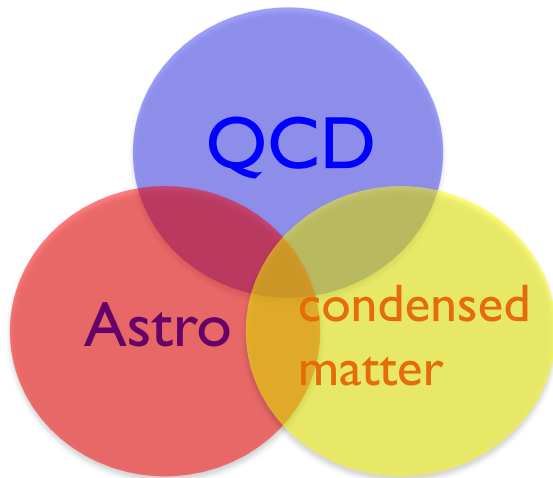


Delineating the properties of matter in cold, dense QCD

Toru Kojo (CCNU)



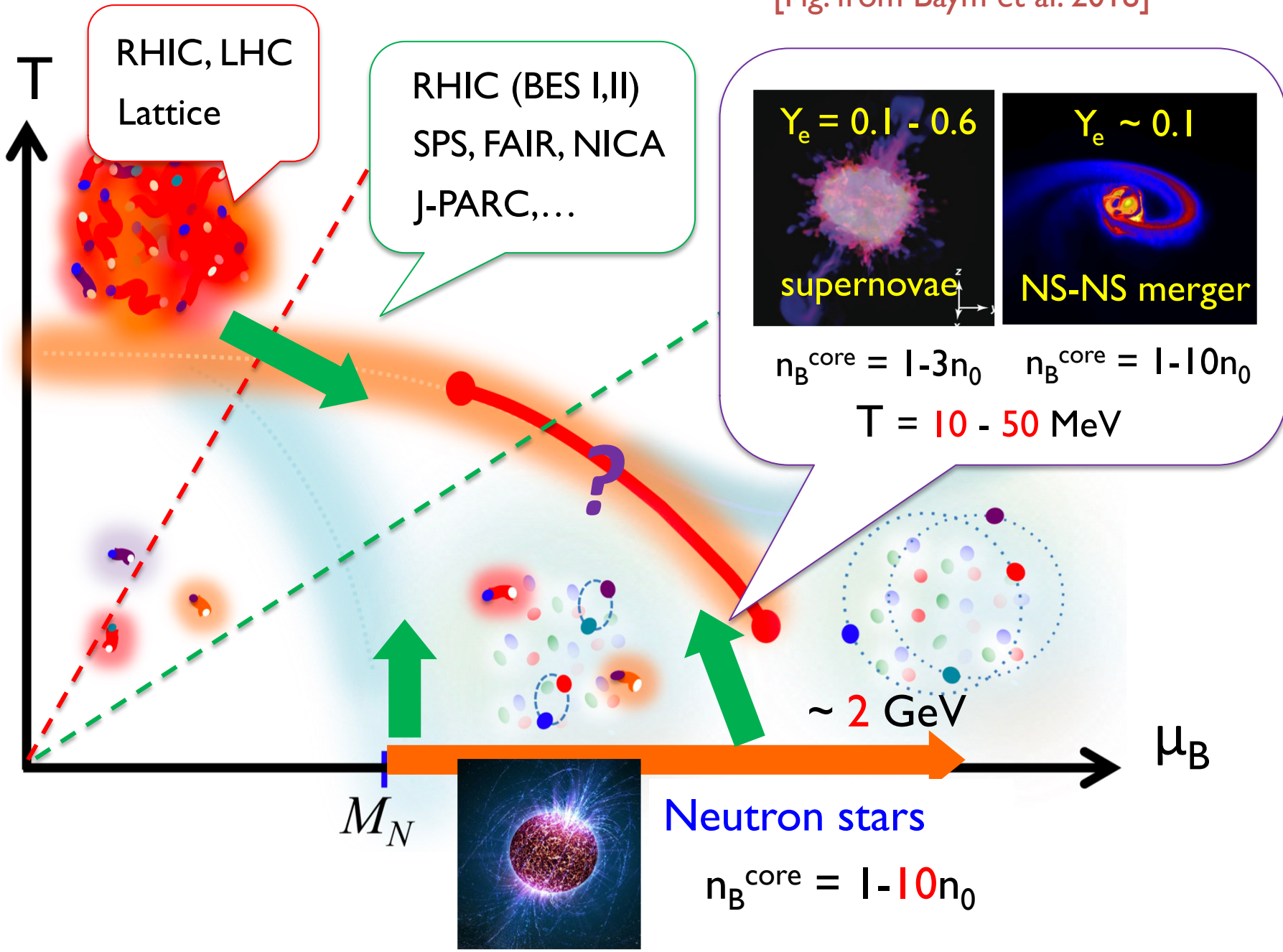
based on works in collaborations with

G. Baym, K. Fukushima, S. Furusawa, T. Hatsuda,
P. Powell, Y. Song, H. Togashi, T. Takatsuka

▪ Review) Rept. Prog. Phys. 81 (2018) no.5, 056902

- 1, Theoretical orientation from QCD
- 2, Quick review of constraints from NSs
- 3, Microphysics based on quark descriptions

[Fig. from Baym et al. 2018]



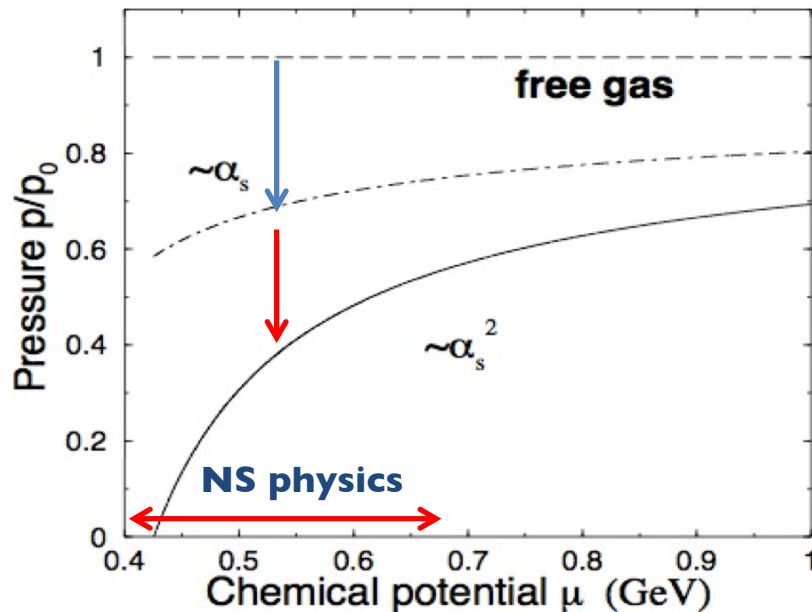
Cold, dense EoS : High density

3-loop pQCD : Freedman-McLerran 78; Baluni 78; Kurkela-Romatschke-Vuorinen 09

[some **4-loop** contributions: E. Sappi et al.]

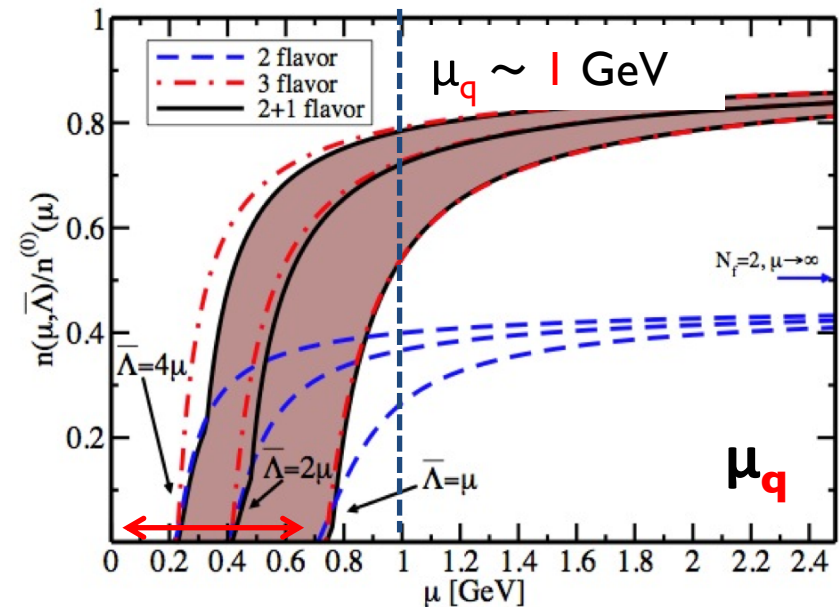
check of **convergence**

(Fraga-Pisarski-Schaffner-Bielich 01)



check of **renorm. scale dep.**

(Kurkela-Romatschke-Vuorinen 09)



