QCD Phase Transitions & one of their Astronomical Observable

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Outline

Ⅰ. Introduction

Ⅱ. Theoretical View of the PTs

Ⅲ. Observables of the PTs

Ⅳ. GW… in newly born NSs

Ⅴ. Remarks

CUSTIPEN Workshop on the EOS of Dense Neutron-Rich Matter in the Era of GWA, Xiamen, 03-07/01/2019

I. Introduction

Physics: to reveal the origin of matter & mass, etc.

$M_N \cong 939$ MeV,

$M_{u,d} >= 100 m_{u,d}$,

DCSB ! How?

Why & How Confined ?

Quarks

Non-confined quarks and gluons and leptons

Nuclear Synthesis → Nuclei

Comp. → Atoms

Galaxy Formed

Present Universe

Chiral Sym.

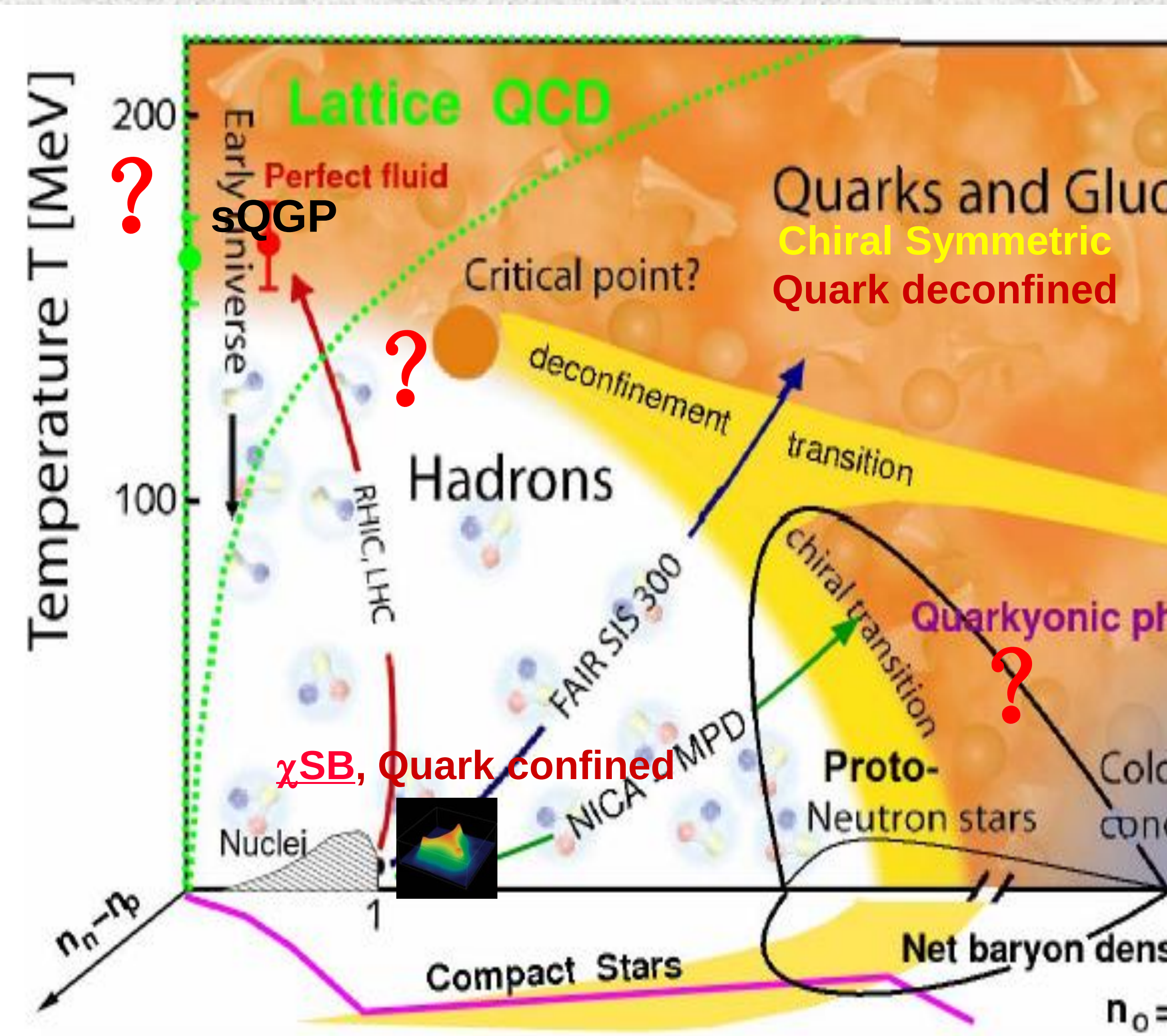
$m_q = 0$ (FS)

Explicit CSB

$m_{u,d} \cong 3\text{-}6$ MeV

I. Introduction

Strong int. matter evolution in early universe can be attributed to QCD Phase Transitions



Phase Transitions involved:

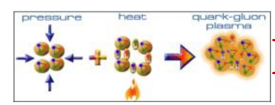
Flavor Sym. – F.S. Breaking

Deconfinement–confinement

DCS – DCSB

Items Influencing the Phase Transitions:

Medium: Temperature T ,



Density ρ (or μ)

Size

Intrinsic: Current mass,

Coupling Strength,

Color-flavor structu

••• •••

♠ Points commonly concerned

- Relation between the chiral phase transition and the confinement;
- Existence and location of the CEP;
- Characteristic of the matter at the T above but near the $T_{\chi c}$;
- Observables ; ••• •••
- Approaches should be nonperturbative QCD ones involving simultaneously the charters of the DCSB & its Restoration , the Confinement & Deconfinement ; since the aspects appear at NP QCD scale (10^2 MeV) .

Ⅱ. Theoretical View of the Phase Trans.

1. Overview of the theoretical Approaches

Theory The Frontiers of Nuclear Science — A LONG RANGE PLAN — December 2007

The primary goal of the RHIC scientific program in the coming years is to progress from qualitative statements to rigorous quantitative conclusions. Quantitative conclusions require sophisticated modeling of relativistic heavy-ion collisions and rigorous comparison of such models with data of greater precision and extended reach. A successful quantitative interpretation of the heavy-ion data will require close collaboration of the experimental data analysis with the theoretical modeling effort. Without such an effort, the

Thus, an essential requirement for the field as a whole is strong support for the ongoing theoretical studies of QCD matter, including finite temperature and finite baryon density lattice QCD studies and phenomenological modeling, and an increase of funding to support new initiatives enabled by experimental and theoretical breakthroughs. The success of this effort mandates significant additional investment in theoretical resources in terms of focused collaborative initiatives, both programmatic and community oriented.

♠ **Discrete FT (L-QCD):**

Running coupling behavior,
Vacuum Structure,
Temperature effect,
"**Small chemical potential**";
…

♠ **Continuum FT:**

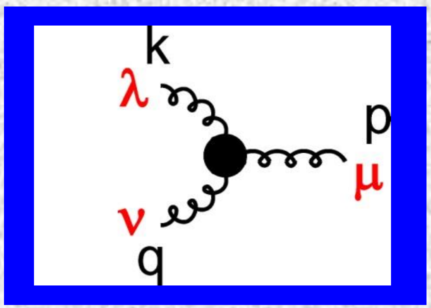
(1) **Phenomenological models**
(p)NJL、(p)QMC、QMF、

(2) **Field Theoretical**
Chiral perturbation,
QCD sum rules,
Instanton(liquid) model,
Functional Renomlt. Group
DS equations ,
AdS/CFT,
HD(T)LpQCD ,
…

2. Dyson-Schwinger Equations — A Nonperturbative QCD Approach

(1) Outline of the DS Equations

Slavnov-Taylor Identity

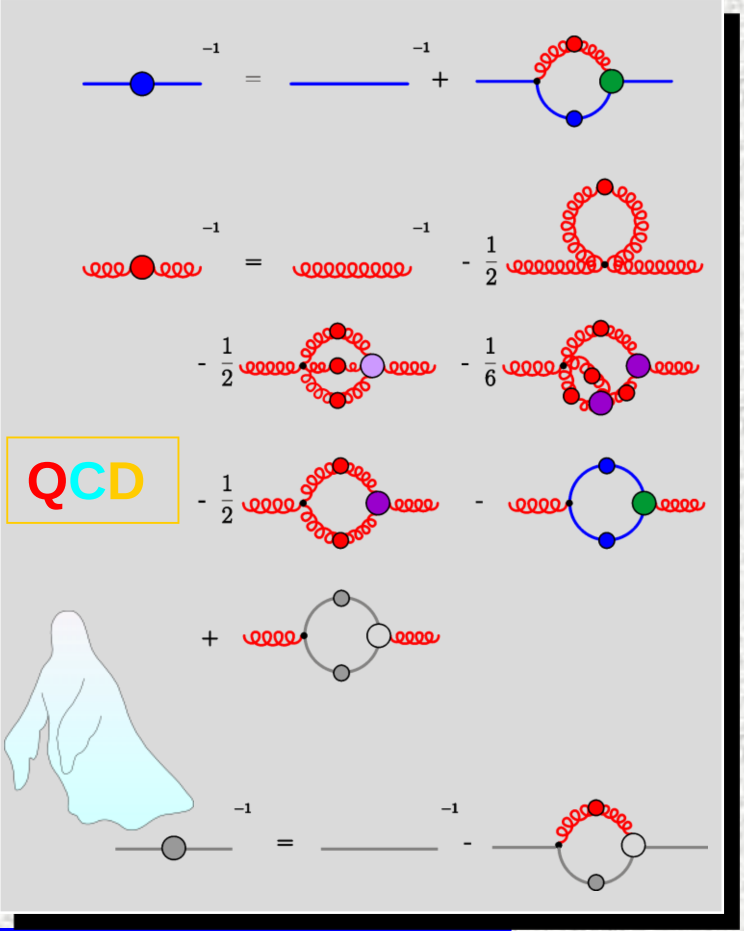


axial gauges **BBZ**

$$k_\lambda \, \Gamma^{\lambda\mu\nu}(k,p,q) = \Pi^{\mu\nu}(p) - \Pi^{\mu\nu}(q)$$

covariant gauges

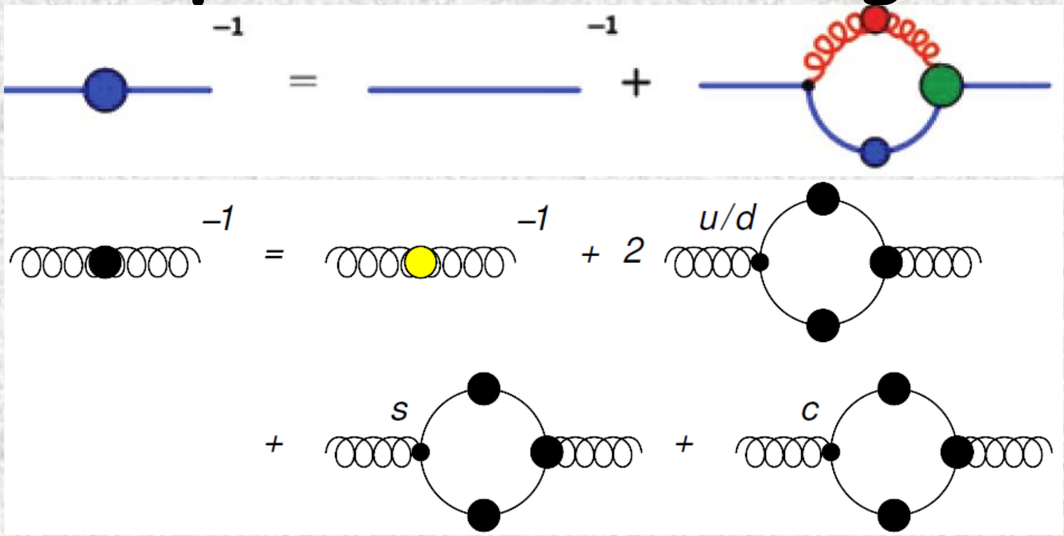
$$k_\lambda \, \Gamma^{\lambda\mu\nu}(k,p,q) = H(k^2) \, [G_{\mu,\sigma}(q,-k) \, \Pi^{T}_{\sigma,\nu}(p) - G_{\nu\sigma}(p,-k) \, \Pi^{T}_{\sigma\mu}(q)]$$



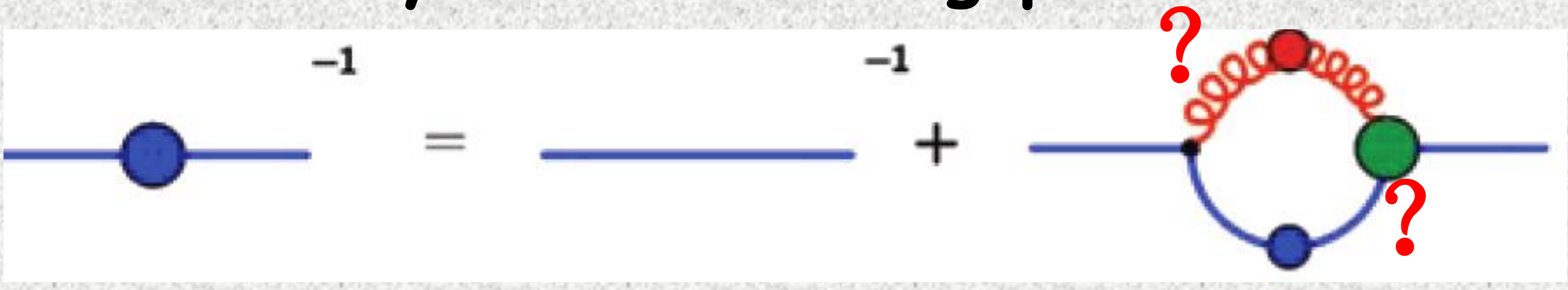
C. D. Roberts, et al, PPNP 33 (1994), 477; 45-S1, 1 (2000); EPJ-ST 140(2007), 53;
R. Alkofer, et. al, Phys. Rep. 353, 281 (2001); LYX, Roberts, et al., CTP 58 (2012), 79; ·

♠ Algorithms of Solving the DSEs of QCD

- **Solving the coupled** quark, ghost and gluon **equations** (parts of the diagrams) :



- **Solving the truncated quark equation** with the symmetries being preserved.



