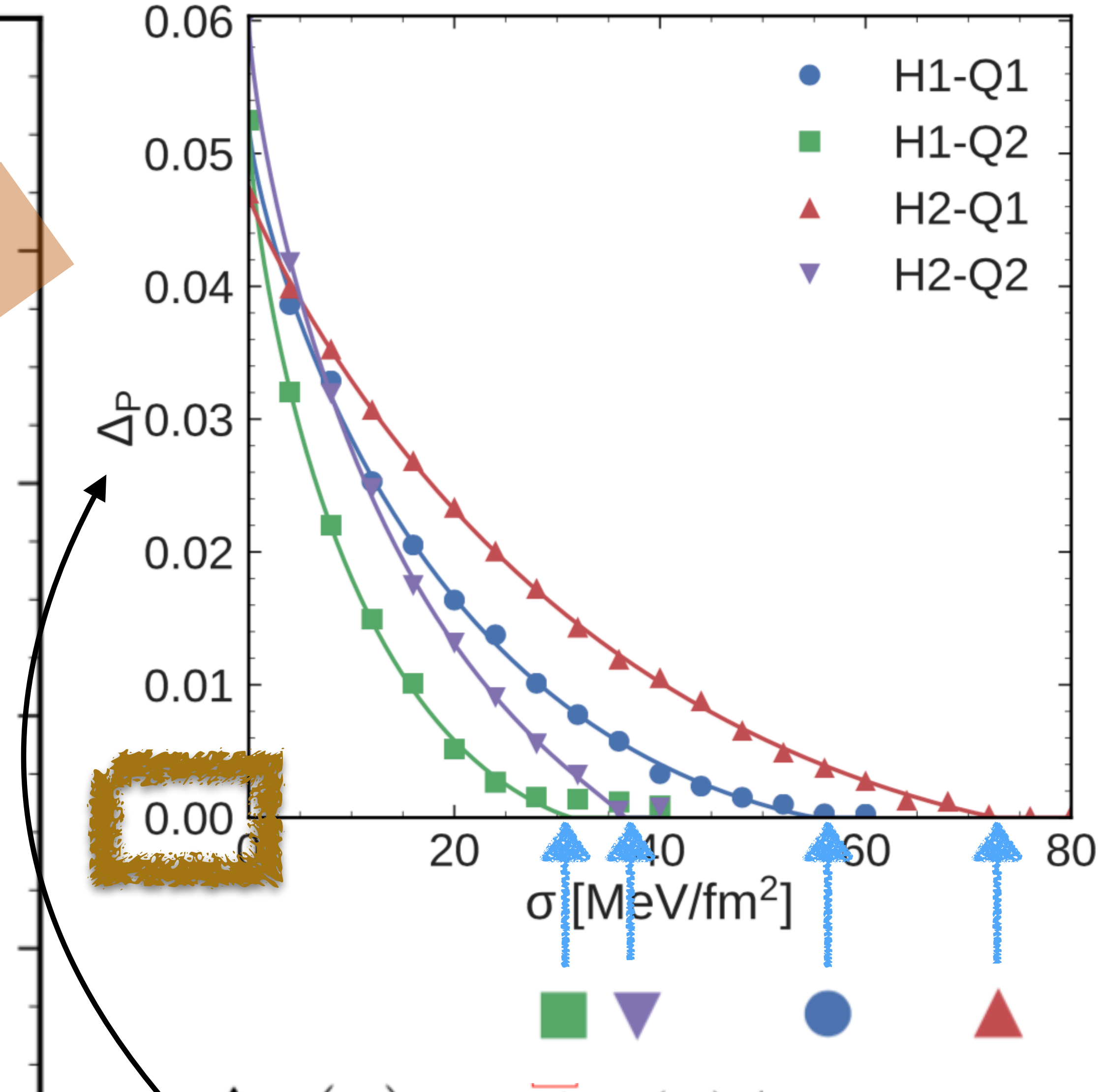
with ***finite-size effects* (pasta).**

$\sigma_C - P_C$ relation

Critical surface energy by full calculations of pasta



Onset pressure of Maxwell construction



$$\Delta_P(\sigma) = \Delta P(\sigma)/P_c$$

Pressure difference between **Pasta** and **Maxwell**

□▽○△ → Analytic estimation of critical surface energy (next slide)

Analytic estimation of critical surface energy

Voskresensky Yasuhira Tatsumi 2003 NPA

$$\widetilde{\sigma}_c = \lambda_D^{(Q)} \frac{\widetilde{\alpha}\widetilde{\beta}(\widetilde{\alpha}+4/3)}{3(1+\widetilde{\alpha})^2}, \quad \widetilde{\alpha} = \frac{\lambda_D^{(Q)}}{\lambda_D^{(H)}}, \quad \widetilde{\beta} = \frac{3(U_0^{\mathrm{II}} - U_0^{\mathrm{I}})^2}{8\pi e^2 (\lambda_D^{(Q)})^2},$$

Here, $$\left(\frac{1}{\lambda_D^{(p)}}\right)^2 = -4\pi e^2 \left(\frac{\partial \rho_{\mathrm{ch}}^{(p)}}{\partial \mu_e}\right)_{\mu_B} \qquad (p) = Q, H$$

$$U_0^{\mathrm{I}} = -4\pi e^2 (\lambda_D^{(Q)})^2 n_{\mathrm{ch}}^{(Q)} (\mu_B = \mu_c, \mu_e = 0)$$

$$U_0^{\mathrm{II}} \simeq -\mu_{e,\mathrm{bulk}}^{(H)}$$

$\sigma_C - P_C$ relation can be roughly estimated without pasta calculations.