- Interactions crucial already at $\mu_q \sim 1$ GeV or $n_B \sim 100 n_0$

Cold, dense EoS : **Low** density

For NS applications ($n_B = 2-10n_0$), the fundamental question is:

convergence of many-body forces

e.g.1) parameterized **pure neutron** matter EoS [Gandolfi+, 2009]

$$\varepsilon = n_0 \left[\overset{\sim\text{kin.} + 2\text{-body}}{(12 \pm 1 \text{ MeV}) \left(\frac{n_B}{n_0}\right)^{1.45 \pm 0.05}} + \overset{\sim 3\text{-body}}{(4 \pm 2 \text{ MeV}) \left(\frac{n_B}{n_0}\right)^{3.3 \pm 0.3}} \right]$$

e.g.2) Akmal-Pandharipande-Ravenhall EoS (**APR 98**) [Table V of APR paper]

**pure
neutron
matter**

n_B	2 -body int.		3 -body int.	
	$\langle v_{ij}^\pi \rangle$	$\langle v_{ij}^R \rangle$	$\langle V_{ijk}^{2\pi} \rangle$	$\langle V_{ijk}^R \rangle$
n_0	-4.1	-29.9	1.2	4.5
2 n_0	-25.1	-36.4	-17.4	30.6
3 n_0	-35.7	-44.7	-34.1	78.0
4 n_0	-52.2	-41.1	-76.9	160.3

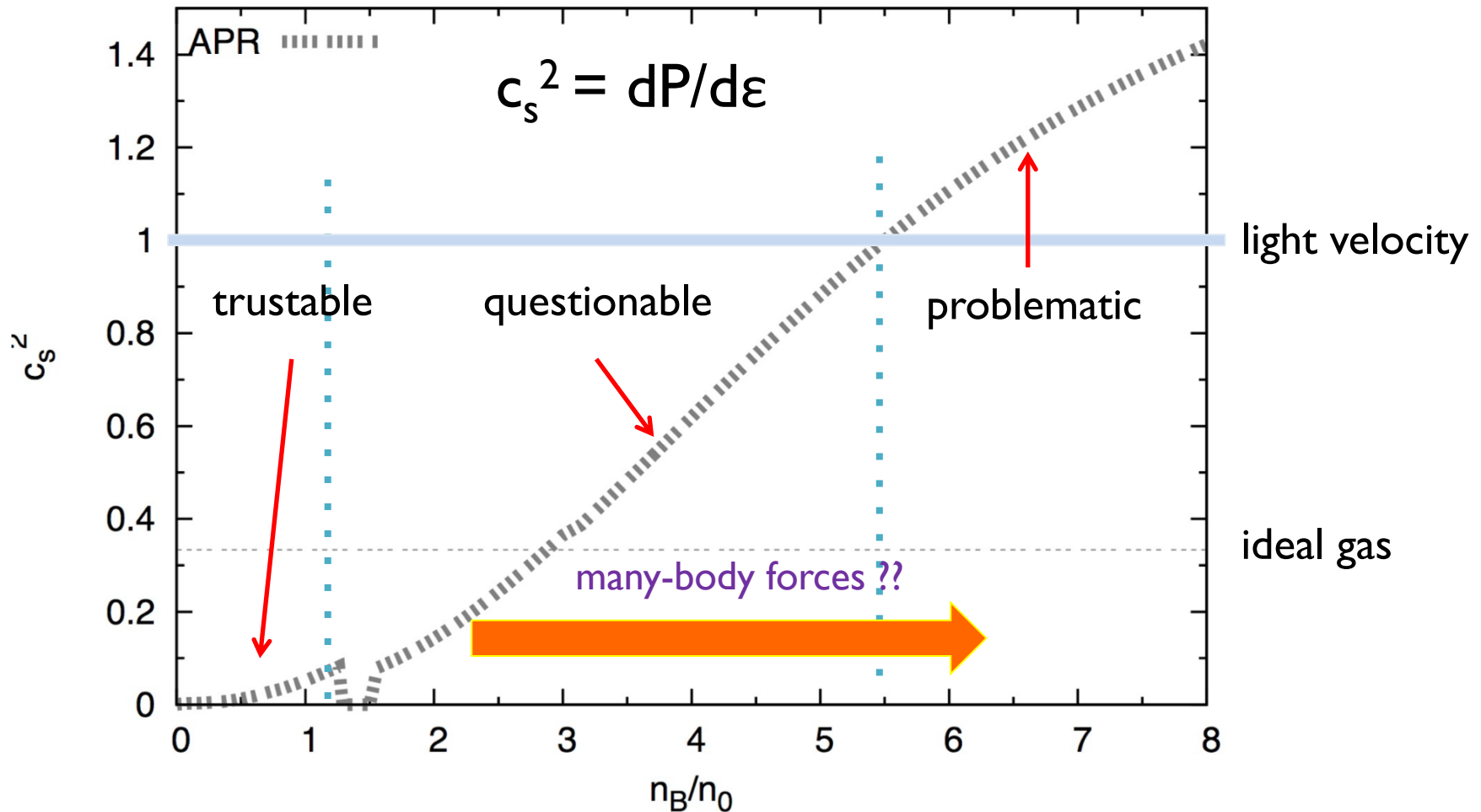
grow
rapidly!

4-, 5- or more-body forces
should be important as well
beyond $\sim 2n_0$

$$\langle V_{N\text{-body}} \rangle \sim (n_B/n_0)^N$$

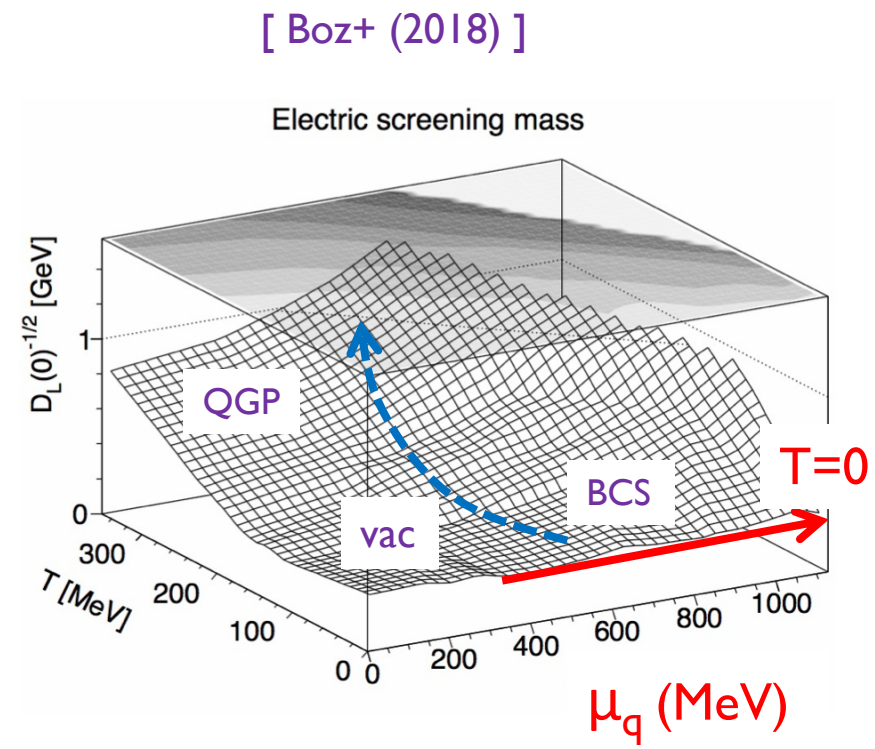
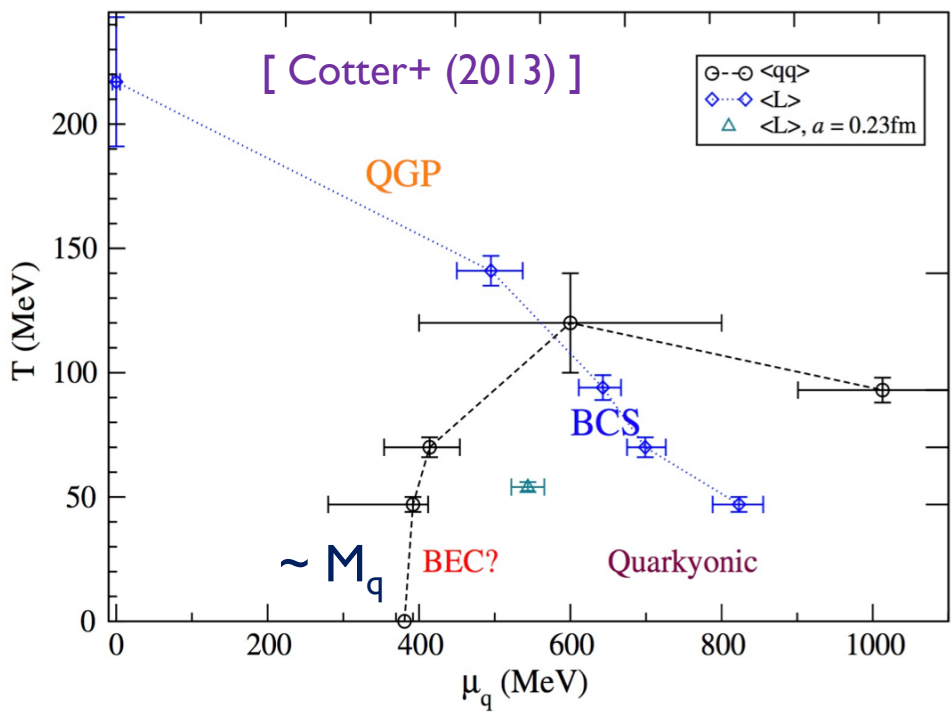
Cold, dense EoS : **Low** density