✦ Expression of the quark gap equation

- Truncation: Preserving Symm. ➔ Quark Eq.

$$S^{-1}(p) = Z_2(-i\not p + Z_m m) + Z_1 g^2 \int \frac{d^4q}{(2\pi)^4} [t^a \gamma_\mu S(q) \Gamma^b_\nu(p,q) D^{ab}_{\mu\nu}(p-q)]$$

- Decomposition of the Lorentz Structure
  - Quark Eq. in Vacuum :

$$S^{-1}(p) = i\not p A(p^2, \Lambda^2) + B(p^2, \Lambda^2)$$

➔

$$\begin{cases} A(x) = 1 + \dfrac{1}{6\pi^3} \displaystyle\int dy \dfrac{yA(y)}{yA^2(y) + B^2(y)} \Theta_A(x,y) \\ B(y) = \dfrac{1}{2\pi^3} \displaystyle\int dy \dfrac{yB(y)}{yA^2(y) + B^2(y)} \Theta_B(x,y) \end{cases}$$

• Quark Eq. in Medium

Matsubara Formalism

Temperature T : ➔ Matsubara Frequency

$$\omega_n = (2n+1)\pi T$$

Density ρ : ➔ Chemical Potential μ

$$S^{-1}(p) \Longrightarrow S^{-1}(p, \omega_n, \mu)$$

Decomposition of the Lorentz Structure

$$S^{-1}(p) = i\gamma \cdot p A(p^2) + B(p^2) , \quad \Longrightarrow$$

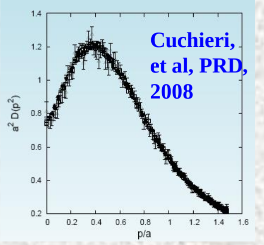
$$S^{-1}(p, \omega_n, \mu) = iA(p, \omega_n, \mu)\vec{\gamma} \cdot \vec{p} + iC(p, \mu)\gamma_4(\omega_n + i\mu) + B(\tilde{p}) + \cdots$$

✦ Models of the eff. gluon propagator

$$g^2 D_{\rho\sigma}(k) = 4\pi \frac{\mathcal{G}(k^2)}{k^2} \left(\delta_{\rho\sigma} - \frac{k_\rho k_\sigma}{k^2}\right)$$

- **Commonly Used: Maris-Tandy Model (PRC 56, 3369)**

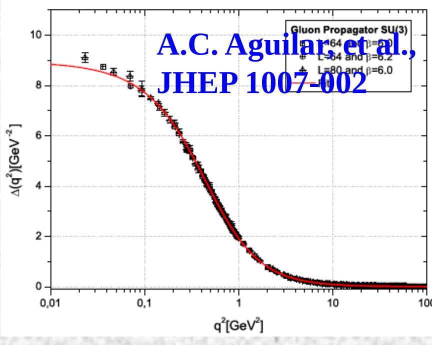
$$\frac{\mathcal{G}(t)}{t} = \frac{4\pi^2}{\omega^6} D\, t e^{-t/\omega^2} + \frac{8\pi^2 \gamma_m}{\ln\left[\tau + \left(1 + t/\Lambda_{QCD}^2\right)^2\right]} \frac{1 - \exp(-t/[4m_{\mathcal{F}}^2])}{t} \quad (3)$$



- **Recently Proposed: Infrared Constant Model**

(Qin, Chang, Liu, Roberts, Wilson, Phys. Rev. C 84, 042202(R), (2011).)

Taking $t/\omega^2 = k^2/\omega^2 = 1$ in the coefficient of the above expression



- Derivation and analysis in PRD 87, 085039 (2013) show that **the one in 4-D should be infrared constant.**

✦ Models of quark-gluon interaction vertex

$$\Gamma^a_\mu(q,p) = t^a \Gamma_\mu(q,p)$$

- **Bare Ansatz**

$$\Gamma_\mu(q,p) = \gamma_\mu$$ (Rainbow Approx.)

- **Ball-Chiu (BC) Ansatz**

$$\Gamma^{BC}_\mu(p,q) = \frac{A(p^2)+A(q^2)}{2}\gamma_\mu + \frac{(p+q)_\mu}{p^2-q^2}\{[A(p^2)-A(q^2)]\frac{(\gamma\cdot p+\gamma\cdot q)}{2} - i[B(p^2)-B(q^2)]\}$$

Satisfying W-T Identity, L-C. restricted

- **Curtis-Pennington (CP) Ansatz**

$$\Gamma^{CP}_\mu(p,q) = \Gamma^{BC}_\mu(p,q) + \frac{1}{2}(A(p^2)-A(q^2))\frac{\gamma_\mu(p^2-q^2)-(k+p)_\mu\gamma\cdot(p+q)}{d(p,q)},$$

$$d(p,q) = \frac{(p^2-q^2)^2+[M^2(p^2)+M^2(q^2)]^2}{p^2+q^2}。$$

Satisfying Prod. Ren.

- **CLR** (BC+ACM, Chang, etc, PRL 106,072001('11), Qin, etc, PLB 722,384('13))

$$\Gamma^{\mathrm{acm}}_\mu(p_f,p_i) = \Gamma^{\mathrm{acm}_4}_\mu(p_f,p_i) + \Gamma^{\mathrm{acm}_5}_\mu(p_f,p_i),$$

♠ A theoretical check on the CLR model for the quark-gluon interaction vertex

Physics Letters B 742 (2015) 183–188

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Bridging a gap between continuum-QCD and *ab initio* predictions of hadron observables

CrossMark

Daniele Binosi [a], Lei Chang [b], Joannis Papavassiliou [c], Craig D. Roberts [d,*]

$$\mathcal{I}_{\hat{d}}(k^2) := k^2 d(k^2) = \frac{\alpha_s(\zeta^2)\Delta(k^2;\zeta^2)}{[1+G^2(k^2;\zeta^2)]^2}$$

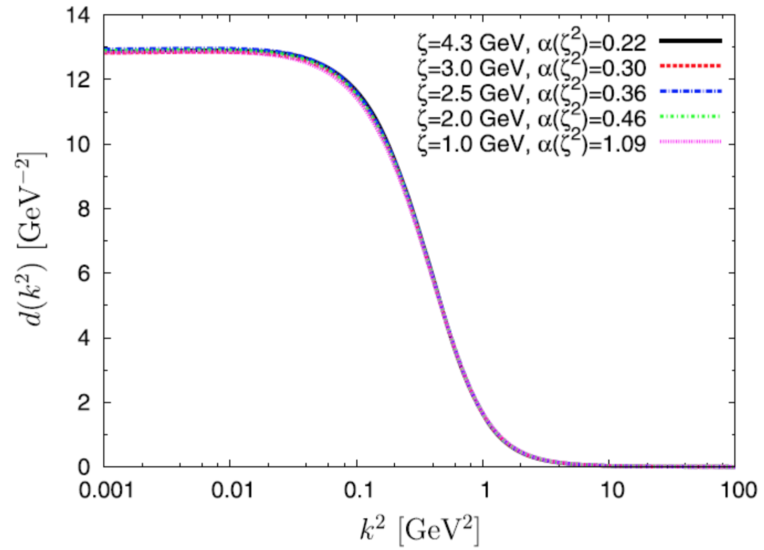


Fig. 1. RGI running interaction strength, $d(k^2)$ in Eq. (19), computed via a combination of DSE- and lattice-QCD results, as explained in Ref. [25]. We display the

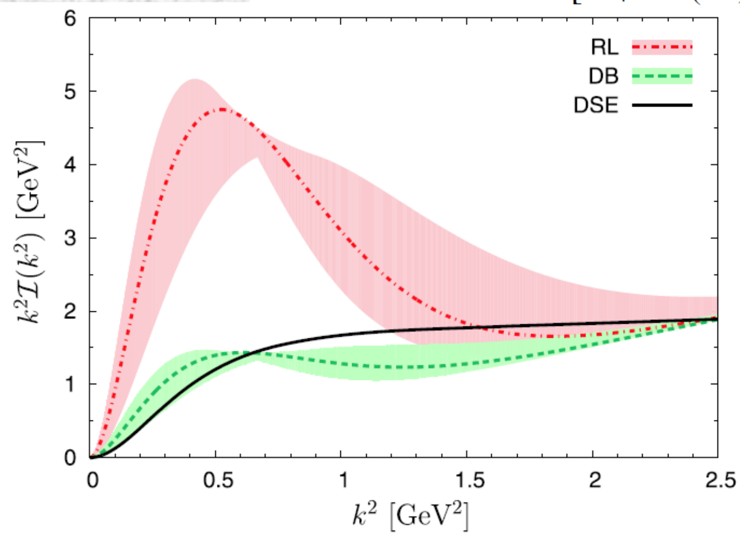


Fig. 2. Comparison between top-down results for the gauge-sector interaction [Eqs. (19), (22), Fig. 1] with those obtained using the bottom-up approach based on hadron physics observables [Eqs. (4)–(8)]. *Solid curve* – top-down result for the

(2) DSE meets the requirements for an approach to describe the QCD PTs

♠ Dynamical chiral symmetry breaking

$$\mathcal{M}(p) \simeq m_0 \left[\ln p/\Lambda_{QCD}\right]^d + C \frac{-\langle \bar{q}q \rangle}{p^2 \left[\ln p/\Lambda_{QCD}\right]^d}$$

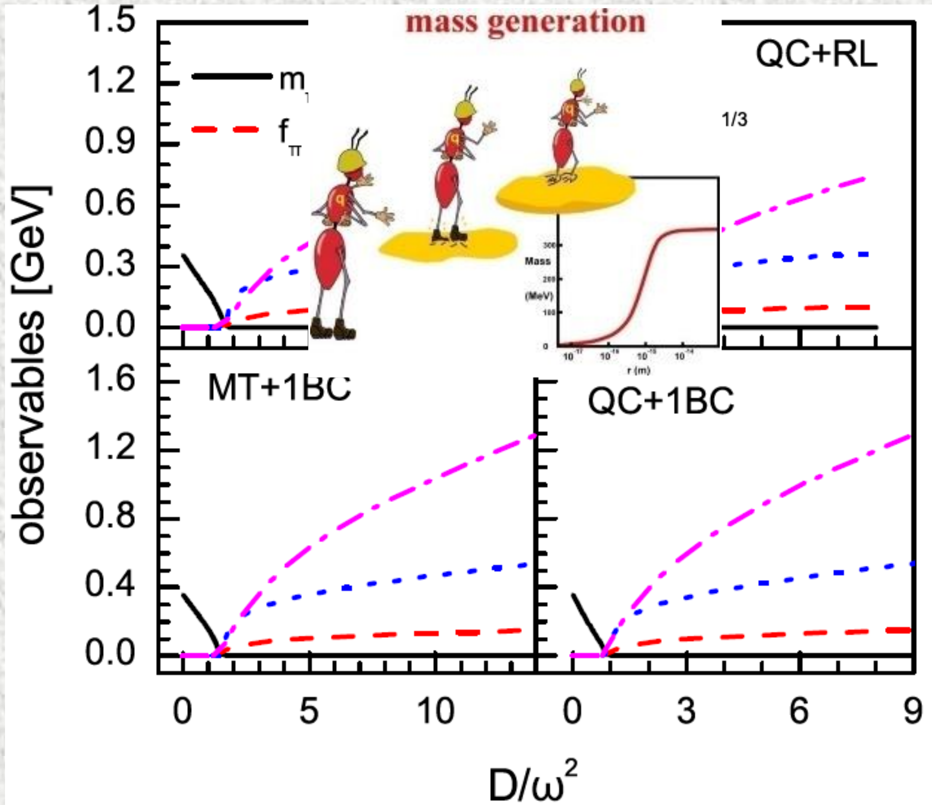
$\langle \bar{q} q \rangle \neq 0$ ➡ **DCSB**

In DSE approach

$$M(p^2) = \frac{B(p^2)}{A(p^2)}$$

Numerical results in chiral limit

➔ **Increasing the interaction strength induces the dynamical mass generation**

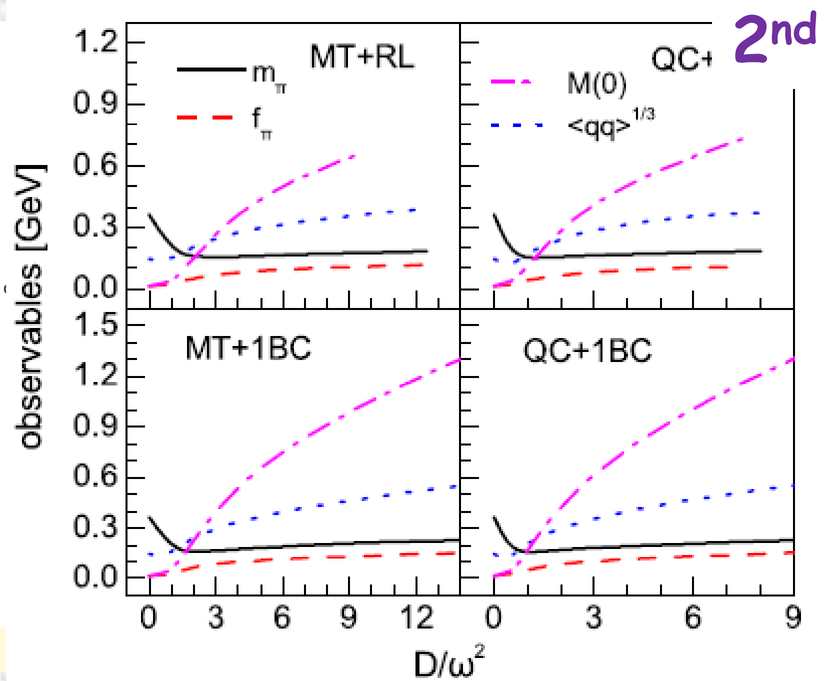


K.L. Wang, YXL, et al., PRD 86,114001('12);

✦ Dynamical Chiral Symmetry Breaking (DCSB) still exists beyond chiral limit

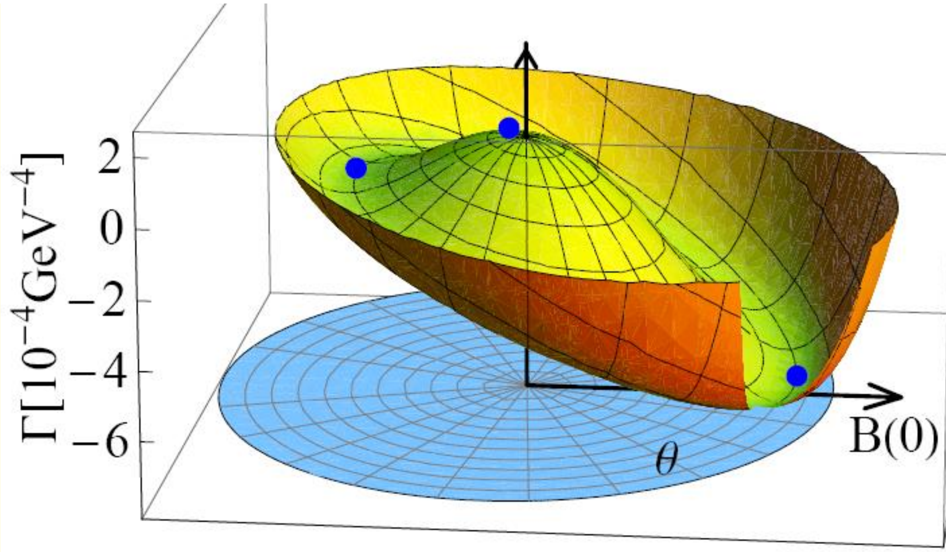
L. Chang, Y. X. Liu, C. D. Roberts, et al, arXiv: nucl-th/0605058;
R. Williams, C.S. Fischer, M.R. Pennington, arXiv: hep-ph/0612061;
K. L. Wang, Y. X. Liu, & C. D. Roberts, Phys. Rev. D 86, 114001 (2012).