What can we know from $\sigma_C - P_C$ relation ?

① EOS tables with pasta

From Maxwell construction(without pasta), we can estimate σ_C.

We do not need to prepare EOS tables over σ_C.

② Observations

If they suggest sharp density jump of EOS (1st order PT),

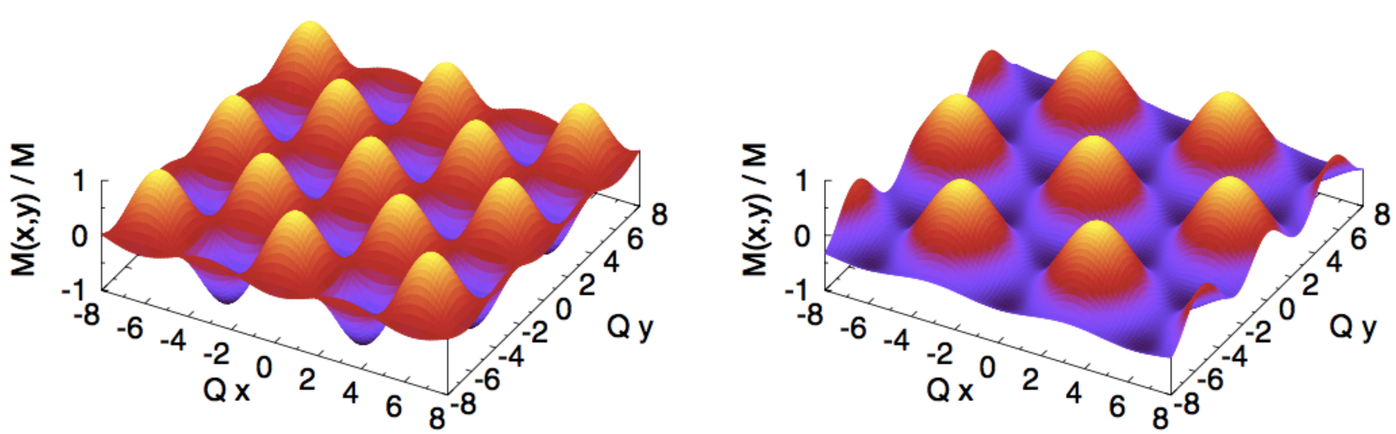
$\sigma_S > \sigma_C$ else then, $\sigma_S < \sigma_C$ (or Cross over?).

$\sigma_S = \sigma_{Strong} + \sigma_D$

- Condensations can have spatial dependency, generally.

$$\langle \bar{\psi}\psi \rangle = S(\mathbf{x}), \qquad \langle \bar{\psi} i\gamma^5 \tau_3 \psi \rangle = P(\mathbf{x})$$

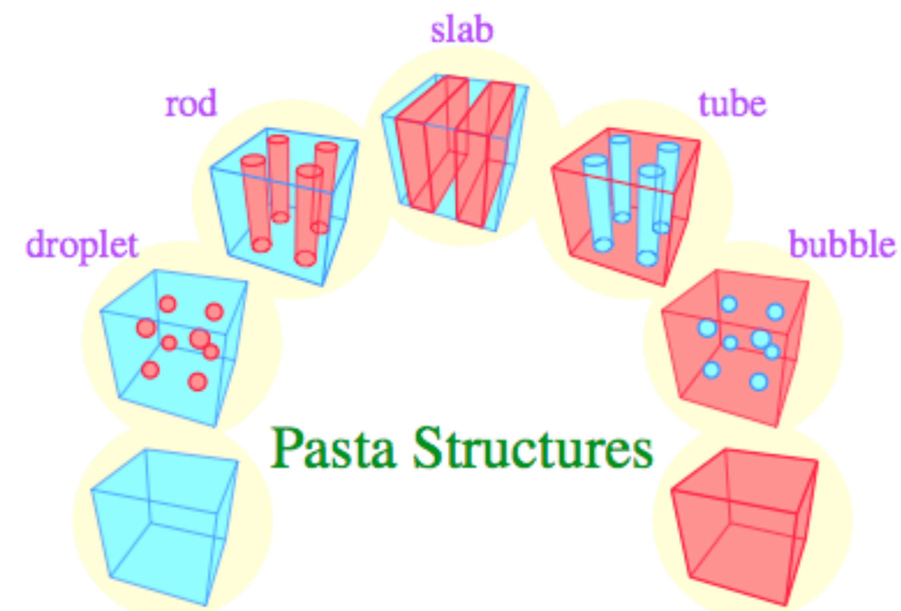
- Beyond 1+1D, we need numerical approaches.
- SU(3)? Moreira et al.(2014) etc.
- Magnetic filed ? Yoshiike, Nishiyama, Tatsumi (2015)etc.
- Iso-spin ? Thies(2016)etc.