Akmal-Pandharipande-Ravenhall EoS (**APR 98**)



QCD with 2-colors ($m_{\pi} \sim 700 \text{ MeV} \sim 2M_q$)

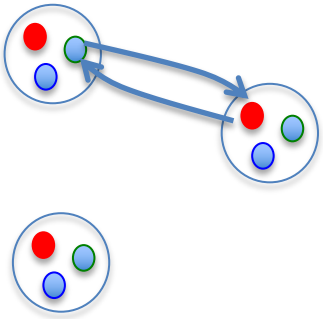
lattice QCD simulations doable



- $T_c^{\text{BCS}} \sim 100 - 120 \text{ MeV} \rightarrow \Delta \sim 1.75 T_c \sim \mathbf{175 - 210 \text{ MeV}}$
- Gluons are **insensitive** to medium effects for $\mu_q < 1 \text{ GeV}$

Overall picture based on QCD

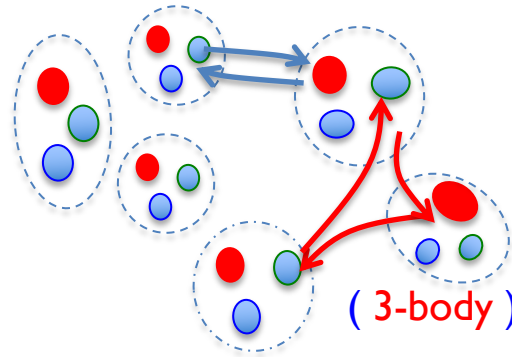
- few meson exchange
- nucleons **only**



ab-initio nuclear cal.
e.g.) ChEFT, variational

steady progress

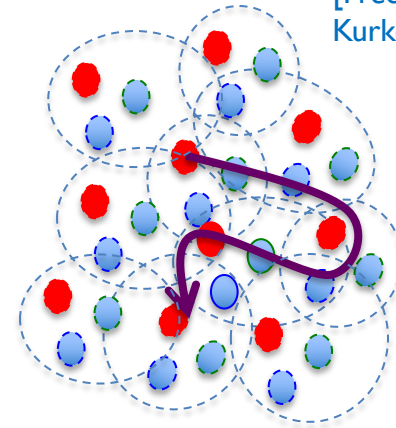
- many-quark exchange
- structural change
- hyperons, Δ , ...



most difficult
(d.o.f ??)

Hints from NSs

- Baryons overlap
- Quark Fermi sea



strongly correlated
(d.o.f : quasi-particles??)

not explored well

→
(pQCD)

$\sim 2n_0$

$\sim 5n_0$
($p_F \sim 400$ MeV)

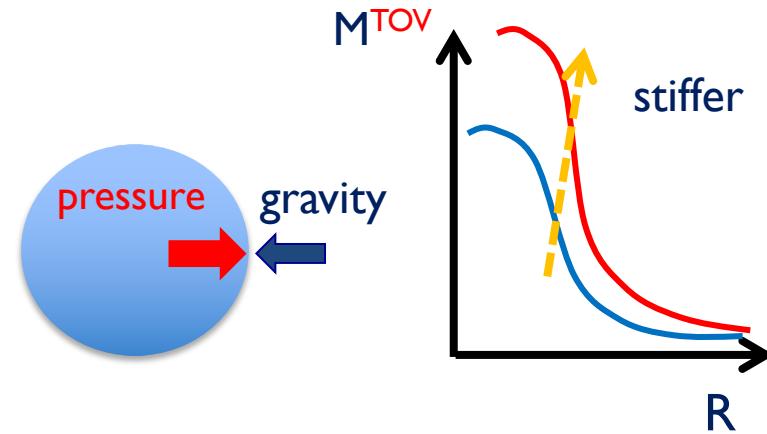
$\sim 100n_0$

n_B

Terminology (in this talk)

1) **Stiff** EoS : P is large at given ϵ

(not necessarily large $c_s^2 = dP/d\epsilon$)