Solutions of the DSE with MT model and QC model for the effective gluon propagator and bare model and 1BC model for the quark-gluon interaction vertex :



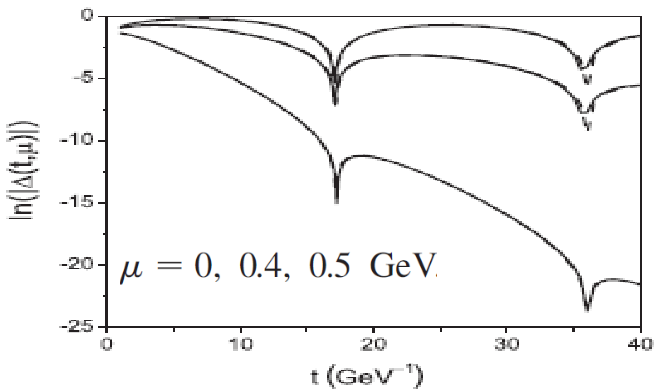
With ω = 0.4 GeV

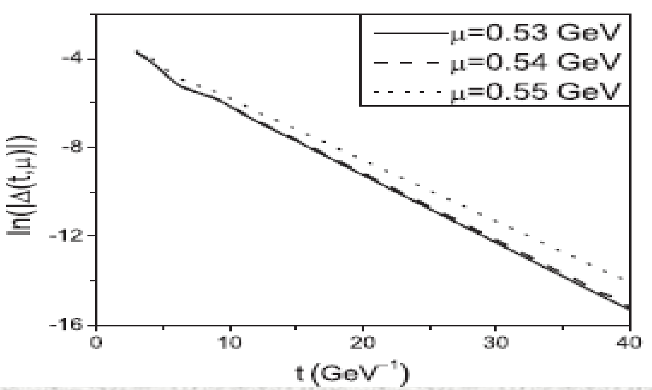
2nd order PT. shifts to crossover.



with $D = 16$ GeV^2, $\omega = 0.4$ GeV

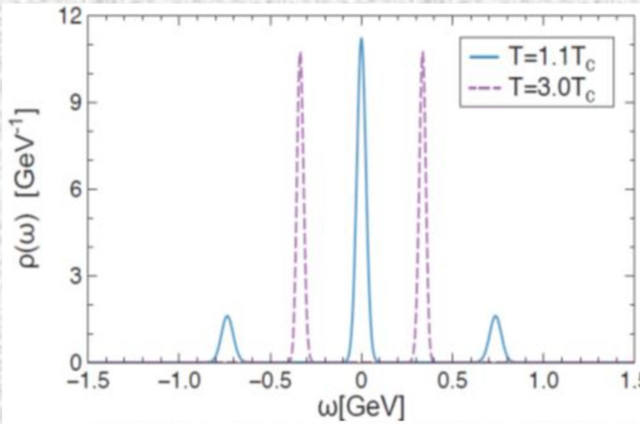
♠ Analyzing the spectral density function indicate that the quarks are confined at low temperature and low density

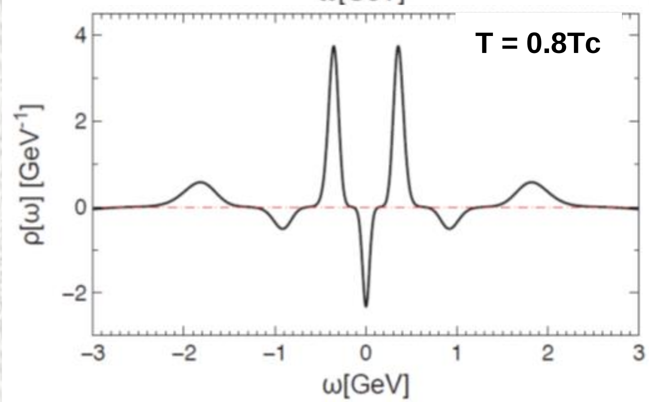




$$\Delta(\tau,\mu) = \int \frac{d^4p}{(2\pi)^4} e^{i\vec{p}\cdot\vec{x}+ip_4\tau}\delta(\vec{p})\sigma_B(p;\mu).$$

H. Chen, YXL, et al., Phys. Rev. D 78, 116015 (2008)





S.X. Qin, D. Rischke, Phys. Rev. D 88, 056007 (2013)

$$S^R(\omega,\vec{p}) = S(i\omega_n,\vec{p})|_{i\omega_n\to\omega+i\epsilon}$$

$$S(i\omega_n,\vec{p}) = \int_{-\infty}^{+\infty}\frac{d\omega'}{2\pi}\frac{\rho(\omega',\vec{p})}{i\omega_n-\omega'}$$

$$\rho(\omega,\vec{p}) = -i\vec{\gamma}\cdot\vec{p}\,\rho_v(\omega,\vec{p}^2) + \gamma_4\omega\,\rho_e(\omega,\vec{p}^2) + \rho_s(\omega,\vec{p}^2)$$

In MEM

$$P[\rho|M(\alpha)] = \frac{1}{Z_S}e^{\alpha S[\rho,m]},$$

$$S[\rho,m] = \int_{-\infty}^{+\infty} d\omega\left[\rho(\omega)-m(\omega)-\rho(\omega)\ln\frac{\rho(\omega)}{m(\omega)}\right]$$

$$m(\omega) = m_0\theta(\Lambda^2-\omega^2).$$

♠ Hadrons via DSE

♣ Approach 1: Soliton/bag model in DSE

$$E_B(T,\mu) = N_q \overline{\varepsilon_j} + \tfrac{4}{3}\pi R^3 B - \frac{Z_0}{R}$$

$$\overline{\varepsilon_j(T,\mu)} = g \sum_{j=0}^{\infty} \frac{\varepsilon_j(T,\mu)}{1+e^{\varepsilon_j/T}},$$

$$\varepsilon_j(T,\mu) = \frac{\kappa}{R(T,\mu)},$$

$$\begin{aligned}\mathscr{B}(T) &\equiv P[\mathcal{G}^{NG}] - P[\mathcal{G}^{W}] \\ &= 4N_c \sum_m \int \frac{d^3p}{(2\pi)^3} \left\{ ln\left[\frac{\Delta_{NG}}{\Delta_W}\right] \right. \\ &\quad \left. + \frac{\vec{p}^2 A_{NG} + \omega_m^2 C_{NG}}{\Delta_{NG}} - \frac{\vec{p}^2 A_W + \omega_m^2 C_W}{\Delta_W} \right\}\end{aligned}$$

♠ Hadrons via DSE

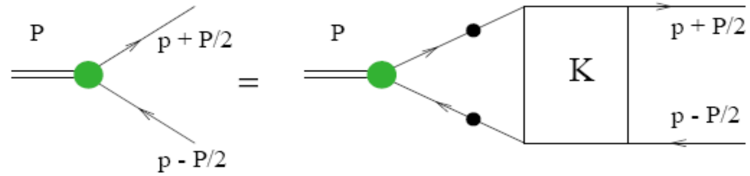
♣ Approach 2: BSE + DSE

- **Mesons**

  BSE with DSE solutions being the input

Quantum field theory bound states: BSE

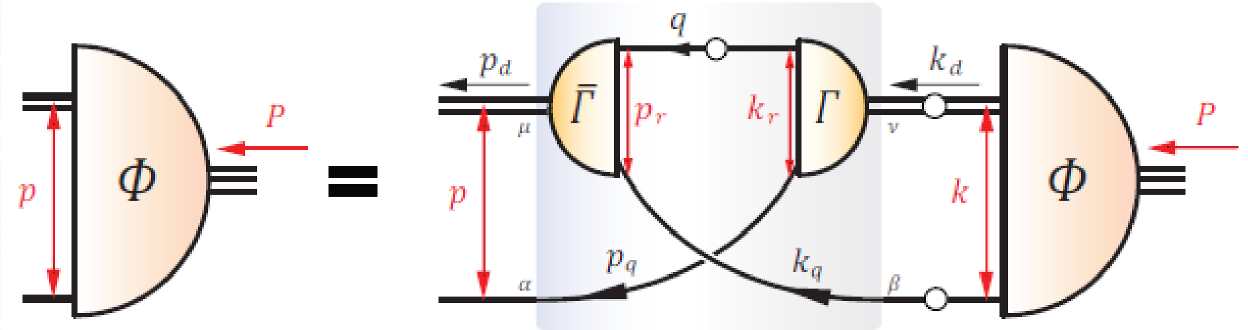
$$\Gamma_M(p;P) = \int_k^{\Lambda} K(p,k;P)\, S(k_+)\, \Gamma_M(k;P)\, S(k_-)$$



L. Chang, C.D. Roberts, PRL 103, 081601 (2009); ……

- **Baryons**

  Faddeev Equation or Diquark model (BSE+BSE)



G. Eichmann, et al., PRL 104, 201601 (2010); ……

✦ Some properties of mesons in DSE-BSE

Solving the 4-dimenssional covariant B-S equation with the kernel being fixed by the solution of DS equation and flavor symmetry breaking, we obtain

	Expt. (GeV)	Calc. (GeV)	Th/
"ρ^0"	0.7755	0.7704	
$\rho^\pm$	0.7755	0.7755	
"ω"	0.7827	0.7806	
$K^{*\pm}$	0.8917	0.8915	
K^{*0}	0.8960	0.8969	
ϕ	1.0195	1.0195	
D^{*0}	2.0067	1.8321	
$D^{*\pm}$	2.0100	1.8387	
$D_s^{*\pm}$	2.1120	1.9871	
J/ψ	3.0969	3.0969	
$B^{*\pm}$		4.8543	
B^{*0}		4.8613	
B_s^{*0}		5.0191	
$B_c^{*\pm}$		6.2047	
Υ	9.4603	9.4603	

	Expt. (GeV)	Calc. (GeV)	Th/Ex-1 (%)
π^0	0.13498	0.13460	-0.3
$\pi^\pm$	0.13957	0.13499	-3.3
$K^\pm$	0.49368	0.41703	-15.5
K^0	0.49765	0.42662	-14.3
η	0.54751	0.45499	-16.9
η'	0.95778	0.91960	-4.0
D^0	1.8645	1.6195	-13.1
$D^\pm$	1.8693	1.6270	-13.0
$D_s^\pm$	1.9682	1.7938	-8.9
η_c	2.9804	3.0171	1.2
$B^\pm$	5.2790	4.7747	-9.6
B^0	5.2794	4.7819	-9.4
B_s^0	5.3675	4.9430	-7.9
$B_c^\pm$	6.286	6.1505	-2.2
η_b	9.300	9.4438	1.5

(L. Chang, Y. X. Liu, C. D. Roberts, et al., Phys. Rev. C 76, 045203 (2007))

✦ Some properties of mesons in DSE-BSE

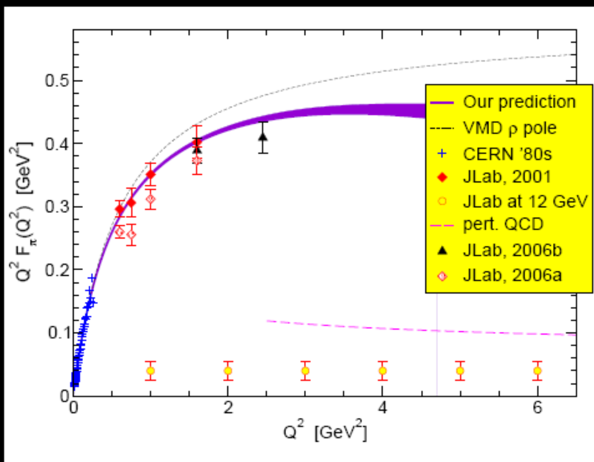
	Present work (with CLR vertex)	Expt.	RL-Padé	RL-direct
m_π	0.138	0.138	0.138	0.137
m_ρ	0.84 ± 0.03	0.777	0.754	0.758
m_σ	1.13 ± 0.01	$0.4 - 1.2$	0.645	0.645
m_{a_1}	1.28 ± 0.01	1.24 ± 0.04	0.938	0.927
m_{b_1}	1.24 ± 0.10	1.21 ± 0.02	0.904	0.912
$m_{a_1} - m_\rho$	0.44 ± 0.04	0.46 ± 0.04	0.18	0.17
$m_{b_1} - m_\rho$	0.40 ± 0.14	0.43 ± 0.02	0.15	0.15

(L. Chang, & C.D. Roberts, Phys. Rev. C 85, 052201(R) (2012))

(S.X. Qin, L. Chang, Y.X. Liu, C.D. Roberts, et al., Phys. Rev. C 84, 042202(R) (2011))

✦ Electromagnetic Property & PDF of hadrons

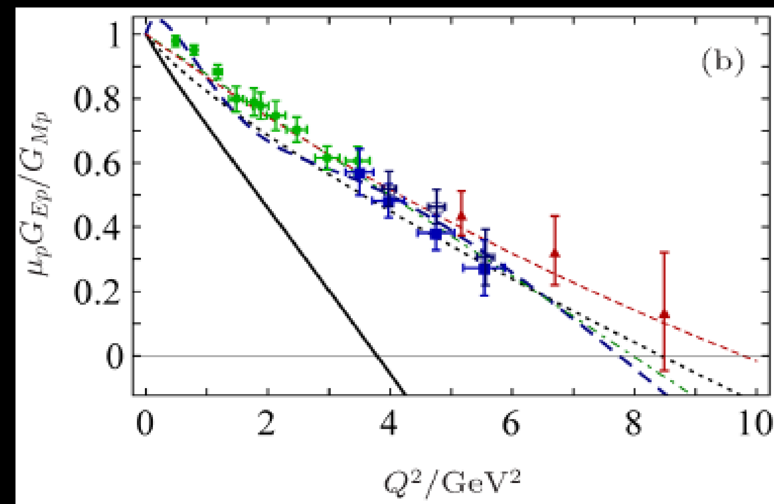
Pion electromagnetic form factor



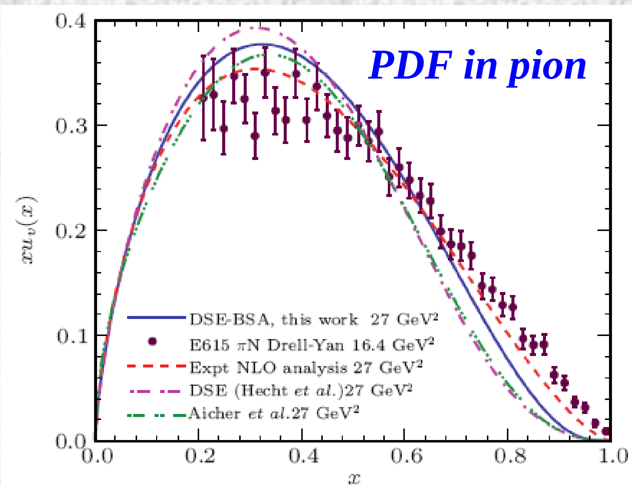
PM and Tandy, PRC62,055204 (2000) [nucl-th/0005015]