$$M(x, y) = M\cos(Qx)\cos(Qy), \qquad M(x, y) = \frac{M}{3}\left[2\cos(Qx)\cos\left(\frac{1}{\sqrt{3}}Qy\right) + \cos\left(\frac{2}{\sqrt{3}}Qy\right)\right],$$

Carignano & Buballa (2012) PRD

? +



NY, Maruyama, Tatsumi (2009) PRD

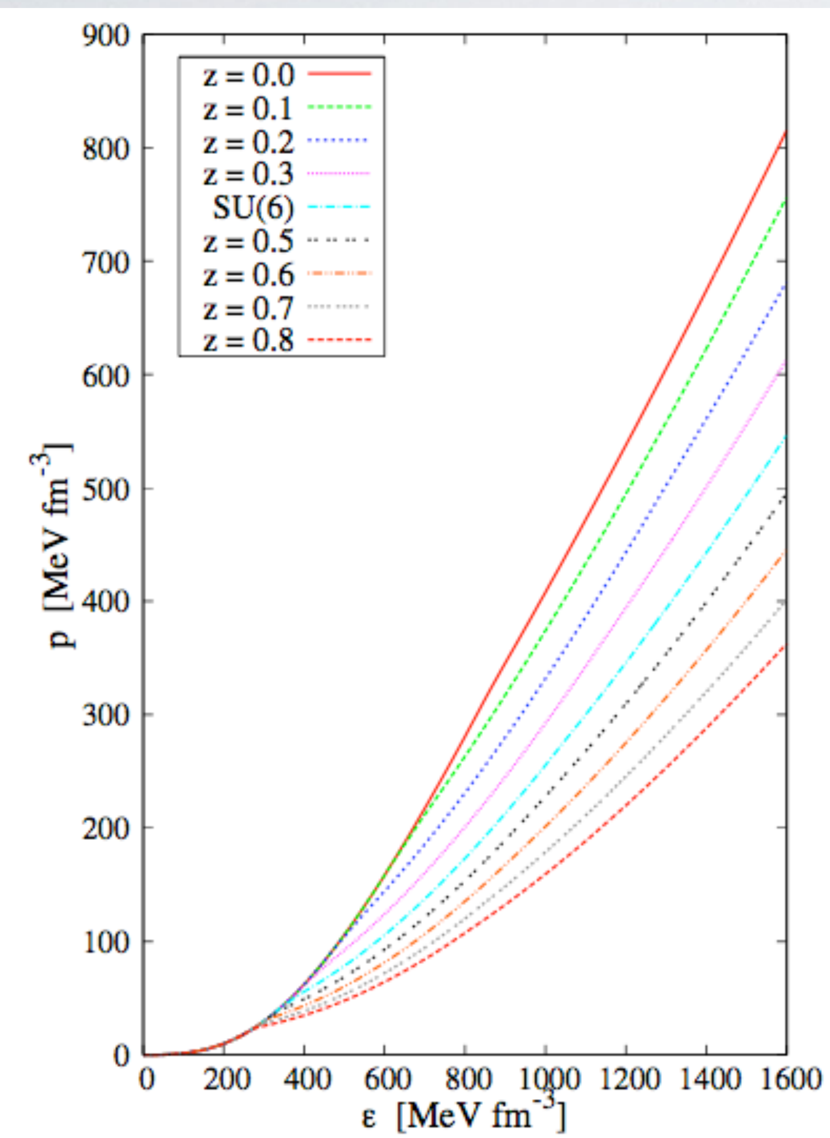
NY, Lastwieski, Sanjin, Blaschke, Maruyama, Tatsumi (2014) PRC

Maslov, **NY**, Ayriyan, Grigorian, Blaschke, Voskresensky, Maruyama, Tatsumi, arXiv: 1812.11889

How to distinguish EOSs without density jumps?