2) Stiffness strongly depends on density; define, e.g.,

Soft-Stiff EoS : **Soft** at $n_B < 2n_0$ & **Stiff** at $n_B > 5n_0$

more specifically,



$$R_{1.4} < \sim 13 \text{ km}$$

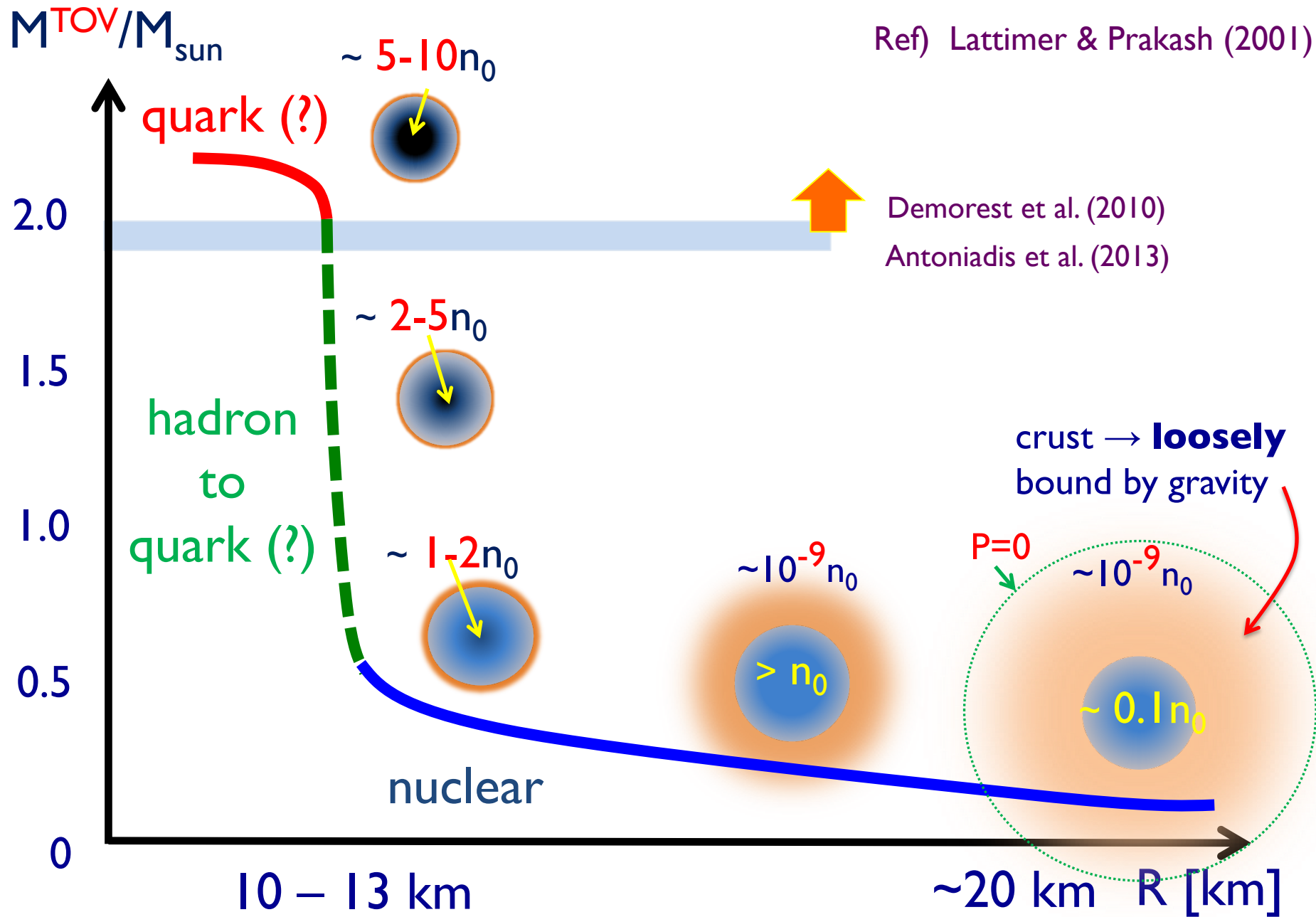


$$M > \sim 2 M_{\text{sun}}$$

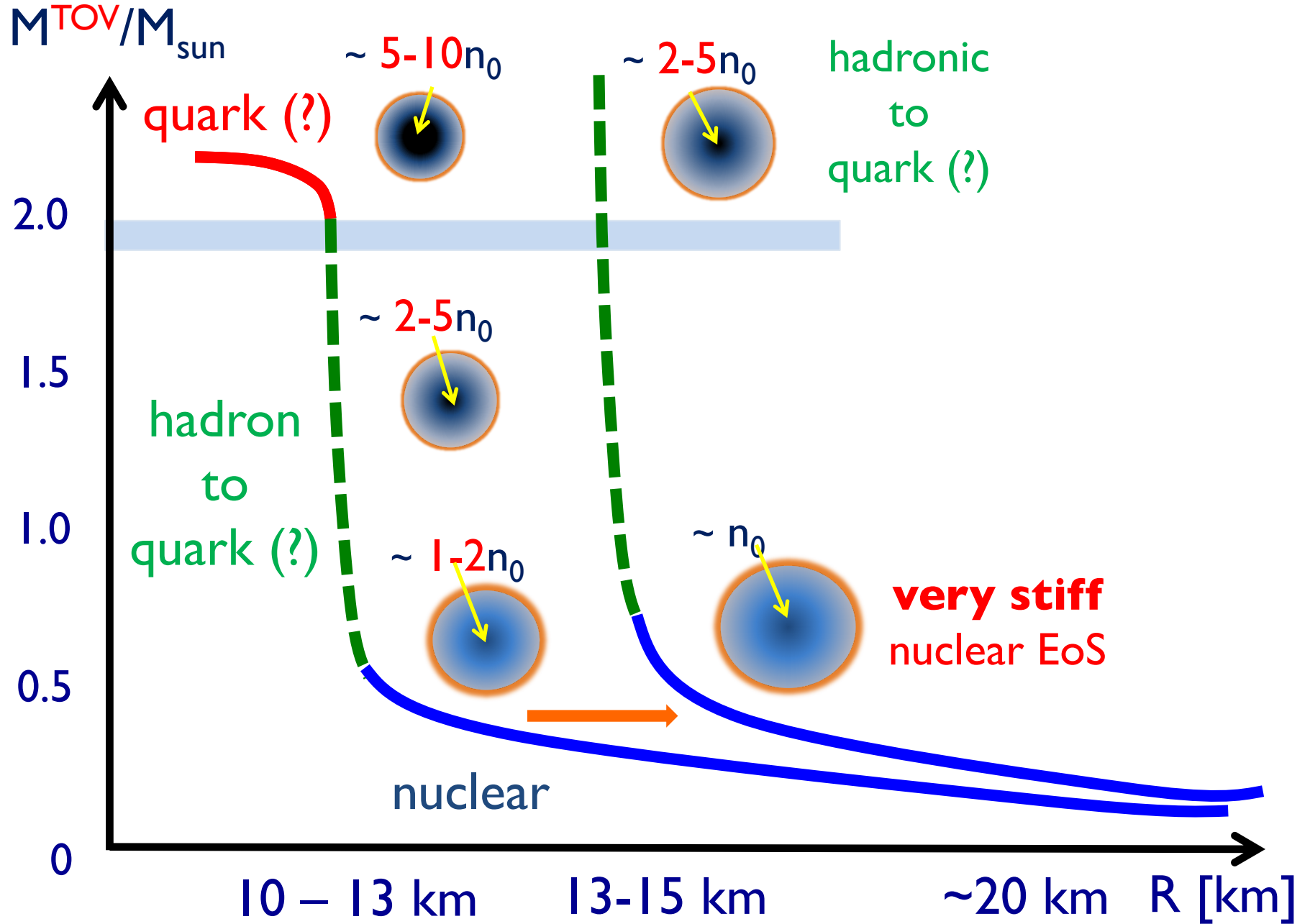
We also use terminology such as stiff-stiff EoS, etc.

Soft-Stiff vs Stiff-Stiff EoS

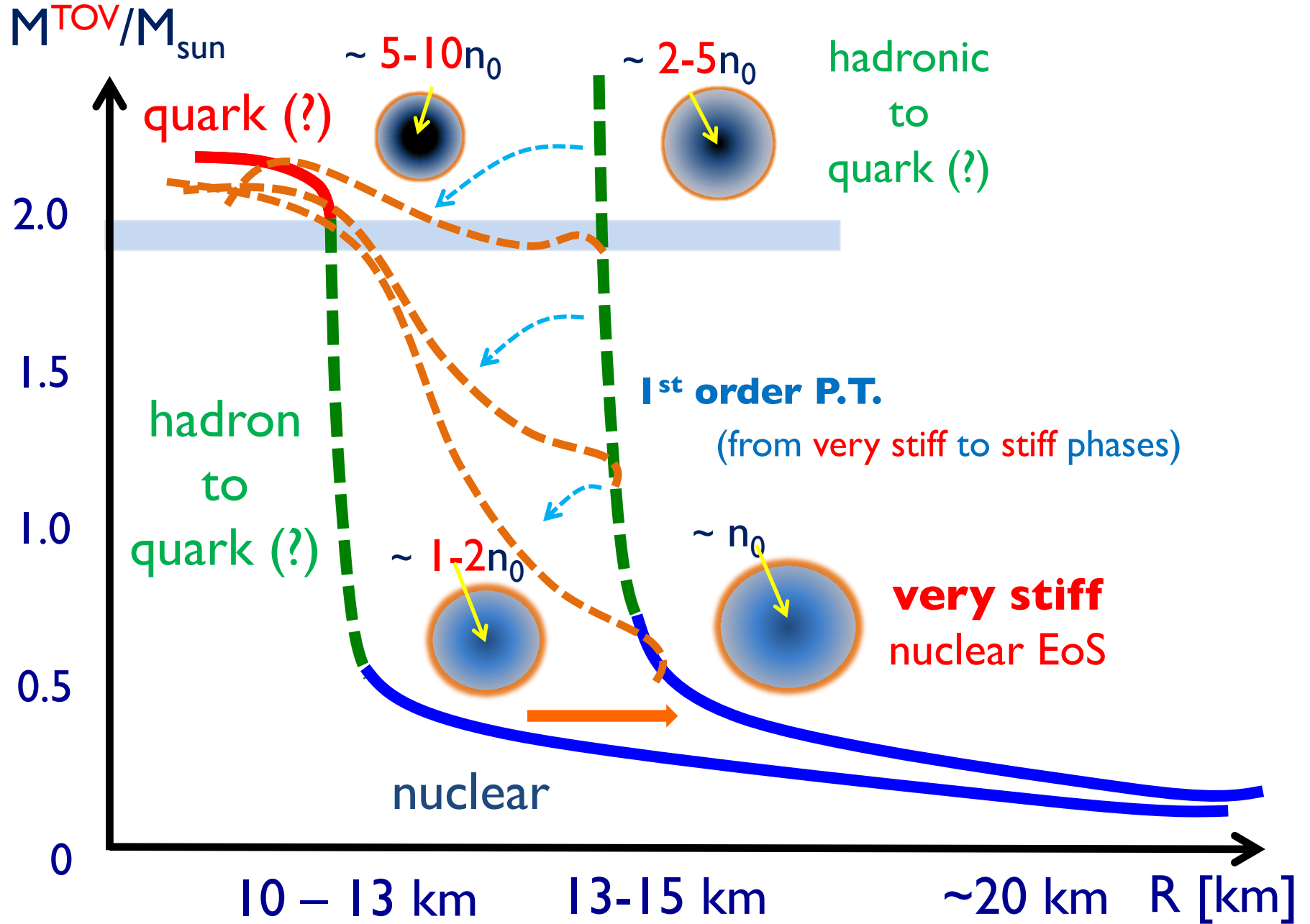
Ref) Lattimer & Prakash (2001)