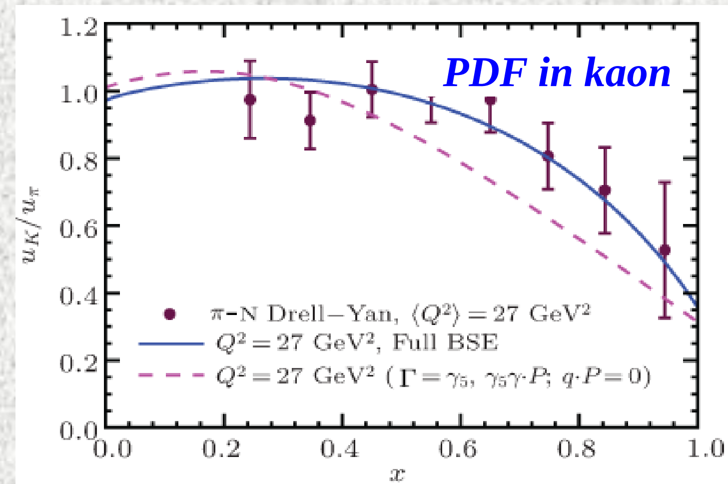
P. Maris & PCT, PRC 61, 045202 ('00)

Proton electromagnetic forma factor



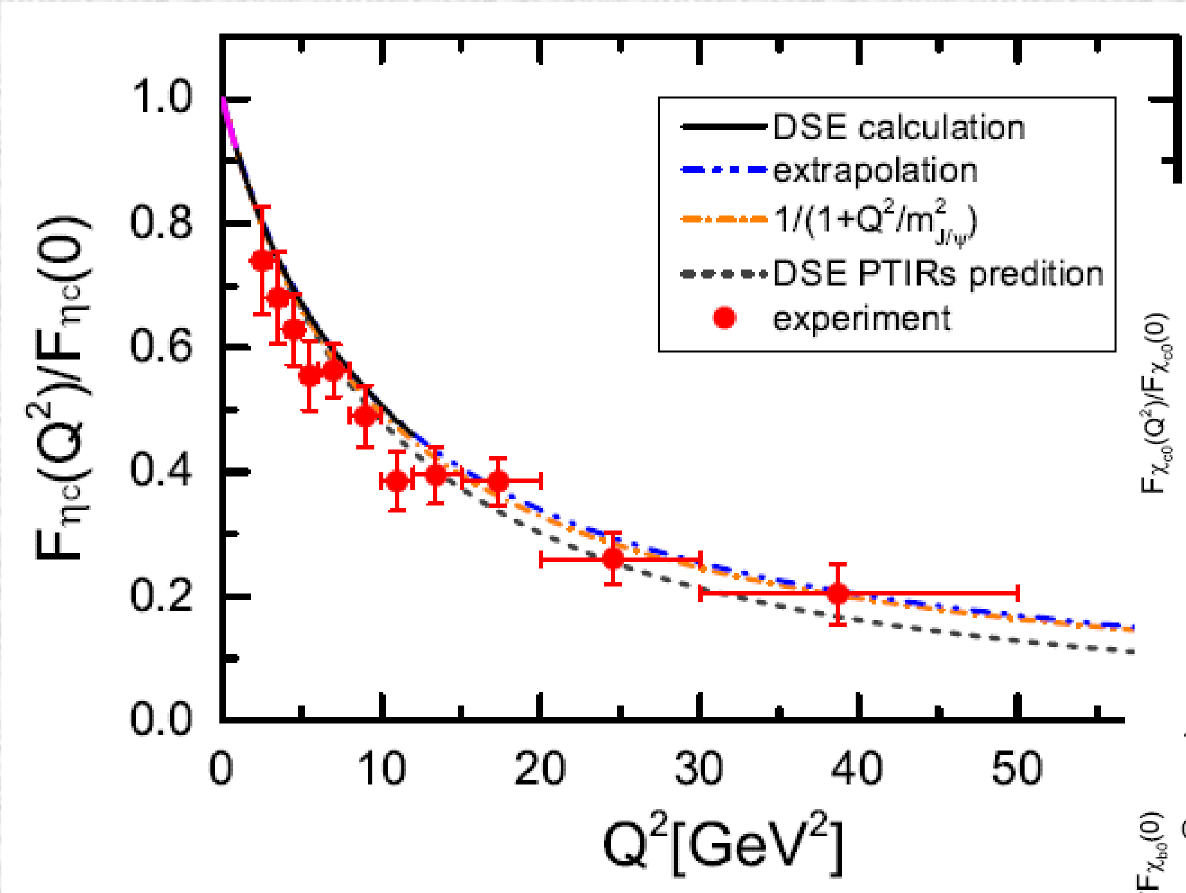
L. Chang et al., AIP CP 1354, 110 ('11)

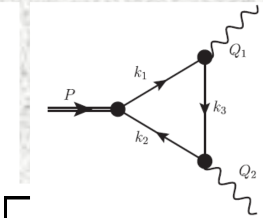


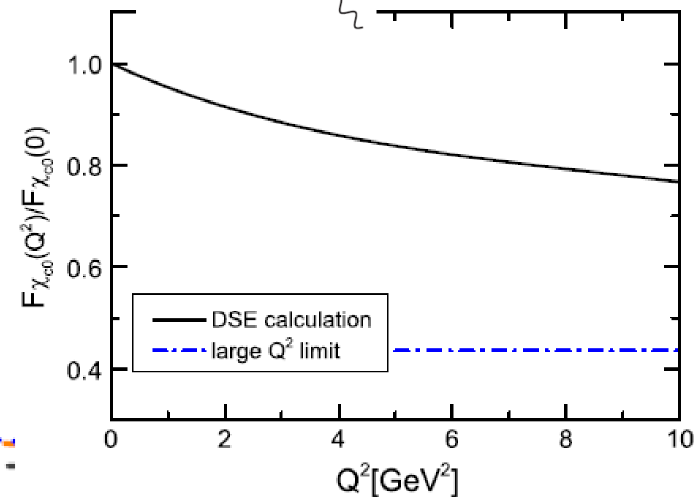


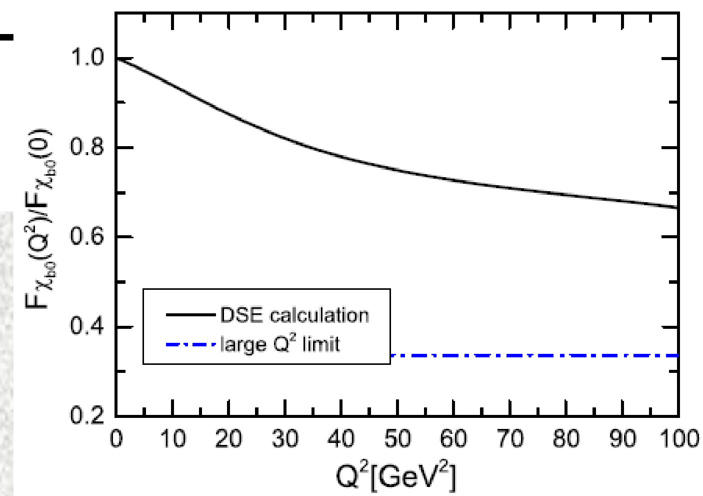
R.J. Holt & C.D. Roberts, RMP 82, 2991(2010); T. Nguyan, CDR, et al., PRC 83, 062201 (R) (2011)

✦ Decay width of $\eta_c \to \gamma^*\gamma$, $\chi_{c0} \to \gamma^*\gamma$, $\chi_{b0} \to \gamma^*\gamma$



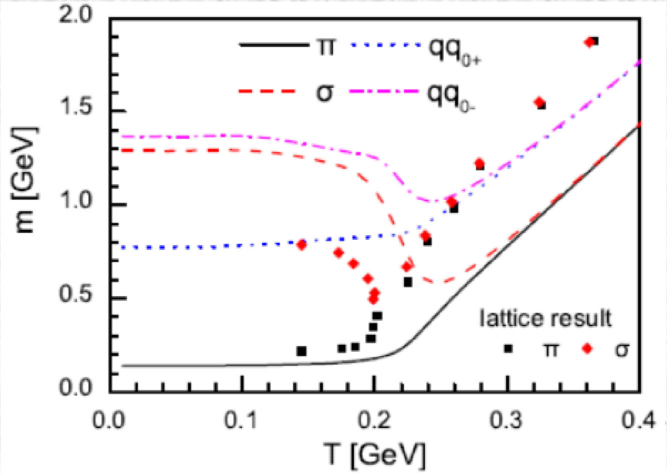


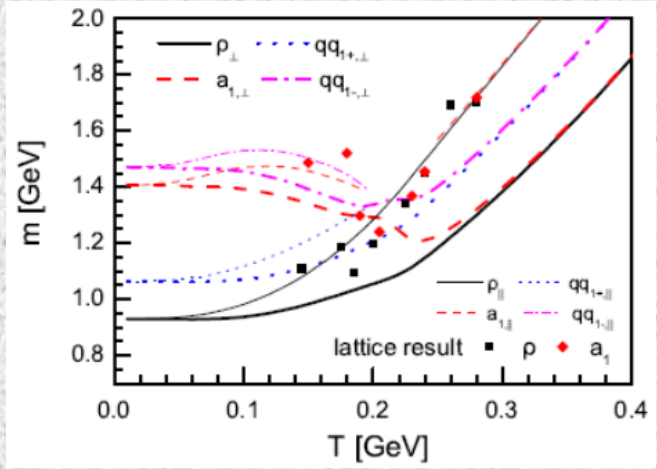




J. Chen, Ming-hui Ding, Lei Chang, and Yu-xin Liu, Phys. Rev. D 95, 016010 (2017)

✦ T-dependence of the screening masses of some hadrons

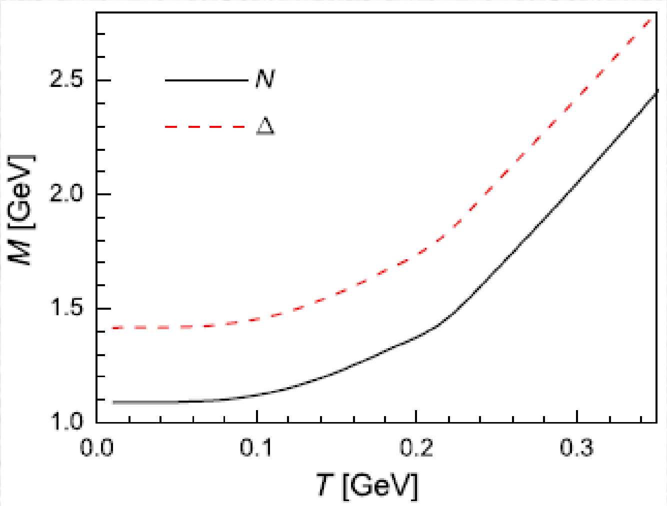




GT Relation

$$M_\sigma^2 = M_\pi^2 + 4M_q^2$$

➔ $M_\sigma \cong M_\pi$ can be a signal of the DCS.



$r_S \propto 1/M_S$, when $r_S < r_{md}$, the color gets deconfined.

Hadron properties provide signals for not only the chiral phase transt. but also the confinement-deconfnmt. phase transition.

Wei-jie Fu, and Yu-xin Liu, Phys. Rev. D 79, 074011 (2009);
K.L. Wang, Y.X. Liu, C.D. Roberts, Phys. Rev. D 87, 074038 (2013);

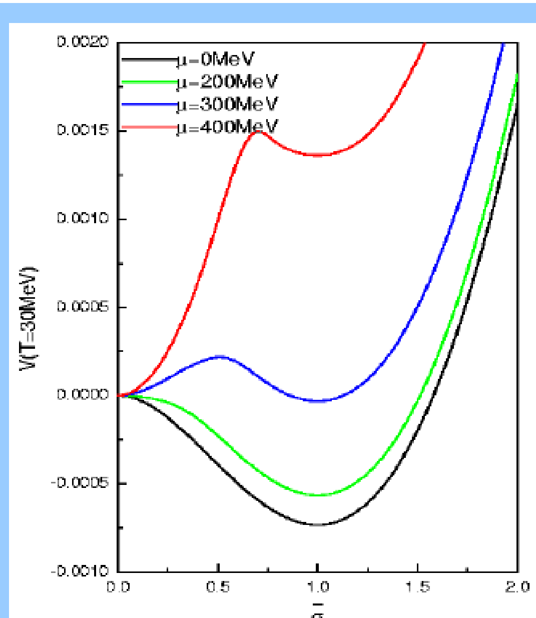
3. Criteria of the Phase Transitions

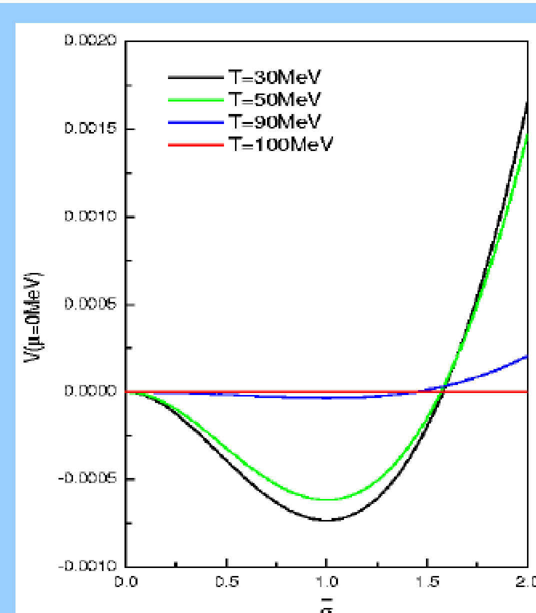
♠ Conventional Criterion

Order Parameter: chiral cond. $\langle \bar{q}q \rangle$!

$$\mathcal{M}(p) \simeq m_0 \left[\ln p/\Lambda_{QCD}\right]^d + C \frac{-\langle \bar{q}q \rangle}{p^2 \left[\ln p/\Lambda_{QCD}\right]^d}$$