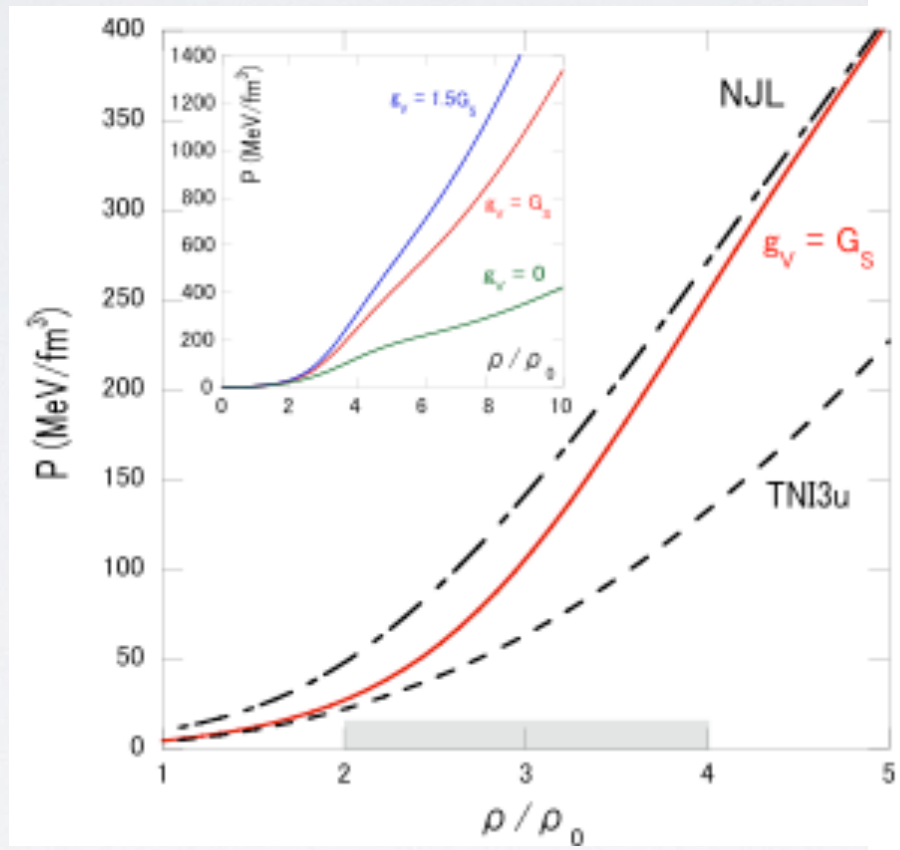
Hyperon matter

S. Weissenborn, et al.
PRC 85, 065802 (2012)



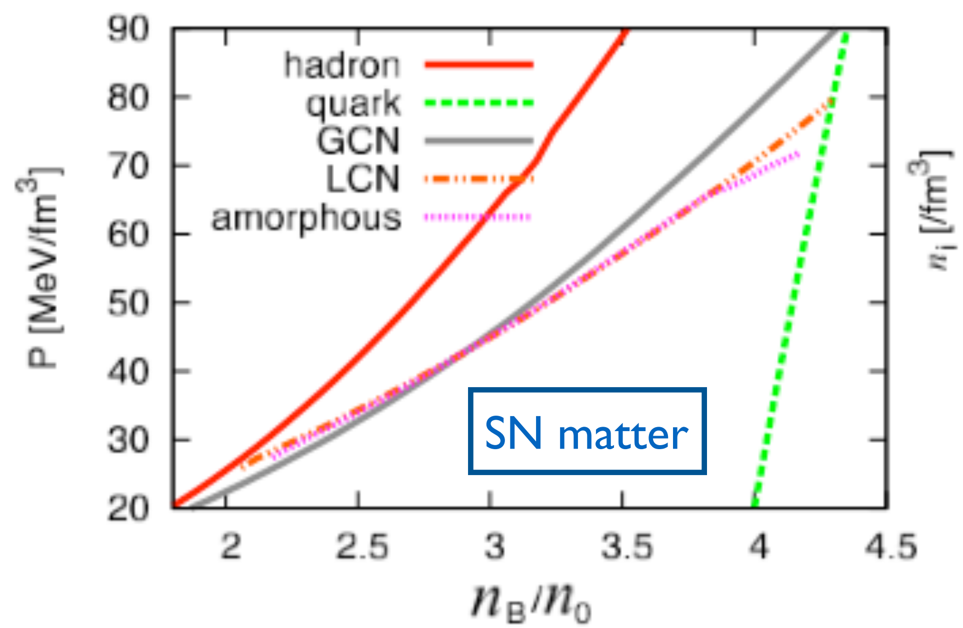
Hyperon + Quark (cross over)