Soft-Stiff vs Stiff-Stiff EoS

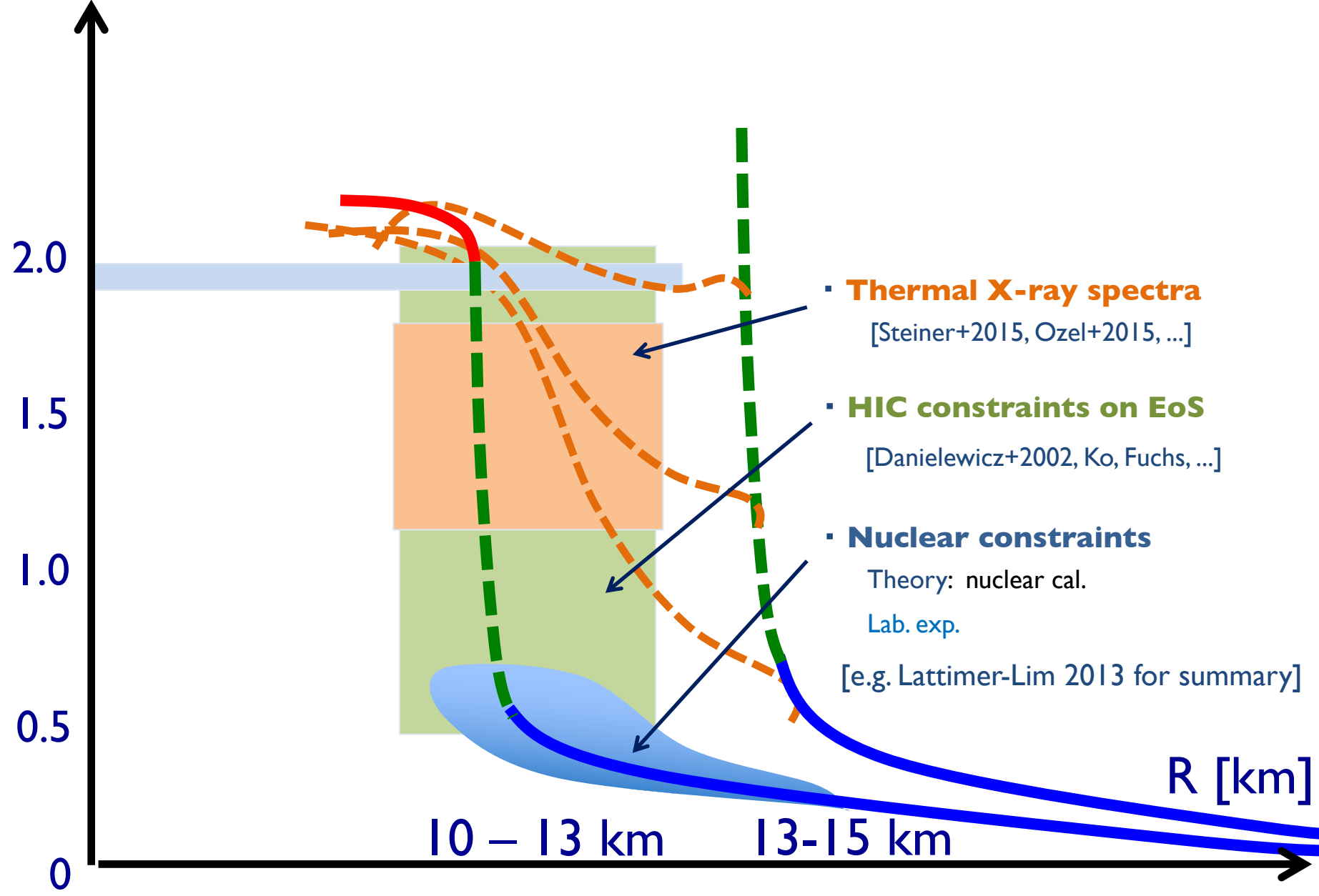


Soft-Stiff vs Stiff-Stiff EoS



Constraints (before GW170817)