Procedure: Analyzing the TD Potential

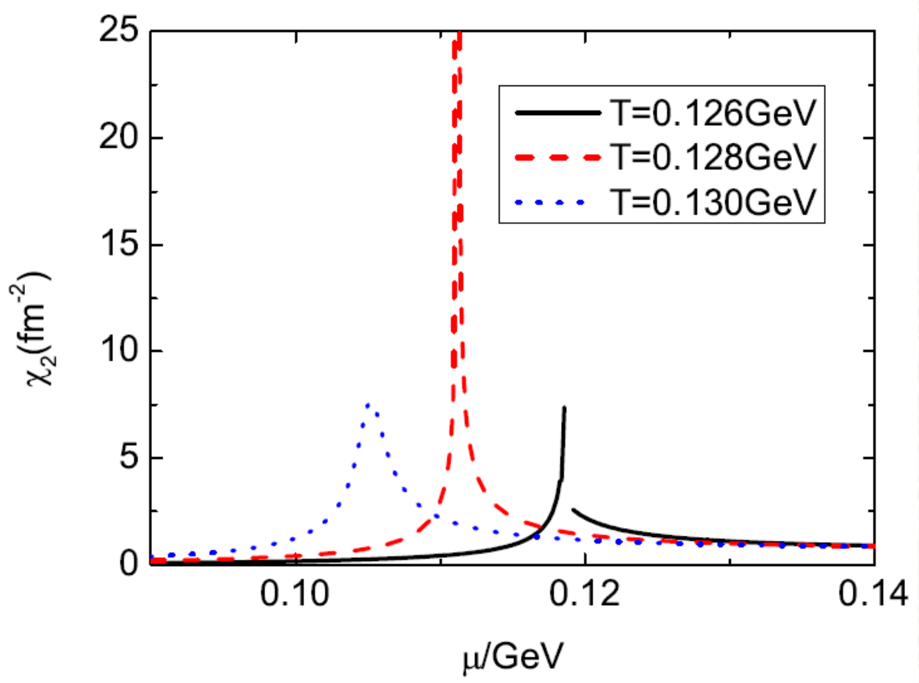




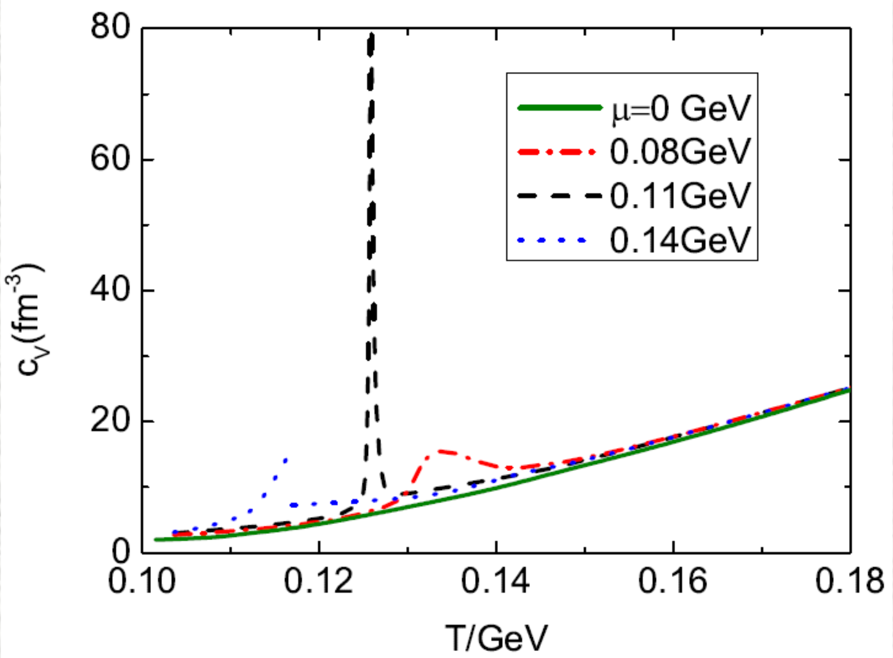
Signature of PT: $\frac{\partial^2 \Omega}{\partial T^2}$, $\frac{\partial^2 \Omega}{\partial \mu^2}$, *etc*, *change sign* .

• Critical phenomenon can be a criterion for CEP

$$\chi = \frac{\partial \rho}{\partial \mu} = \frac{\partial^2 P}{\partial \mu^2}$$

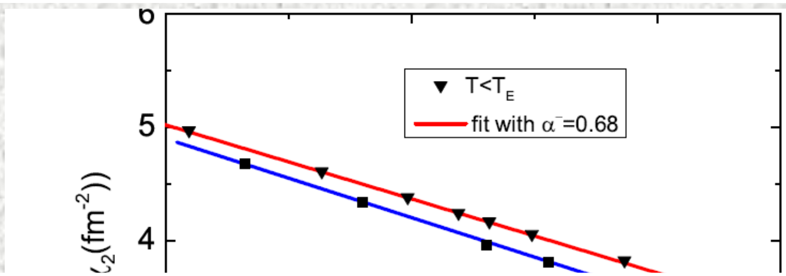


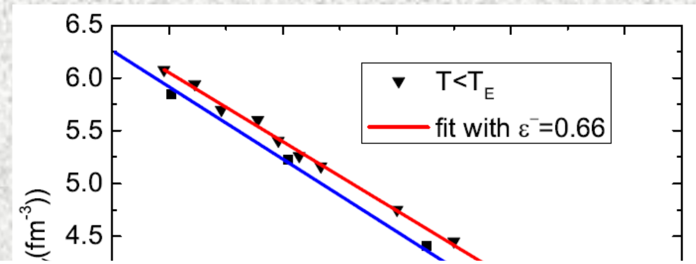
$$\varepsilon = T^2 \frac{\partial P/T}{\partial T} \quad c_V = \frac{\partial \varepsilon}{\partial T}$$



Gao, Chen, Liu, Qin, Roberts, Schmidt, Phys. Rev. D 93, 094019 (2016).

• Locating the CEP with the Critical Behavior





Quantity	$T-T_E\to 0^+$	$T-T_E\to 0^-$	$\mu-\mu_E\to 0^+$	$\mu-\mu_E\to 0^-$
c_V	0.69 ± 0.02	0.66 ± 0.01	0.69 ± 0.01	0.65 ± 0.01
χ_q	0.69 ± 0.01	0.68 ± 0.01	0.68 ± 0.01	0.65 ± 0.02

$$(\mu_E^\chi, T_E^\chi) = (110.9, 127.5)\ \mathrm{MeV}$$

Question: **In complete nonperturbation, one one can not have the thermodynamic potential.**

The conventional criterion fails.

One needs then new criterion!

Gao, Chen, Liu, Qin, Roberts, Schmidt, Phys. Rev. D 93, 094019 (2016).

♠ New Criterion: Chiral Susceptibility

- **Def.: Resp. the OP to control variables**

$$\frac{\partial M}{\partial T}, \quad \frac{\partial M}{\partial \mu}; \qquad \frac{\partial \langle \bar{q} q \rangle}{\partial T}, \quad \frac{\partial \langle \bar{q} q \rangle}{\partial \mu}; \qquad \frac{\partial B}{\partial T}, \quad \frac{\partial B}{\partial \mu}; \qquad \frac{\partial B}{\partial m_0};$$

- **Simple Demonst. Equiv. of NewC to ConvC**

(刘玉鑫,《热学》, 北京大学出版社, 2016年第1版)

TD Potential: $\Omega(T,\eta) = \Omega_0(T) + \frac{1}{2}\alpha\eta^2 + \frac{1}{4}\beta(\eta^2)^2 + \frac{1}{6}\gamma(\eta^2)^3 + \cdots$

Stability Condition: $\frac{\partial \Omega}{\partial \eta} = \alpha\eta + \beta\eta^3 + \gamma\eta^5 = 0$

$$\frac{\partial^2 \Omega}{\partial \eta^2} = \alpha + 3\beta\eta^2 + 5\gamma\eta^4 > 0, \ St.; \ < 0, \ Unst. .$$

Derivative of ext. cond. against control. var.:

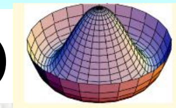
$$[\alpha + 3\beta\eta^2 + 5\gamma\eta^4]\left(\frac{\partial \eta}{\partial \varsigma}\right)_{\varsigma=\zeta_c} + \eta\left(\frac{\partial \alpha}{\partial \varsigma}\right)_{\varsigma=\zeta_c} + \eta^3\left(\frac{\partial \beta}{\partial \varsigma}\right)_{\varsigma=\zeta_c} + \eta^5\left(\frac{\partial \gamma}{\partial \varsigma}\right)_{\varsigma=\zeta_c} = 0$$

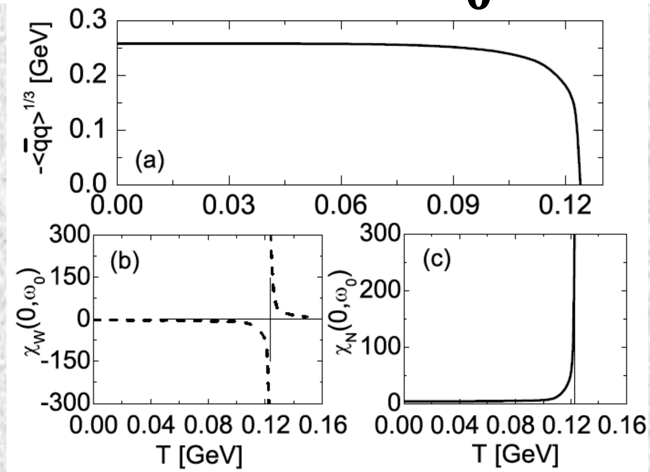
we have: $\chi = \left(\frac{\partial \eta}{\partial \varsigma}\right)_{\varsigma=\zeta_c} = -\frac{\eta\left(\frac{\partial \alpha}{\partial \varsigma}\right)_{\varsigma=\zeta_c} + \eta^3\left(\frac{\partial \beta}{\partial \varsigma}\right)_{\varsigma=\zeta_c} + \eta^5\left(\frac{\partial \gamma}{\partial \varsigma}\right)_{\varsigma=\zeta_c}}{\left(\frac{\partial^2 \Omega}{\partial \eta^2}\right)_{\frac{\partial \Omega}{\partial \eta}=0}}$

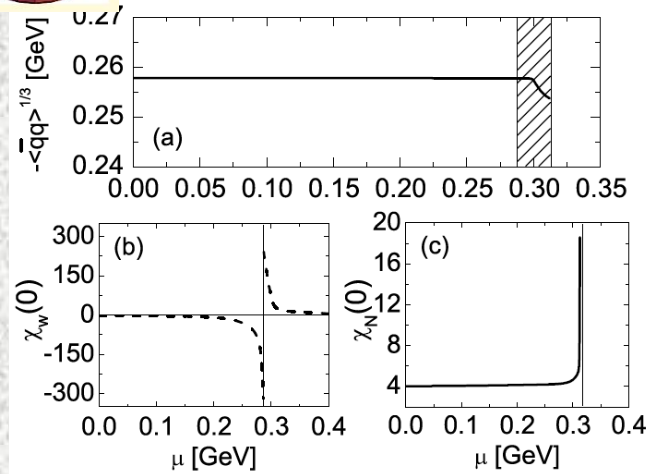
At field theory level, see Fei Gao, Y.X. Liu, Phys. Rev. D 94, 076009 (2016) .

♣ Demonstration of the New Criterion

In chiral limit ($m_0 = 0$)

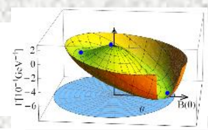




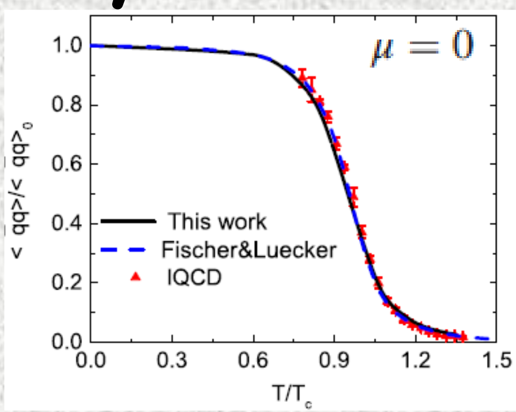


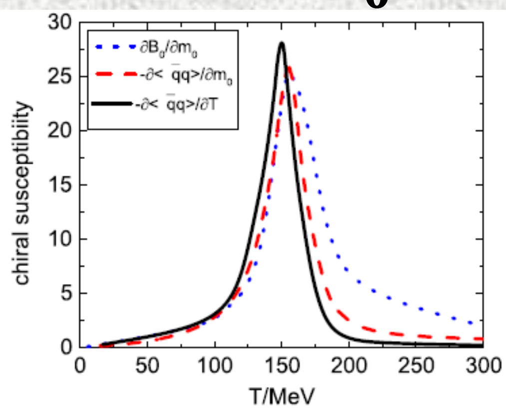
S.X. Qin, L. Chang, H. Chen, YXL, et al., Phys. Rev. Lett. 106, 172301 (2011).

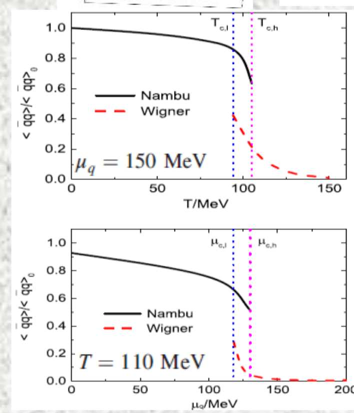
Beyond chiral limit ($m_0 \neq 0$)

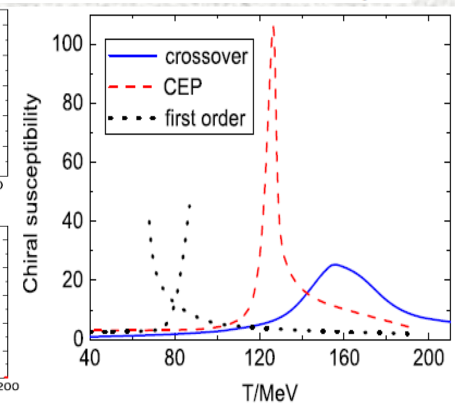


$$\langle\bar{q}q\rangle = \langle\bar{q}q\rangle_{m_0} - m_0 \frac{\partial\langle\bar{q}q\rangle_{m_0}}{\partial m_0}$$









Fei Gao, Y.X. Liu, Phys. Rev. D 94, 076009 (2016) .

♣ Characteristic of the New Criterion

As 2nd order PT (Crossover) occurs,
the χs of the two (DCS, DCSB) phases diverge (take maximum) at same states.

As 1st order PT takes place,
χs of the two phases diverge at dif. states.

→ the χ criterion can not only give the phase boundary, but also determine the position the CEP.

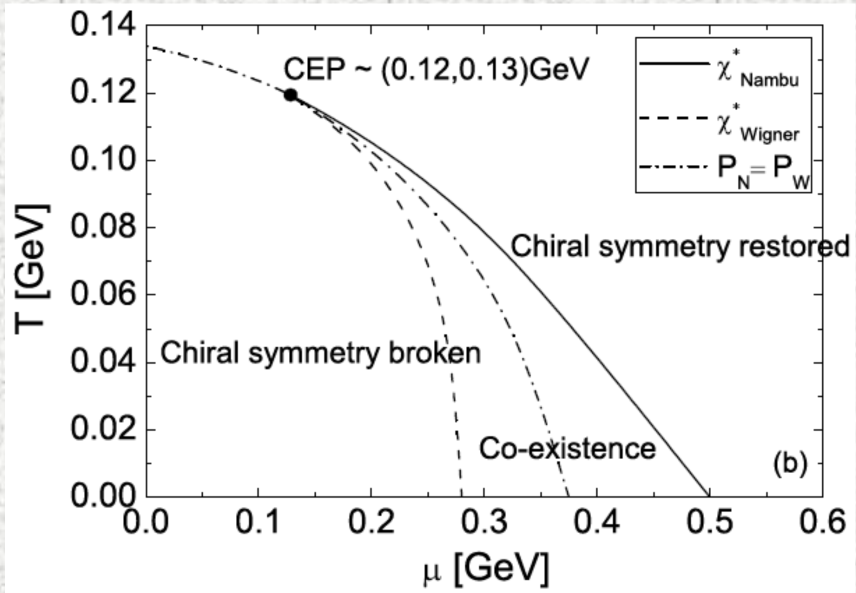
For multi-flavor system,
one should analyze the maximal eigenvalue of the susceptibility matrix (L.J. Jiang, YXL, et al., PRD 88, 016008),
or the mixed susceptibility (F. Gao, YXL, PRD 94, 076009).