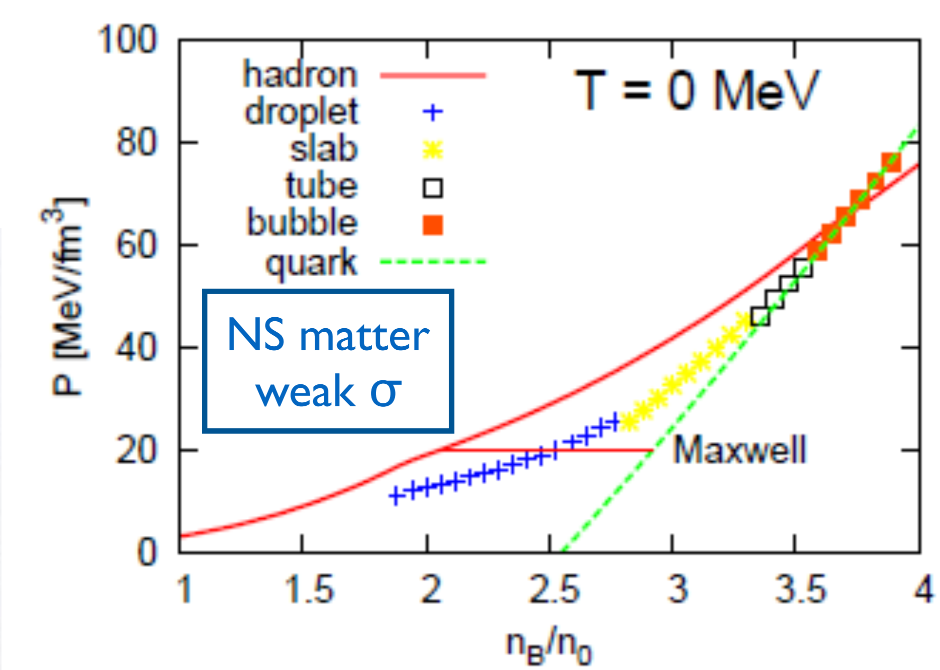
Masuda, Hatsuda, Takatsuka PRD (2013)



Hadron + Quark (pasta)

NY, Maruyama, Tatsumi, PRD 2009a, 2012





Cooling of Neutron Stars

Cooling has many physical properties

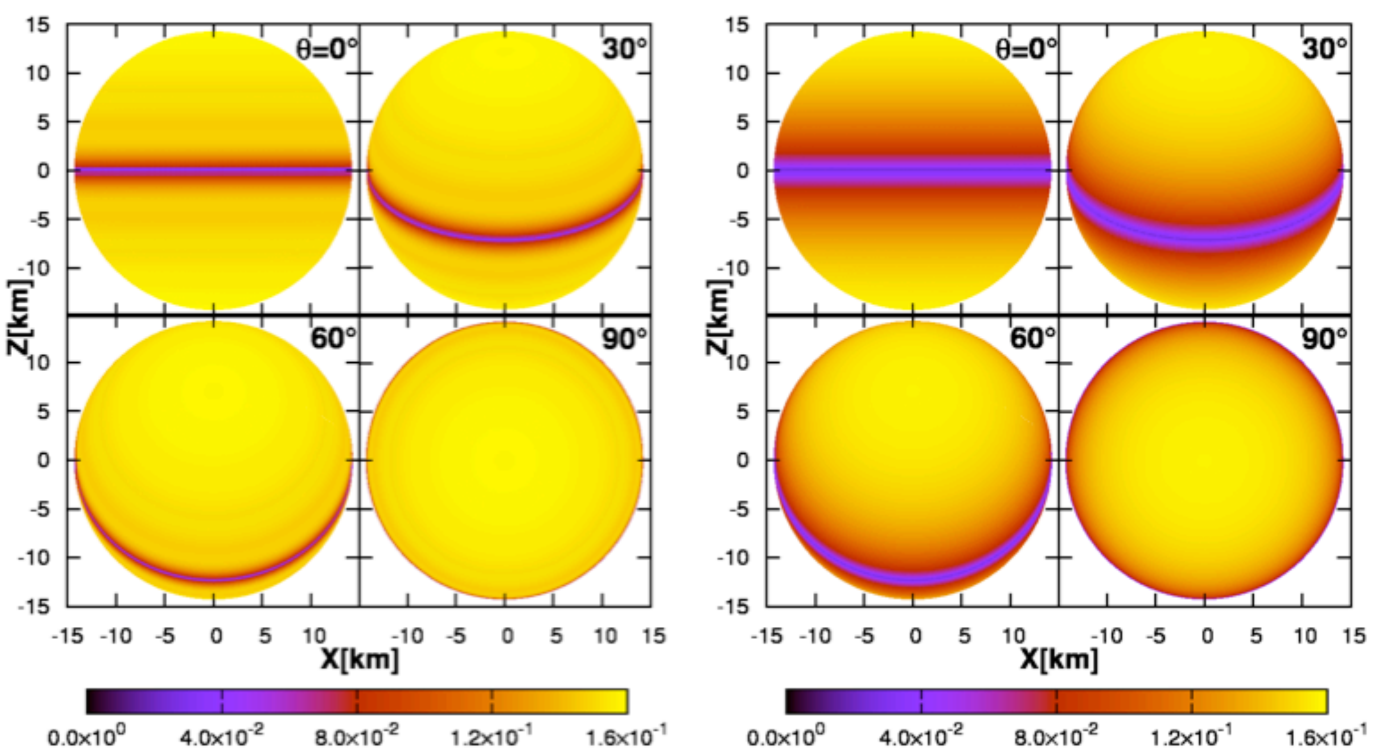
$$c_v e^{\Phi} \frac{\partial T}{\partial t} + \nabla \cdot (e^{2\Phi} F) = e^{2\Phi} Q$$

thermal diffusion eq.