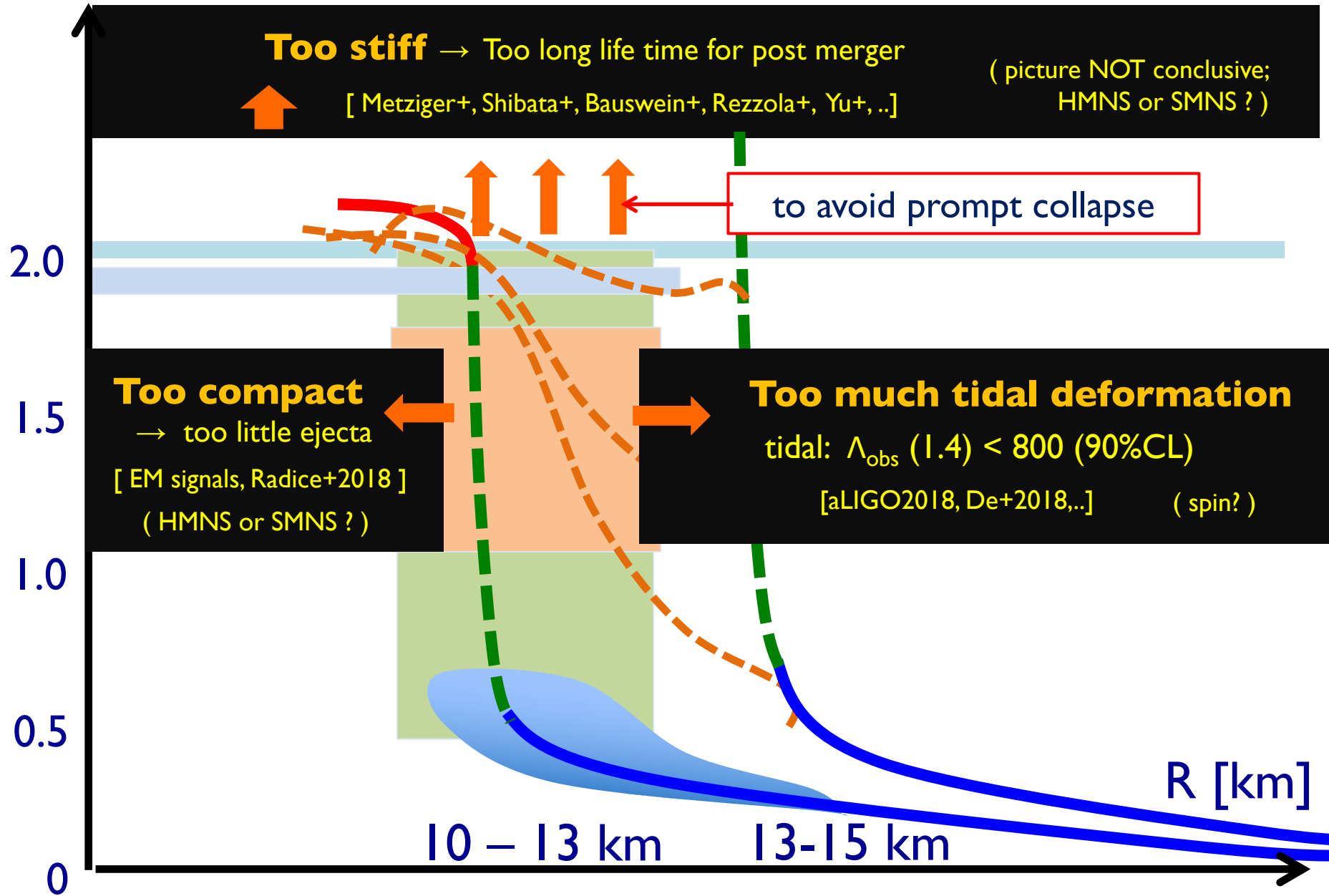
$M^{\text{TOV}}/M_{\text{sun}}$



$M^{\text{TOV}}/M_{\text{sun}}$

New Constraints

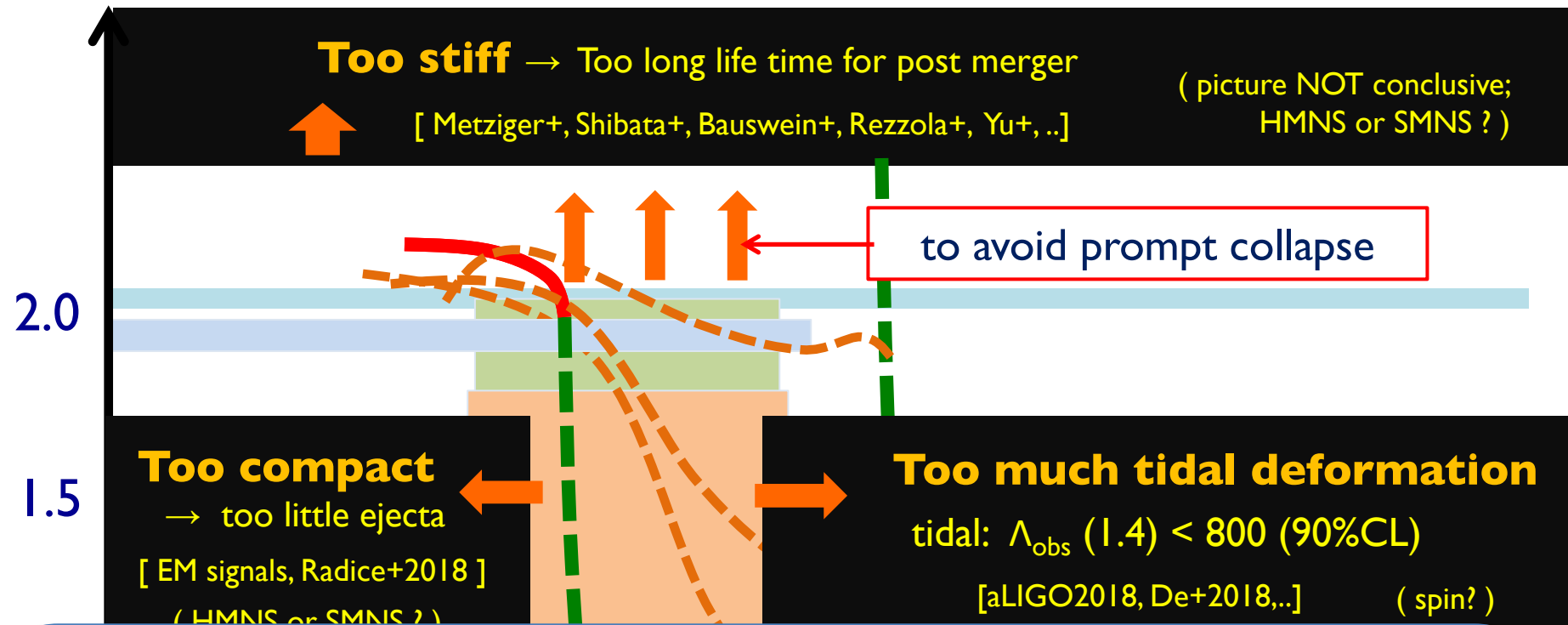
(GW170817; GRB170817A; AT2017gfo) 12/21



M^{TOV}/M_{sun}

New Constraints

(GW170817; GRB170817A; AT2017gfo)

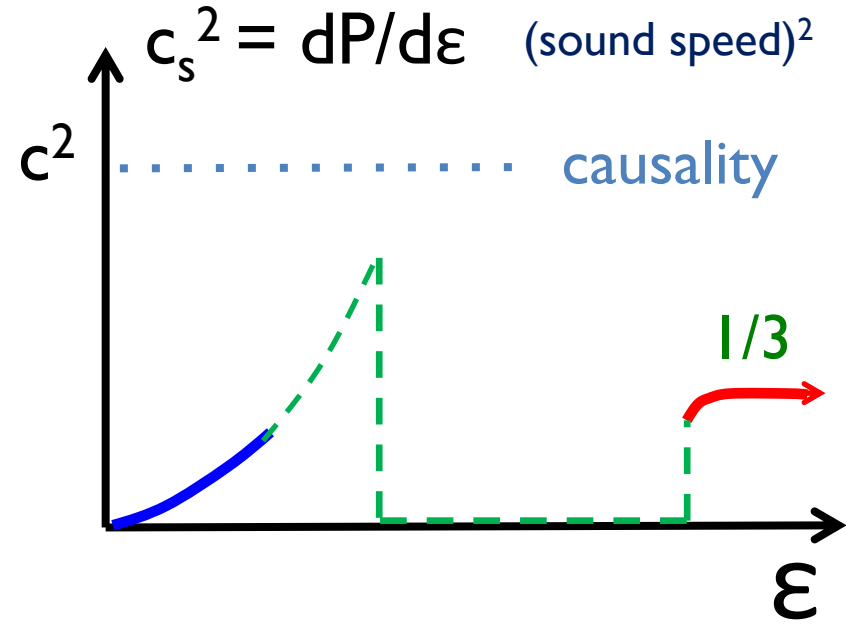
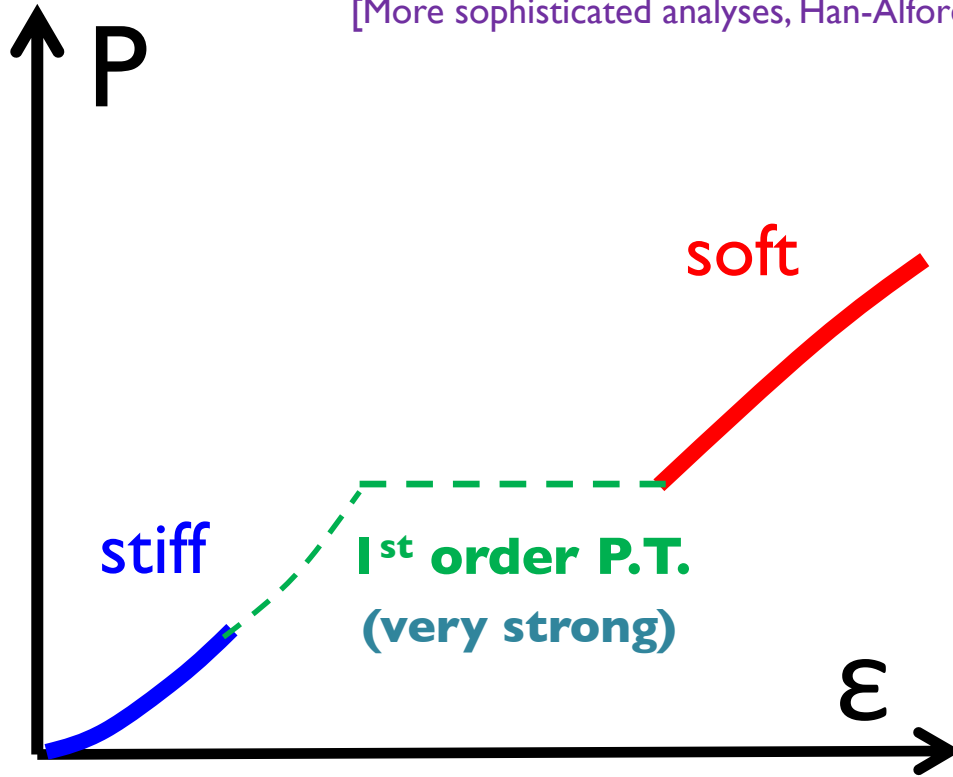


Hints for soft-stiff EoS

	$R(1.4)$ [km]		M^{TOV}_{max}/M_{sun}	
11.2 - 13.4	[Radice+2018]	<	2.17	[Metziger+2018]
9.0 - 13.6	[Tews+2018]		2.15 - 2.25	[Shibata+2018]
8.9 - 13.2	[De+2018]		1.97 - 2.33	[Rezzola+2018]
11.1 - 13.4	[Annala+2018]	>	2.25	[Yu+2018]