♠ Some Numerical Results

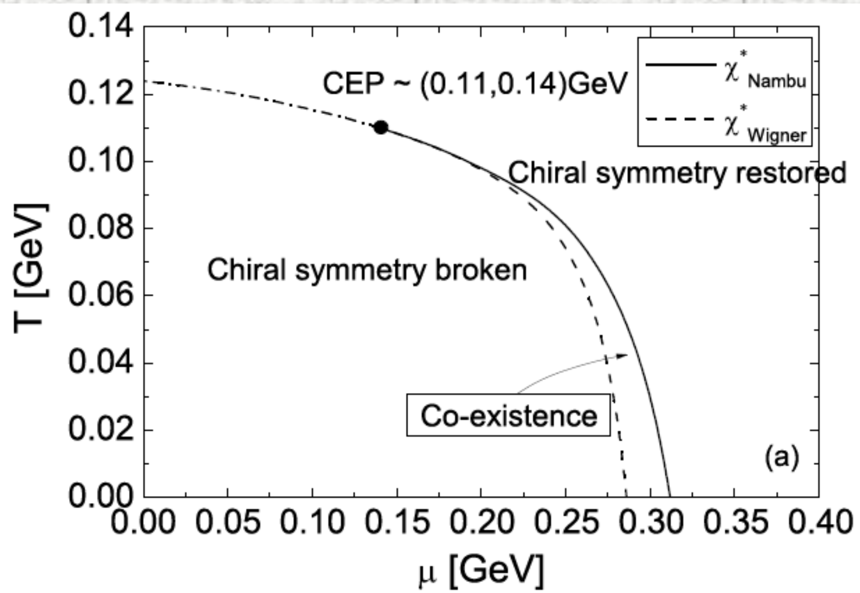
♣ QCD Phase Diagrams and the position of the CEP have been given in the DSE

- In chiral limit

With bare vertex
(ETP is available, the PB is shown as the dot-dashed line)



With Ball-Chiu vertex
(ETP is not available, but the coexistence region is obtained)



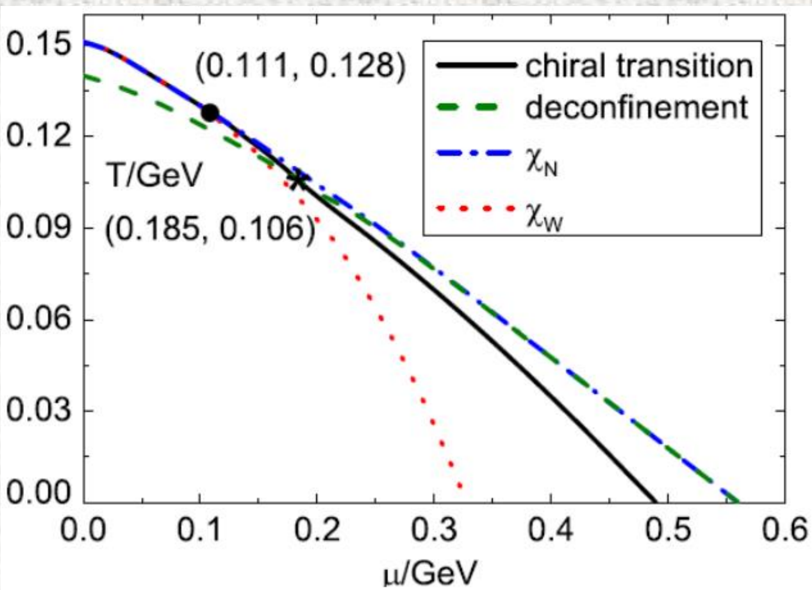
S.X. Qin, L. Chang, H. Chen, YXL, et al., Phys. Rev. Lett. 106, 172301 (2011).

♣ QCD Phase Diagrams and the position of the CEP have been given in the DSE

- **Beyond chiral limit**

With bare vertex

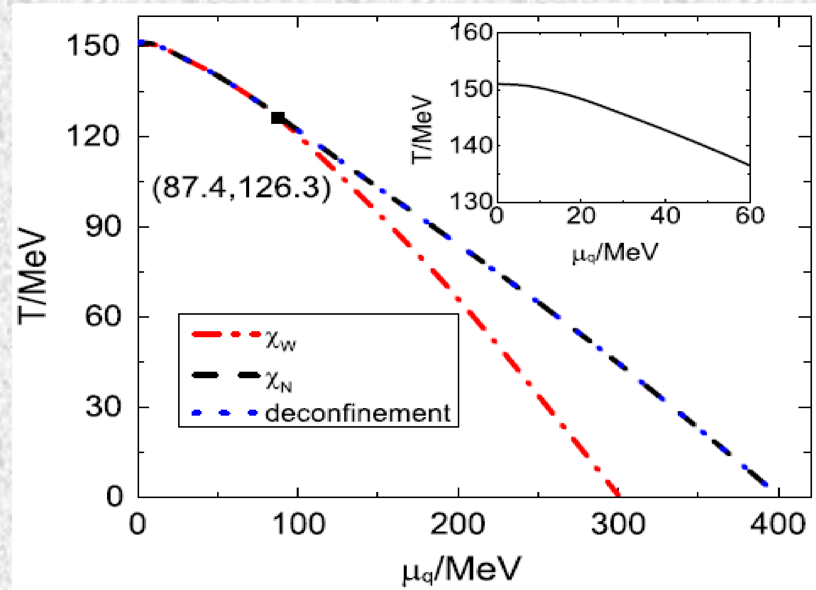
(ETP is available, the PB is shown as the dot-dashed line)



F. Gao, J. Chen, Y.X. Liu, et al., Phys. Rev. D 93, 094019 (2016).

With CLR vertex

(ETP is not available, but the coexistence region is obtained)



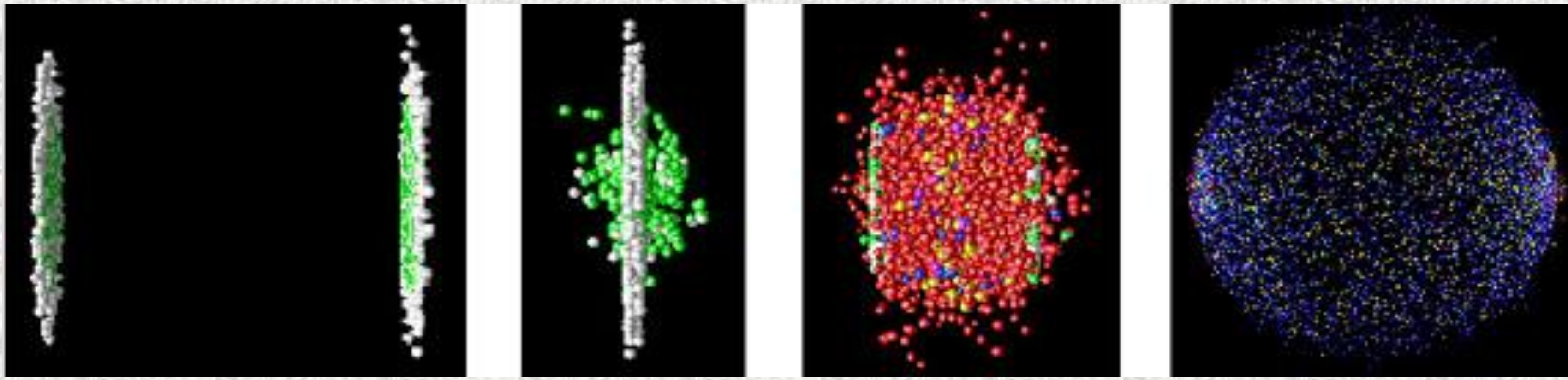
F. Gao, Y.X. Liu, Phys. Rev. D 94, 076009 (2016).

♣ Some Issues highly concerned here

- Interface tension & its effect.
- Viscosities (Shear & Bulk)

Ⅲ. Facilities & Observables

1. Relativistic Heavy Ion Collisions (RHIC)



Thin Pancakes | Nuclei pass | Huge Stretch | The Last Epoch:

However, One can not get very high density matter in Lab.

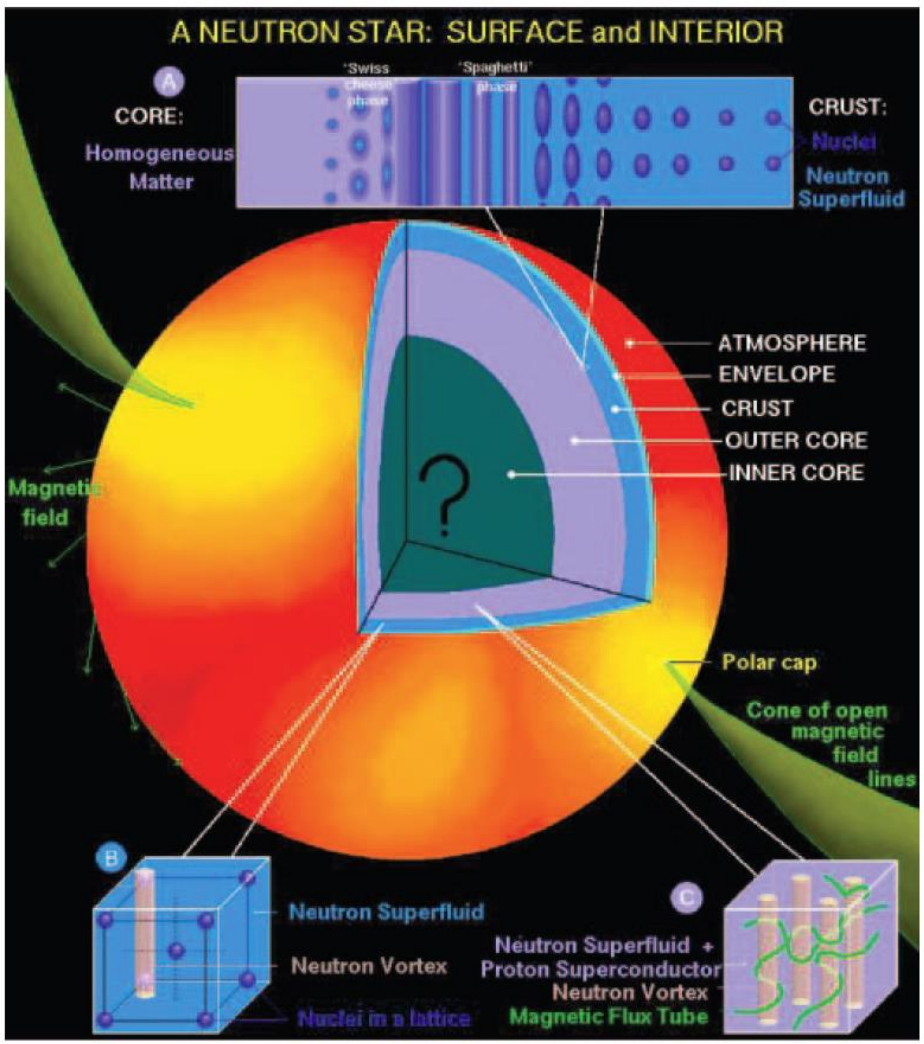
Currently Working: RHIC@BNL, ALICE-LHC@CERN

Under Construction: FAIR@G, NICA@R, HIAF@C

Jet Q., v_2, v_3, Viscosity, CC Fluct. & Correl., Hadron Prop., γ, ···

2. Astronomical Observables: Properties of Compact Stars (Pulses)

Radio Pulses =➔ "Neutron" Stars

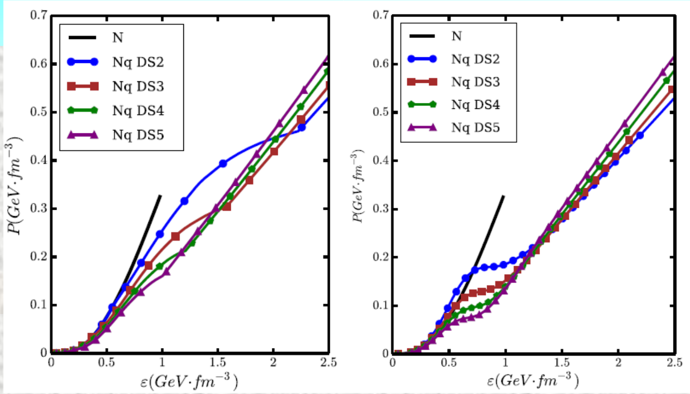


M-R Rel.,
Rad. Spectra,
Inst. R. Oscil.,
Frequency of GW,
...

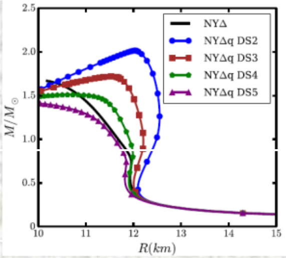
➔ Composition & Structure of NSs !

J. M. Lattimer, *et al.* Science **304**, 536 (2004)

♣ EOS of the matter involving QCD phase transitions & Composition of the NS with $M \geq 2.0 M_{sun}$

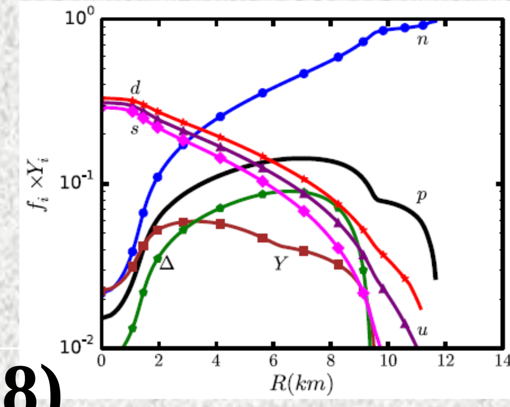


- Hadron Matter: RMF;
  Quark matter: DSE;
  Interplay: Gibbs Construction & 3-window construction➔EOS



- TOV Eq. ➔ MR Relation.