heat capacity

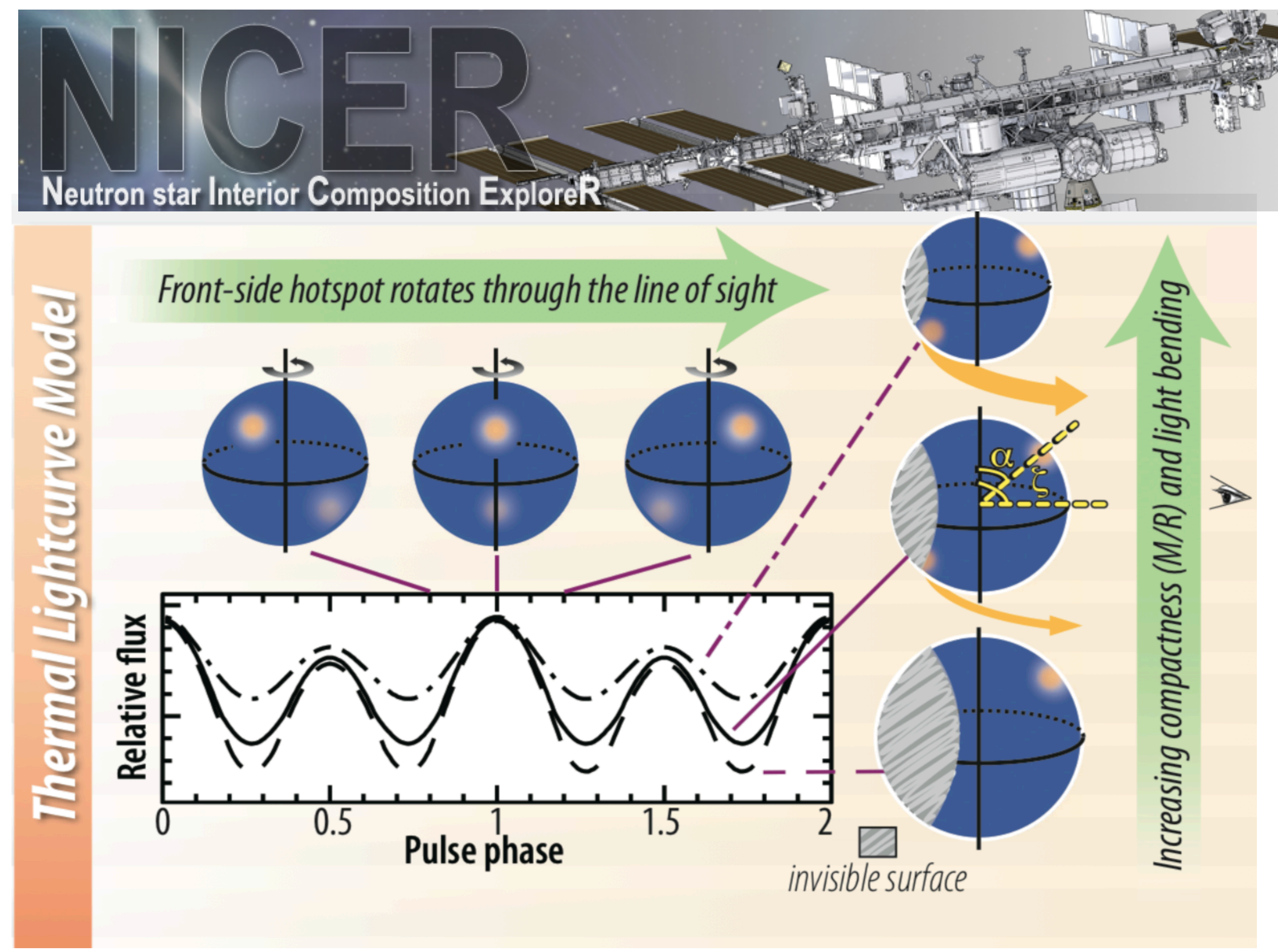
flux (thermal conductivity) (magnetic field)

cooling rate (neutrino) + heating rate (magnetic field)



G. 7: (Color online) Temperature distribution for model "mSUK" after 10^4 years depended on the inclination angle θ . The it of color contour is [keV].