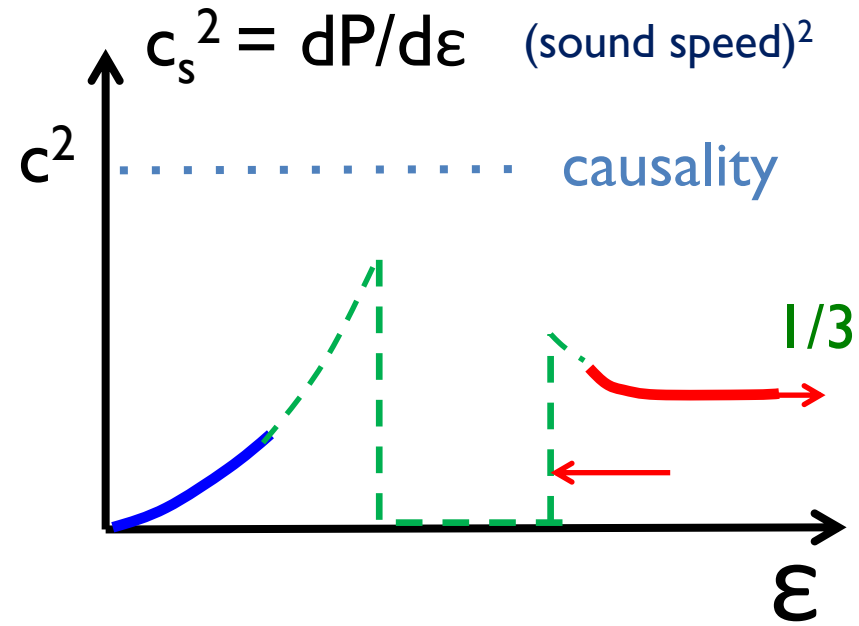
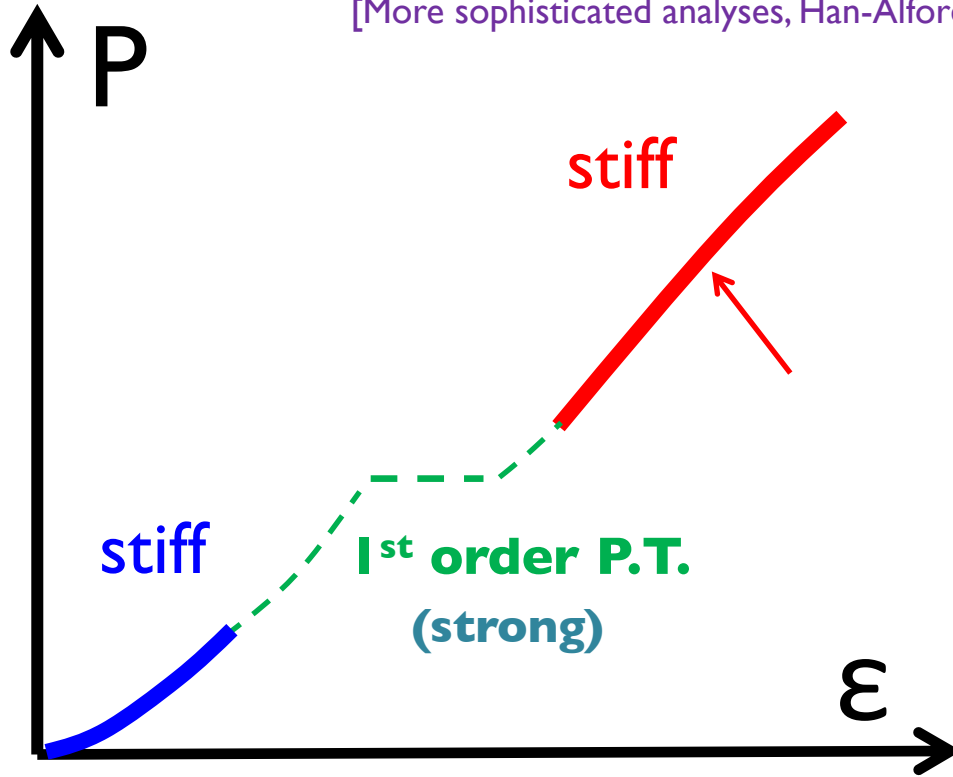
stiff-soft vs stiff-stiff vs soft-stiff

[More sophisticated analyses, Han-Alford-Prakash 2013, Bedaque-Steiner 2015]



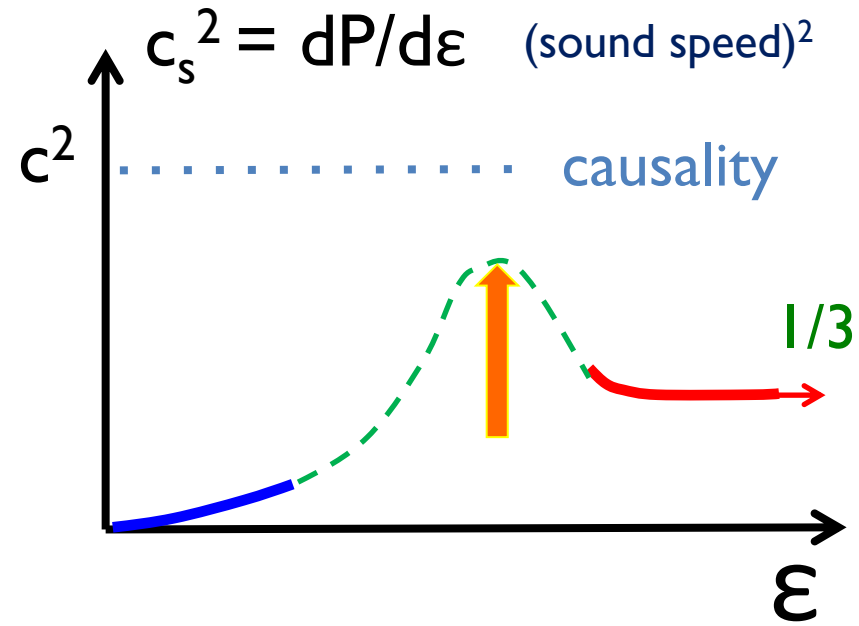
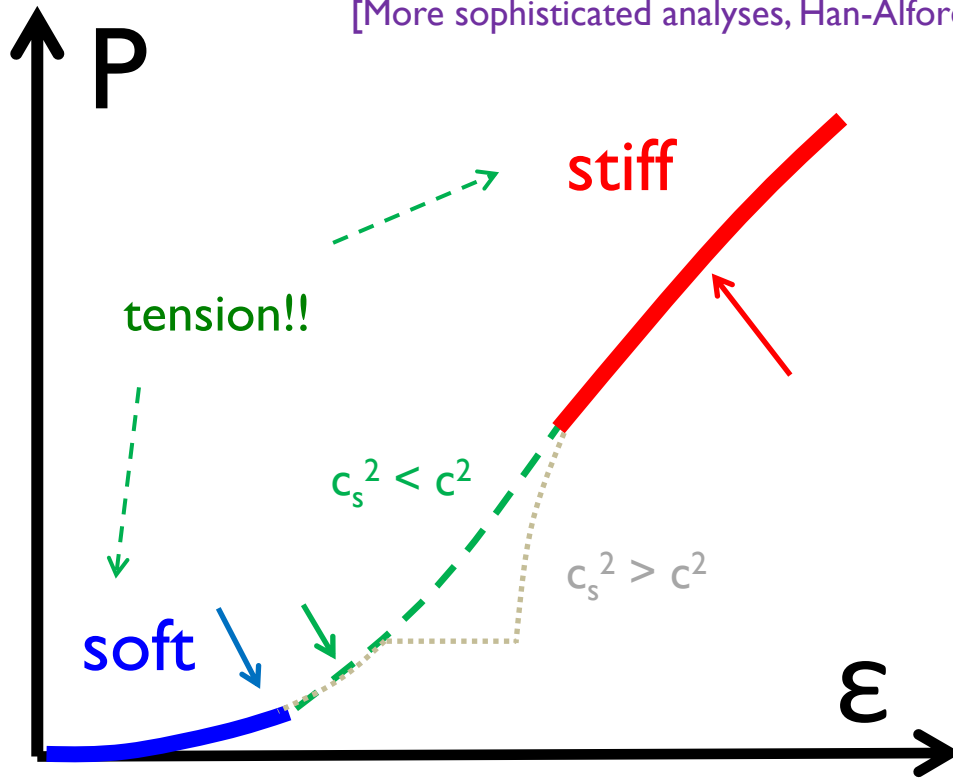
stiff-soft vs stiff-stiff vs soft-stiff