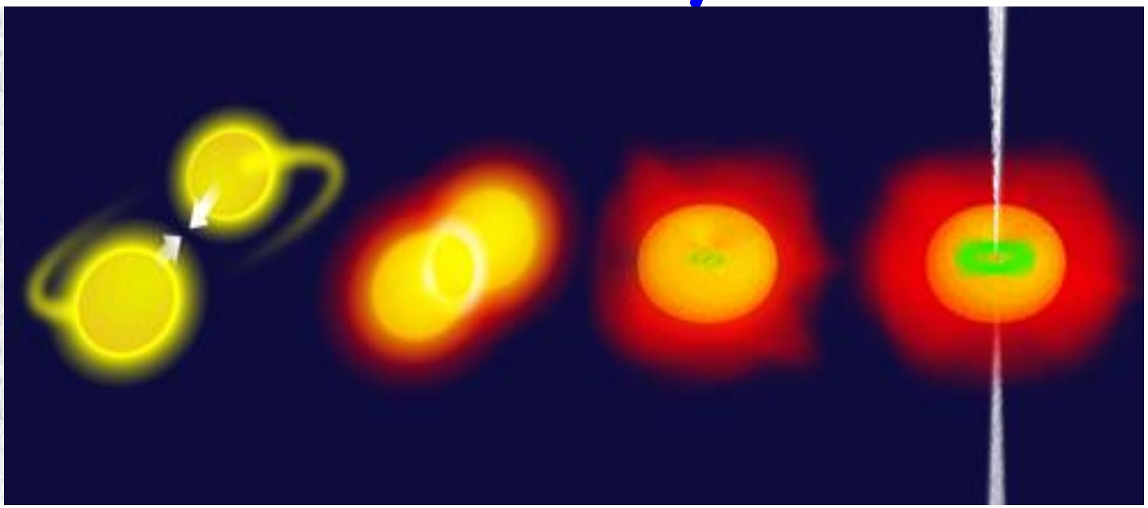
- Distribution of the ingredients in terms of the radius,
  ➔ Massive pulses may be hybrid stars.



Z. Bai & YXL, Phy. Rev. D 97, 023018 (2018)

♠ Gravitational Mode Oscillation Frequency can also be an Excellent Astron. Signal

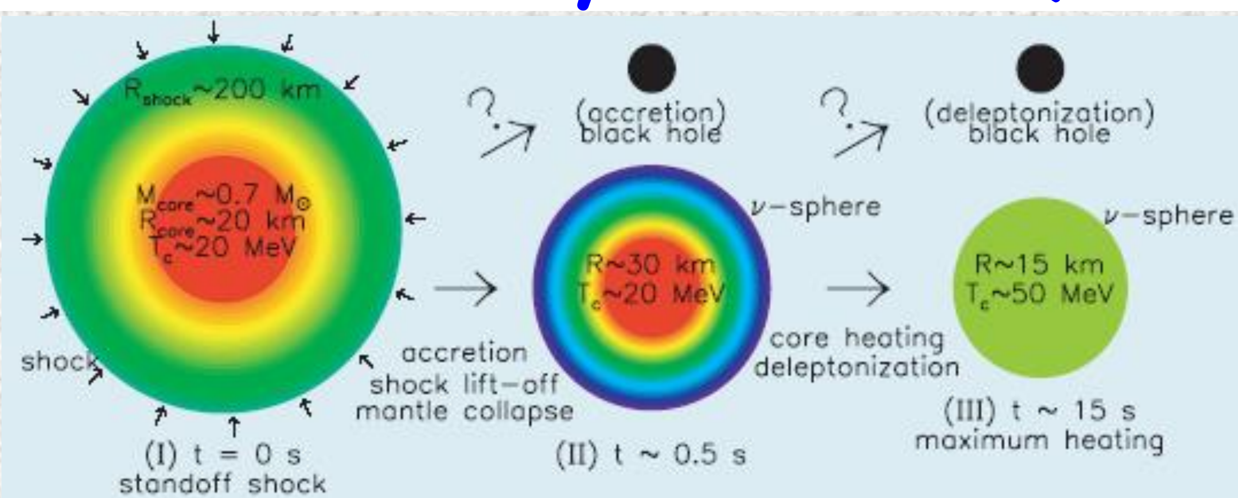
- G-Wave in Binary Neutron Star Merger

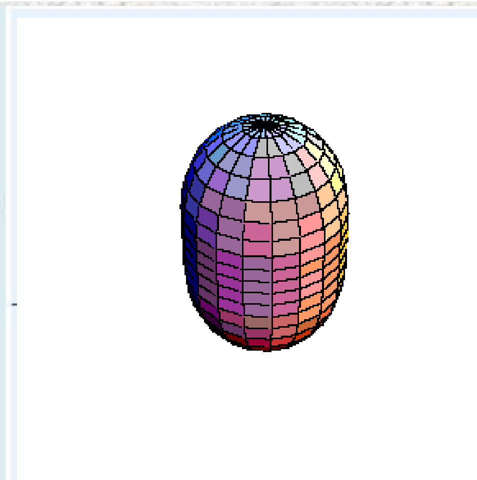


$F_{postmerger} \in (1.84, 3.73)$kHz, with width<200Hz, (PRD 86, 063001(2012))

$F_{spiral} < F_{postmerger}$

- G-Wave in Newly Born NS/QS after the SNE





- Comparison of G-mode Oscillation Frequencies of the two kind nb Stars

Neutron Star: RMF, Quark Star: Bag Model

→ Frequency of the G-mode oscillation

Radial order of g-mode	Neutron Star			Strange Quark Star		
	$t=100$	$t=200$	$t=300$	$t=100$	$t=200$	$t=300$
$n=1$	717.6	774.6	780.3	82.3	78.0	63.1
$n=2$	443.5	467.3	464.2	52.6	45.5	40.0
$n=3$	323.8	339.0	337.5	35.3	30.8	27.8

W.J. Fu, H.Q. Wei, and Y.X. Liu, arXiv: 0810.1084, Phys. Rev. Lett. 101, 181102 (2008)

• Comparison with other modes

Neutron Star: RMF, Quark Star: Bag Model

→ **Frequencies of the f- & p-mode oscillations**

Modes	Neutron Star			Strange Quark Star		
	$t=100$	$t=200$	$t=300$	$t=100$	$t=200$	$t=300$
$_2f$	1103	1133	1176	2980	2997	3016
$_2p_1$	2265	2426	2494	18282	17330	16792
$_2p_3$	5519	5702	5809	38988	36950	35798

Wei Wei & Collaborators' work (1811.11377) Confirms these results qualitatively !

♣ *G-mode oscillation in quark star has very low freq. !*

W.J. Fu, Z. Bai, Y.X. Liu, arXve:1701.00418

• Taking into account the DCSB effect

Newly obtained results for QS in NJL Model

Radial order of g-mode	Neutron Star			Strange Quark Star		
	$t=100$	$t=200$	$t=300$	$t=100$	$t=200$	$t=300$
$n=1$	717.6	774.6	780.3	100.2	115.4	107.4
$n=2$	443.5	467.3	464.2	60.1	57.0	51.8
$n=3$	323.8	339.0	337.5	42.9	40.9	40.2

$\Lambda_{1.4} \approx 320$ $\Lambda_{1.4} \approx 40$

- Work in the DS equation scheme is under progress.

V. Summary & Remarks

♠ DSE, a npQCD approach, is described.

- Characts. npQCD are featured well in DSE

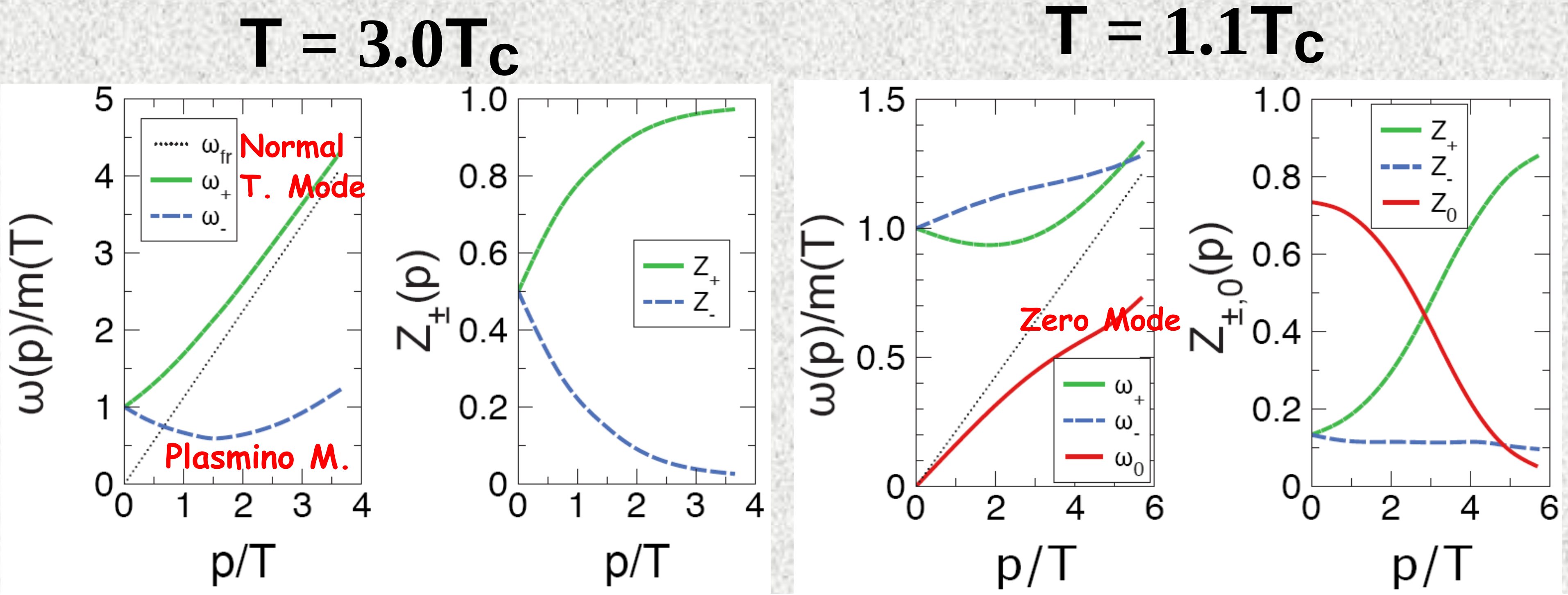
♠ QCD PTs are presented via the DSE

- Dyn. M., Phase Diagram & CEP are given;
- Matter at the T above but near $T_{c\chi}$ is sQGP;
- Hadronization takes place at $T < T_c$;
- Quarkyonic Phase arises from ultrahot effect
- non-radial oscillation frequencies may be a signal of the QCD phase transitions.

Thanks !!

♠ Disperse Relation and Momentum Dep. of the Residues of the Quasi-particles' poles

➔ χs matter at $T\in(1,1.5)T_c$ is in sQGP state

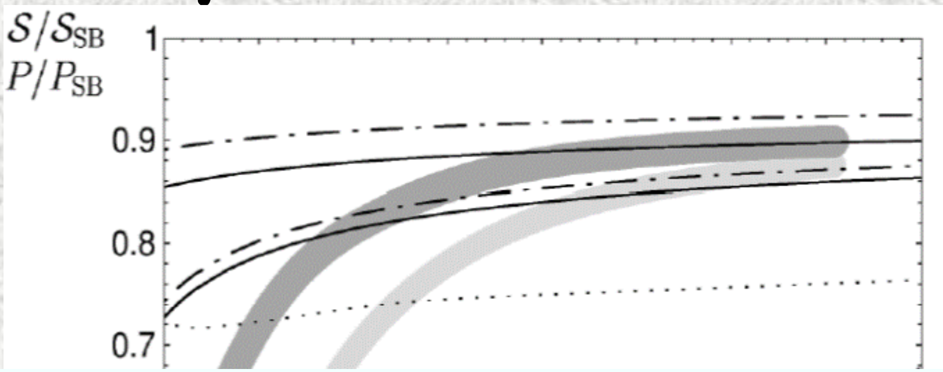


- The zero mode exists at low momentum($<7.0T_C$), and is long-range correlation ($\lambda\sim\omega^{-1}>\lambda_{FP}$) .

S.X. Qin, L. Chang, Y.X. Liu, C.D. Roberts, PRD 84, 014017 (2011);
F. Gao, S.X. Qin, Y.X. Liu, C.D. Roberts, PRD 89, 076009 (2014).

♣ Hadroniz. Process

- For the system at **thermodynamic limit**,



J.-P. Blaizot, et al., PRL 83, 2906

In fact, there exists finite interface, which contributes to the entropy of the system.

The monotonic behavior manifests that the entropy density of the quark-gluon phase is always larger than that of hadron phase.

(Nonaka, et al., Phys. Rev. C **71**, 051901R (2005) ; tec.,)

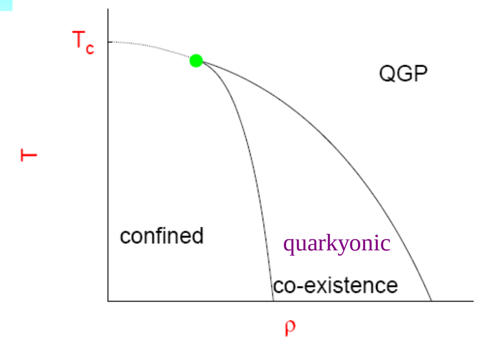
---- Entropy Puzzle.

♠ Relation between the Chiral PT & the Conf.—Deconf. PT

♣ **Lattice QCD Calculation**

de Forcrand, et al.,

Nucl. Phys. B Proc. Suppl. 153, 62 (2006); ⋯

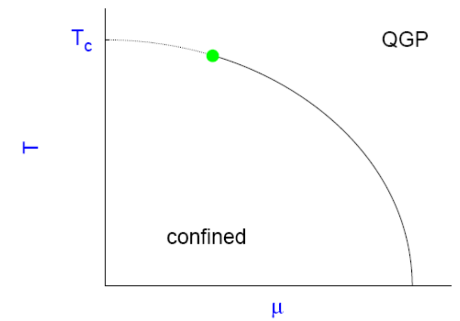


and **General (large-N_C) Analysis**

McLerran, et al., NPA 796, 83 (‘07);

NPA 808, 117 (‘08);

NPA 824, 86 (‘09), ⋯



claim that there exists a quarkyonic phase.

♣ **Coleman-Witten Theorem** (PRL 45, 100 (‘80)):

Confinement coincides with DCSB !!

♣ Inconsistence really exists?!

Nature of the Quarkyonic Phase ?!

♠ Interface effects in the hadroniz. Proc.

- Interface tension between the DCS-unconf. phase and the DCSB-confined phase

With the scheme (J. Randrup, PRC 79, 054911 (2009))

$$F(\vec{r}) = n\mu + \tfrac{1}{2}C(\nabla n)^2 + \cdots\cdots \cong n\mu + \tfrac{1}{2}C(\nabla n)^2,$$

we have $\Delta F_T = F_T(n) - F_M(n),$

with $F_M(n) = F_T(n_L) + \frac{F_T(n_H) - F_T(n_L)}{n_H - n_L}(n - n_L)$

and (EoM) $\Delta F_T + \tfrac{1}{2}C(\frac{\partial n}{\partial r})^2 = 0$.