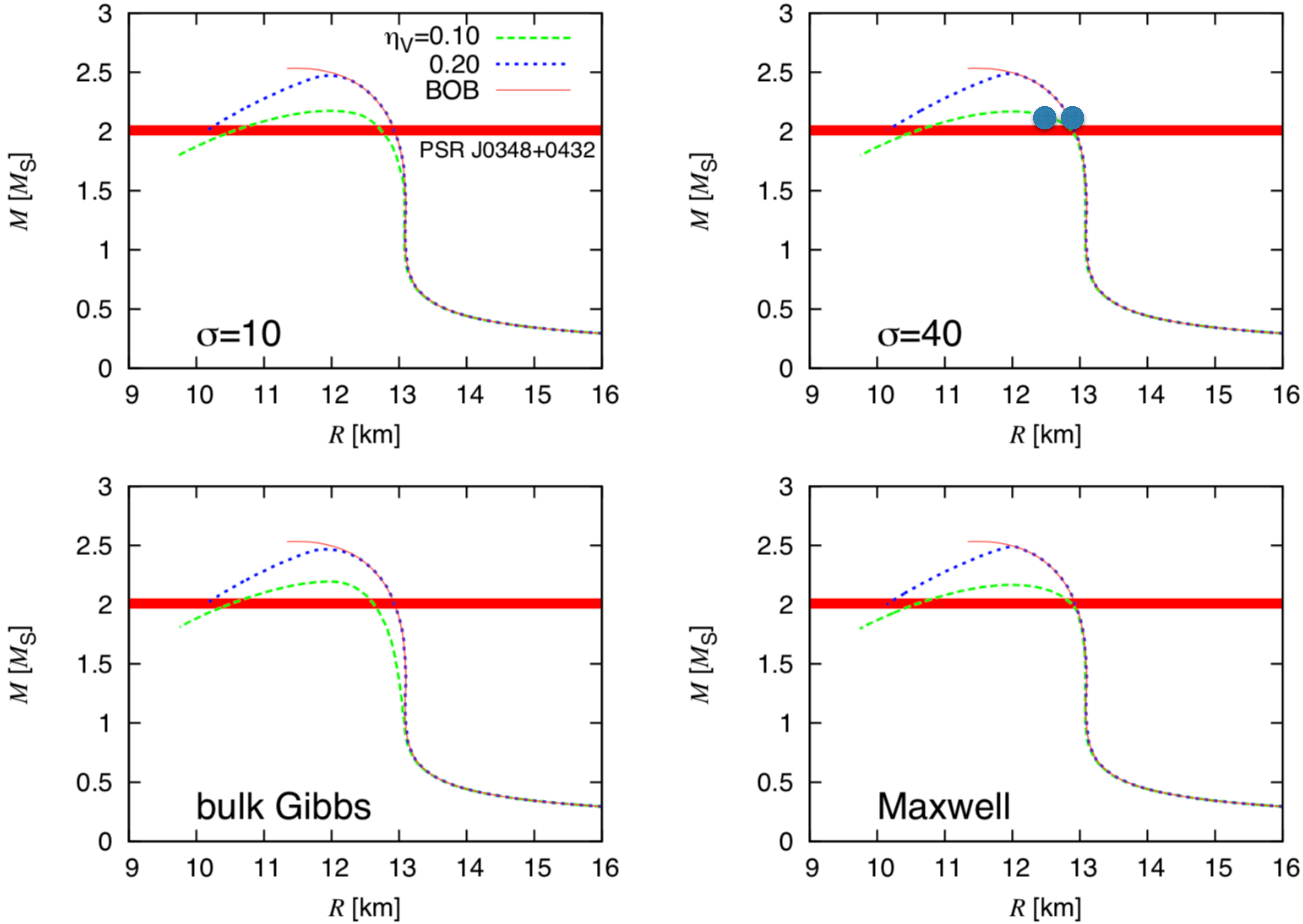
NY, Kotake, Kutsuna,Shigeyama (2014) PASJ



M-R relation with Q-H pasta

Non-local NJL(Nf=2) + BHF(BOB+TBF)

NY, Lastowiecki, Benic, Blaschke, Maruyama, Tatsumi(2014) PRC



Trial calculations of NS cooling(2D) with Q-H pasta

Non-local NJL(Nf=2) + BHF(BOB+TBF)

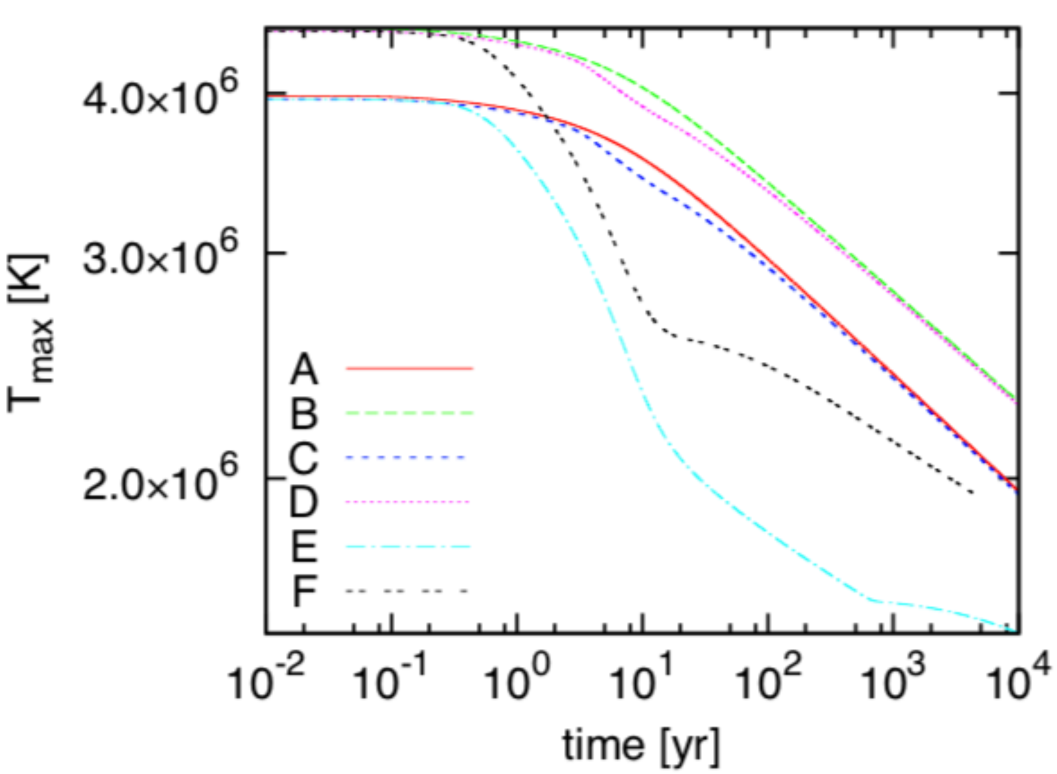
NY, Lastowiecki, Benic, Blaschke, Maruyama, Tatsumi(2014) PRC