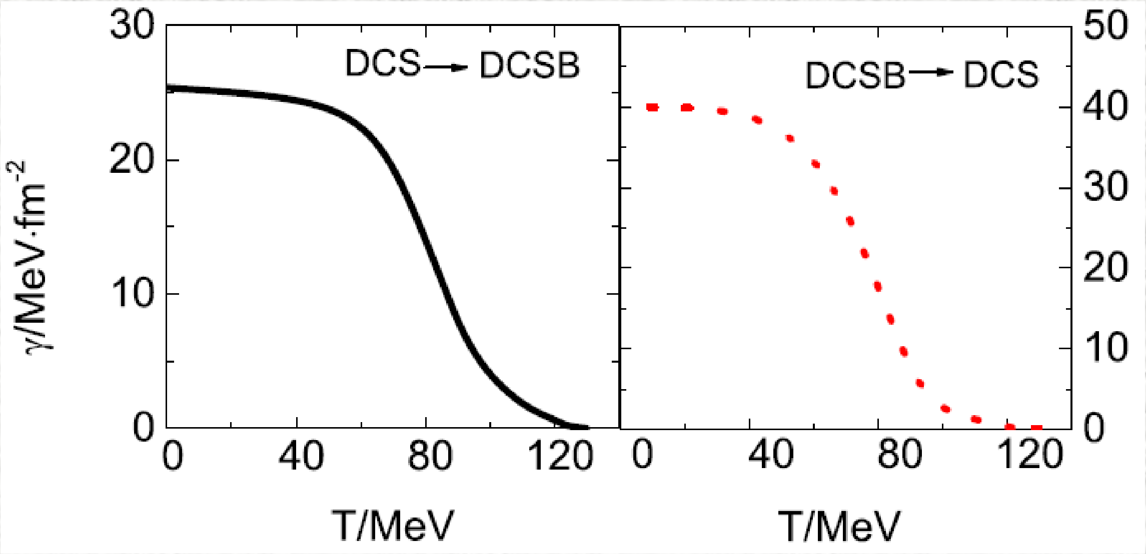
$$\Rightarrow \gamma(T) = \int_{-\infty}^{+\infty} \Delta F_T \, dx = -\tfrac{1}{2}\int_{-\infty}^{+\infty} C(\tfrac{\partial n}{\partial r})^2 dx,$$

$$= \int_{n_L}^{n_H} \sqrt{\tfrac{C}{2}\Delta F_T(n)}\, dn$$

- **Interface tension between the DCS-unconf. phase and the DCSB-confined phase**

$$\gamma(T)=\int_{n_L}^{n_H}\sqrt{\frac{C}{2}\Delta F_T(n)}dn\ .$$

J. Randrup, PRC 79, 054911 (2009)



Parameterized as $\gamma(T)=a+be^{(c/T+d/T^2)}$,

with parameters

	$a/(\mathrm{MeV/fm^2})$	$b/(\mathrm{MeV/fm^2})$	$c/$ MeV	$d/\mathrm{GeV^2}$
DCS → DCSB	25.4	−1.5	736	−0.048
DCSB → DCS	40.0	−8.1	399	−0.025

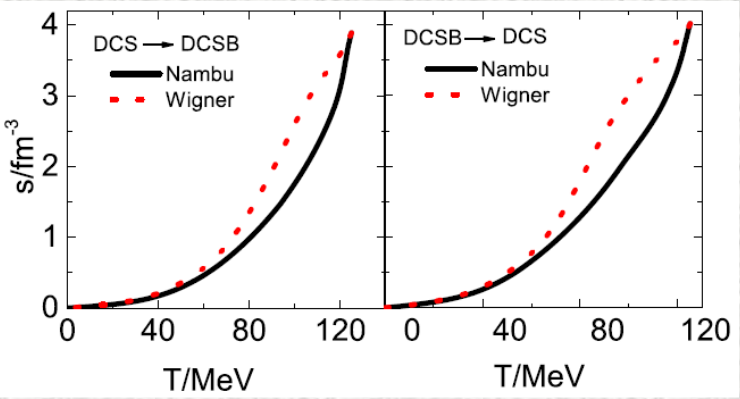
Fei Gao, & Yu-xin Liu，Phys. Rev. D 94, 094030 (2016)

• Interface effects in the hadroniz. Proc.

Solving the entropy puzzle

In thermodynamical limit

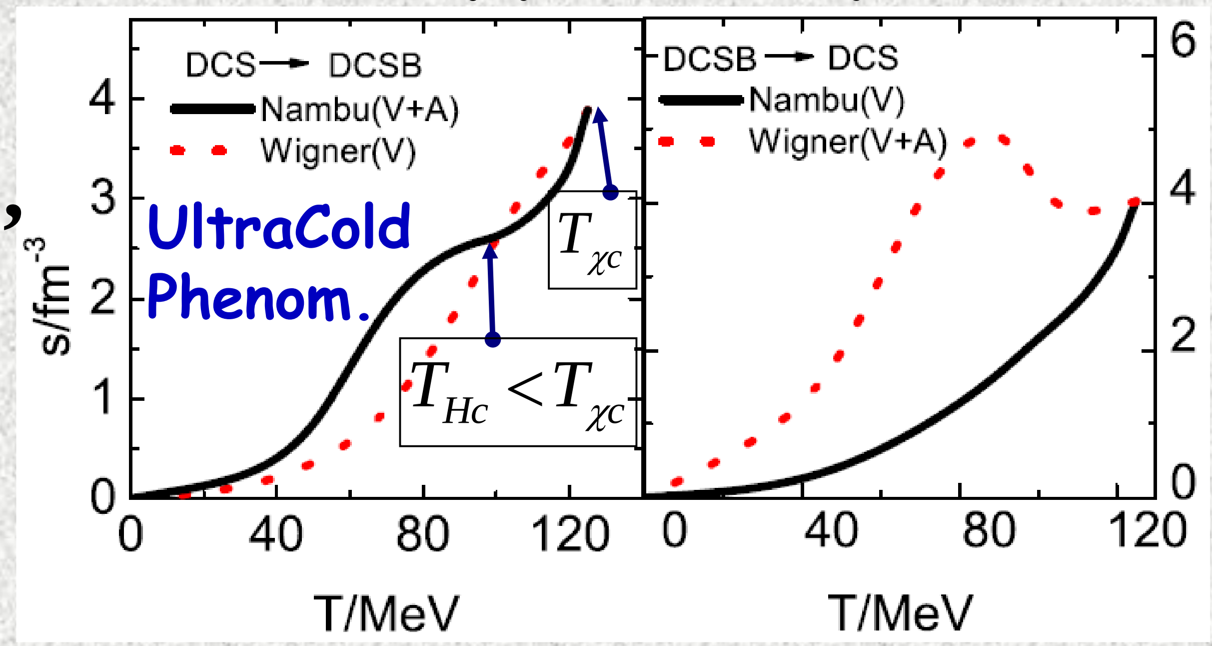
$$s_V = \frac{1}{T}(\epsilon + P - \mu n) = \frac{\partial P}{\partial T}.$$



With the interface entropy density

$$s_A = -(\frac{\partial \gamma}{\partial T})_{VA}$$

being included, we have



F. Gao, & Y.X. Liu, Phys. Rev. D 94, 094030 (2016)

♣ Viscosities & their ratios to the entropy density in medium

- $$\eta = \frac{1}{10\pi^2 T}\int_0^\infty \frac{d|\vec{k}||\vec{k}|^6}{(\omega^\pi_{|\vec{k}|})^2\Gamma^{\rm TW}_\pi(|\vec{k}|)} n_{|\vec{k}|}(\omega^\pi_{|\vec{k}|})\{1+n_{|\vec{k}|}(\omega^\pi_{|\vec{k}|})\},$$

$$\zeta = \frac{3}{T}\int \frac{d^3\vec{k}}{(2\pi)^3}\frac{1}{(\omega^\pi_{|\vec{k}|})^2\Gamma^{\rm TW}_\pi(|\vec{k}|)} n_{|\vec{k}|}(\omega^\pi_{|\vec{k}|})\{1+n_{|\vec{k}|}(\omega^\pi_{|\vec{k}|})\} \times \left\{\left(\frac{1}{3}-c_s^2\right)|\vec{k}|^2 - c_s^2 m_\pi^2\right\},$$

$$n_{|\vec{k}|}(\omega^\pi_{|\vec{k}|}) = 1/(e^{\omega^\pi_{|\vec{k}|}/T}-1), \quad \omega^\pi_{|\vec{k}|} = \sqrt{|\vec{k}|^2 + m_\pi},$$

$$c_s^2 = \frac{\partial P}{\partial \varepsilon}, \quad \& \ \Gamma_\pi^{\rm TW}(k),$$

✦ Width of pion in medium

- With the T-dependent mass and decay constant of π obtained by solving the BS equation as the input of the Roy Eq. of π-π scat.,

➔ $M_\sigma(T)$, $\Gamma_\sigma(T)$, $M_\rho(T)$, $\Gamma_\rho(T)$.

- With

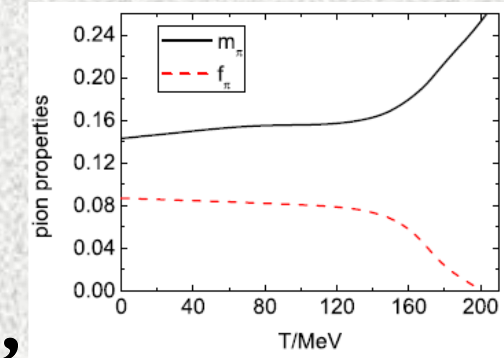
$$\Gamma_\pi^{TW} = \sum_{r=\rho,\sigma} \frac{1}{N_r} \int_{(m_r^-)^2}^{(m_r^+)^2} dM^2 \rho_r(M) \Gamma_{\pi\pi,r},$$

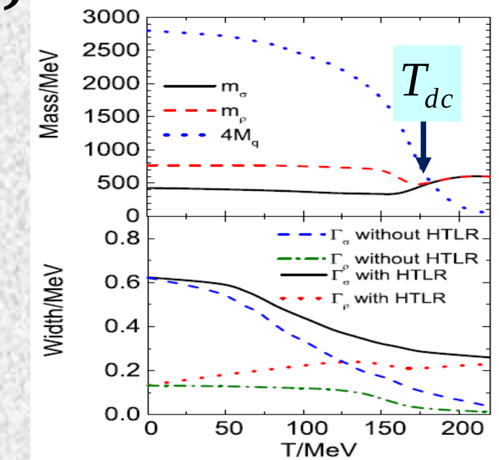
$$\Gamma_{\pi\pi,r} = \frac{1}{16\pi m_\pi |\vec{k}|} \int_{\omega_+}^{\omega_-} d\omega L_r(\omega) [n(\omega) - n(\omega^\pi_{|\vec{k}|} + \omega)],$$

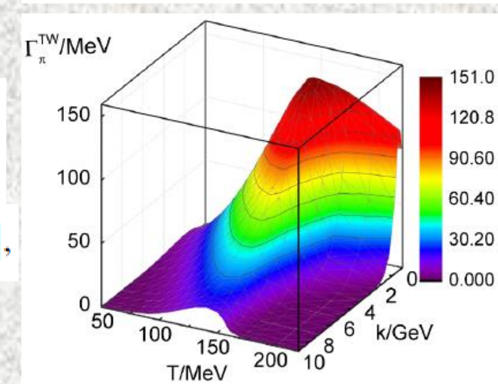
$$N_r = \int_{(m_r^-)^2}^{(m_r^+)^2} dM^2 \rho_r(M), \qquad \rho_r(M) = \frac{1}{\pi} \mathrm{Im}\left[\frac{-1}{M^2 - M_r^2 + i\Gamma_r M_r}\right].$$

$$L_\sigma(\omega) = \frac{-g_\sigma^2 M_\sigma^2}{4} \quad \text{and} \quad L_\rho(\omega) = \frac{-g_\rho^2}{M_\rho^2}[2m_\pi^2(m_\pi^2 - M_\rho^2) - 2(-\omega^\pi_{|\vec{k}|}\omega M_\rho^2 + m_\pi^4)],$$

➔ $\Gamma_\pi^{TW}(k)$.







F. Gao, Y.X. Liu, Phys. Rev. D 97, 056011 (2018) .

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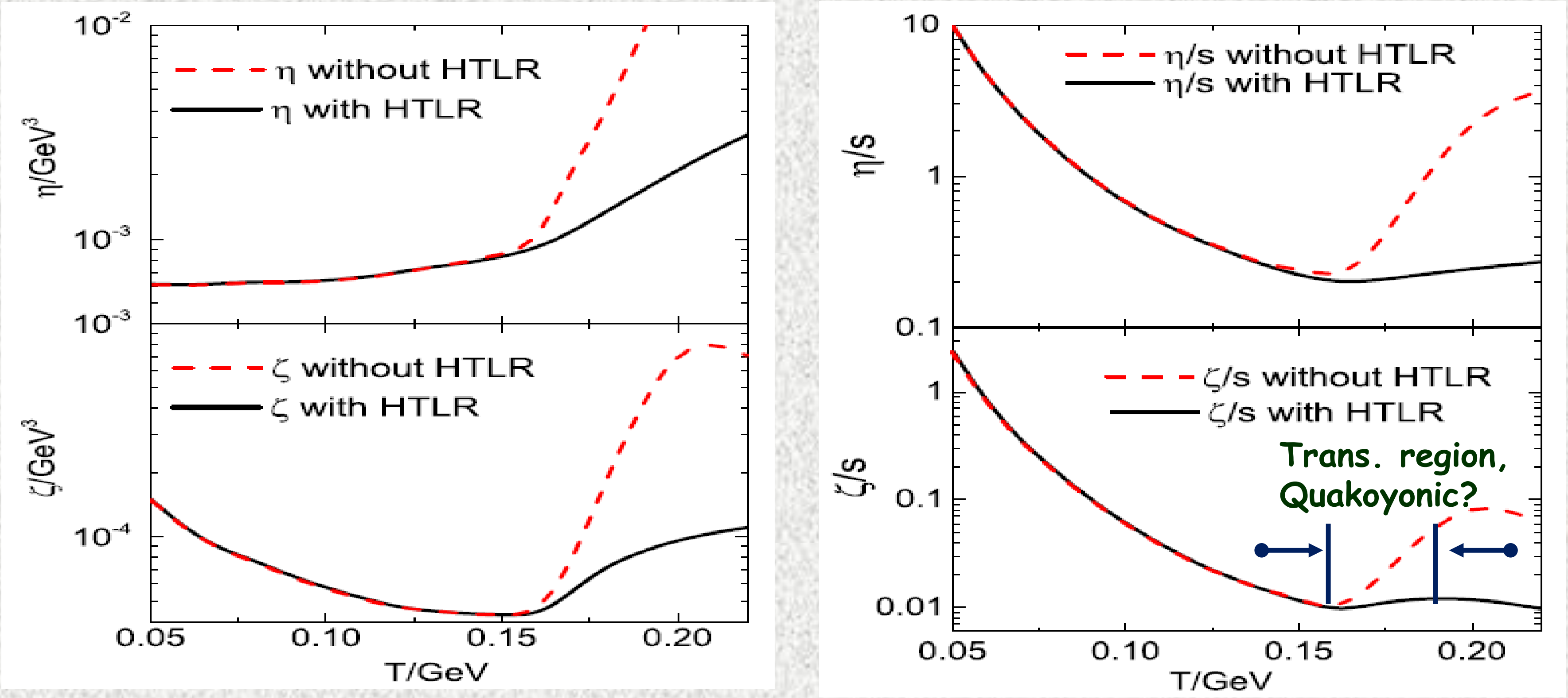
→ The viscosities & their ratios to the s .



→ Quarkyonic Phase is a Demonstration of the Ultrahot effect !