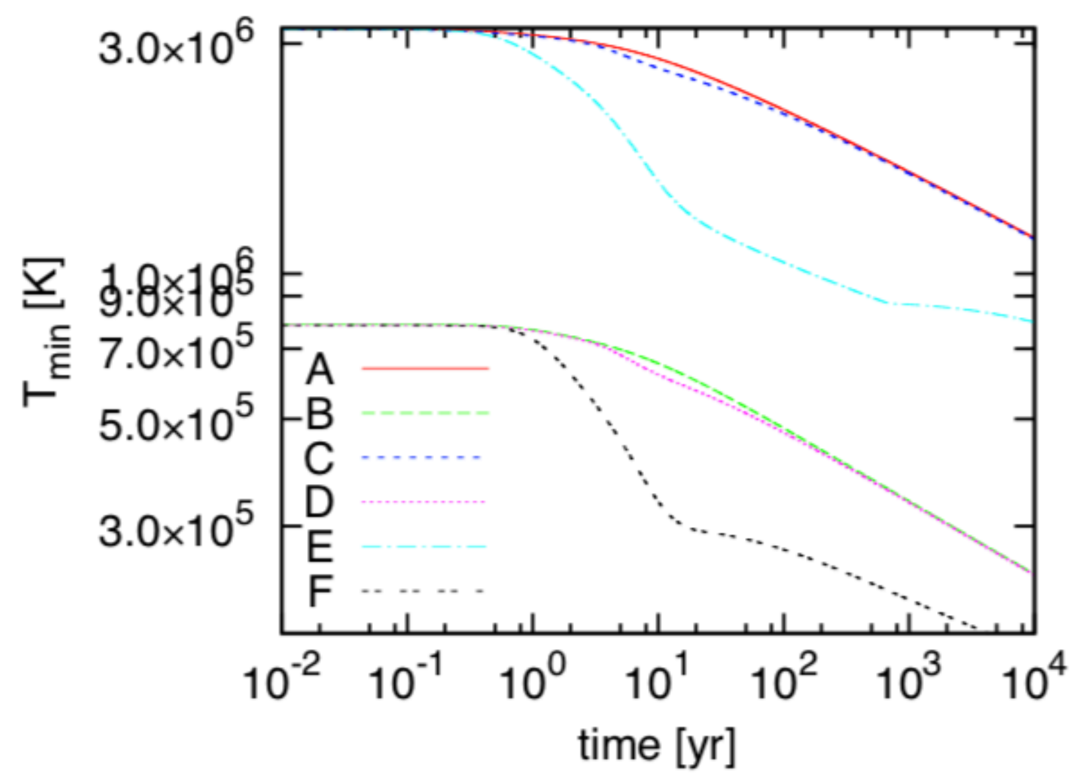
Model	EOS	B_p[G]	B_{max} [G]	ρ_c [10^{15} g cm^{-3}]	Δ[MeV]
A	HM	8.2×10^{11}	3.0×10^{12}	1.0	-
B	HM	8.9×10^{13}	3.3×10^{14}	1.0	-
C	HM+QM	8.2×10^{11}	3.0×10^{12}	1.1	0.1
D	HM+QM	8.9×10^{13}	3.3×10^{14}	1.1	0.1
E	HM+QM	8.2×10^{11}	3.0×10^{12}	1.1	0.5
F	HM+QM	8.9×10^{13}	3.3×10^{14}	1.1	0.5

2SC Gap

D. N. Aguilera, D. Blaschke, and H. Grigorian, A&A 416 (2004) 991-996.

H. Grigorian, D. Blaschke, D. Voskresensky Phys.Rev. C71 (2005) 045801





Summary

- We have to take into account inhomogeneous phase transition to know the realistic phase diagram.
- Gravitational wave and cooling (Xray observation) will give good constraints for EOSs.
- From $\sigma_C - P_C$ relation, we can get some constraints of surface energy.
- Non-local effects of strong interaction ?

謝謝