[More sophisticated analyses, Han-Alford-Prakash 2013, Bedaque-Steiner 2015]



stiff-soft vs stiff-stiff vs soft-stiff

[More sophisticated analyses, Han-Alford-Prakash 2013, Bedaque-Steiner 2015]



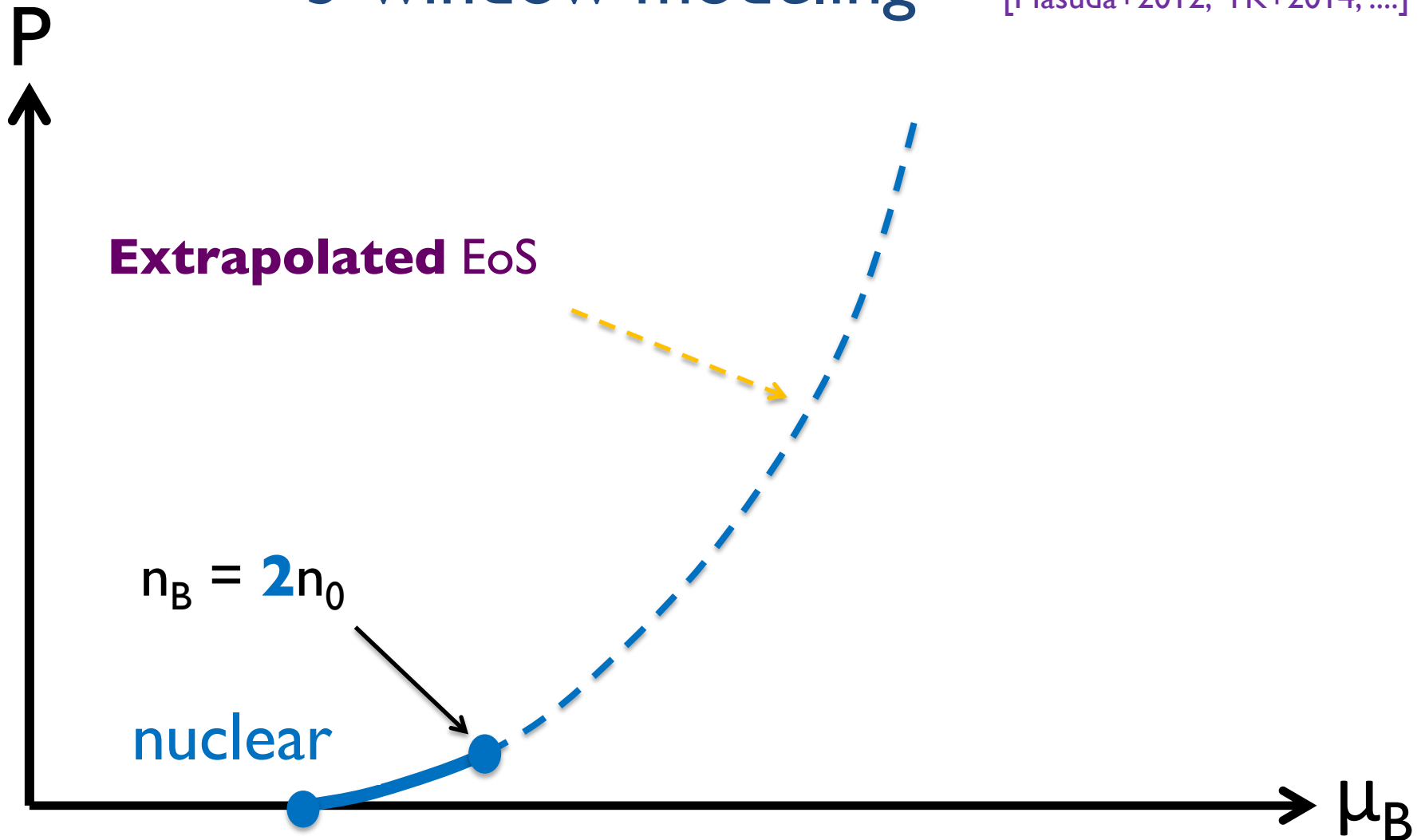
soft-stiff EoS \rightarrow **crossover** or **weak 1st order** for $2-5n_0$

"Hadron-quark continuity" (a new baseline ?)

[Schafer-Wilczek 1998, Hatsuda+ 2006, ...; cf) quarkyonic; McLerran-Pisarski 2007]

3-window modeling

[Masuda+2012, TK+2014, ...]

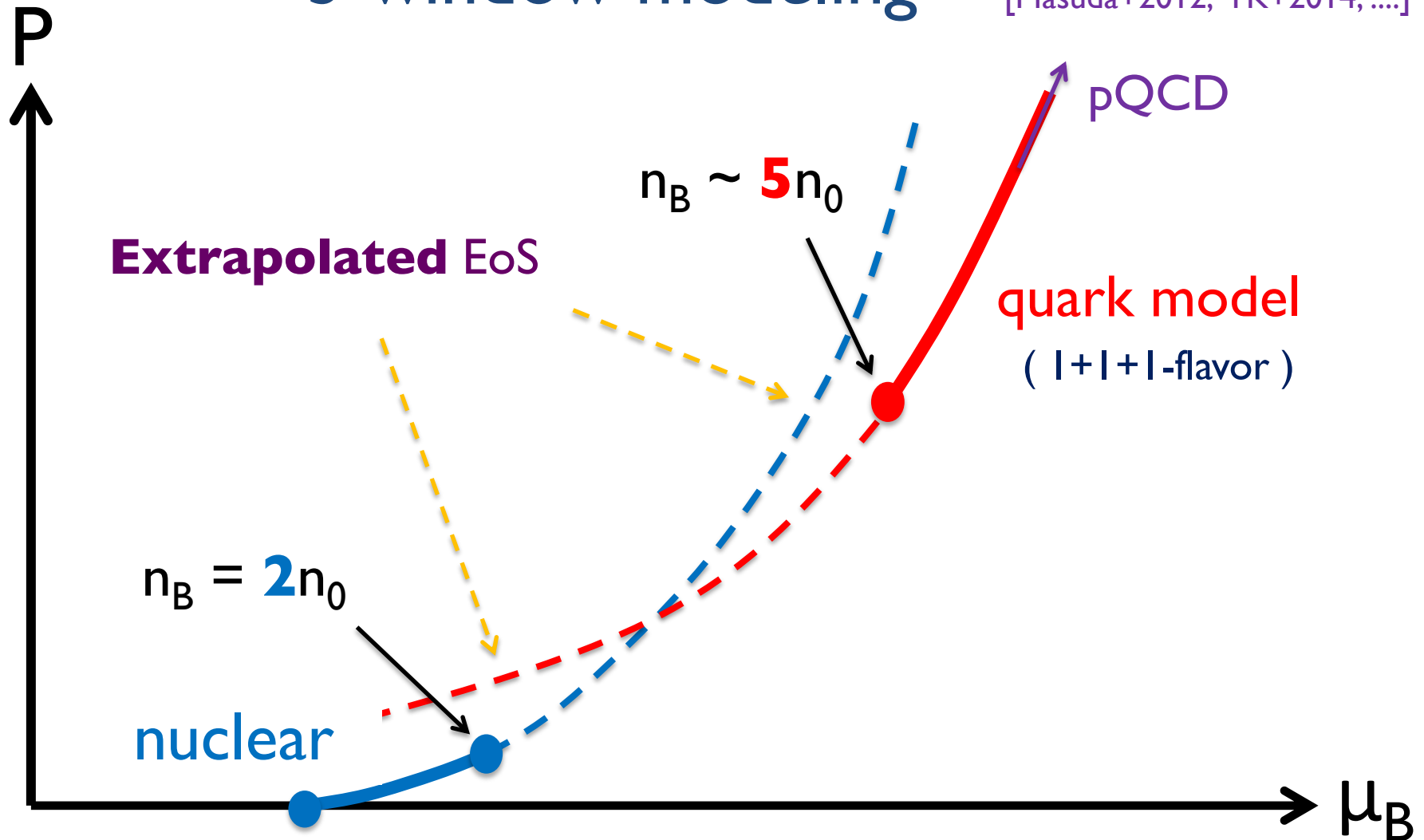


[Akmal+1998, Togashi+2017,
Hebeler+2017, Gandolfi+, ...]



3-window modeling

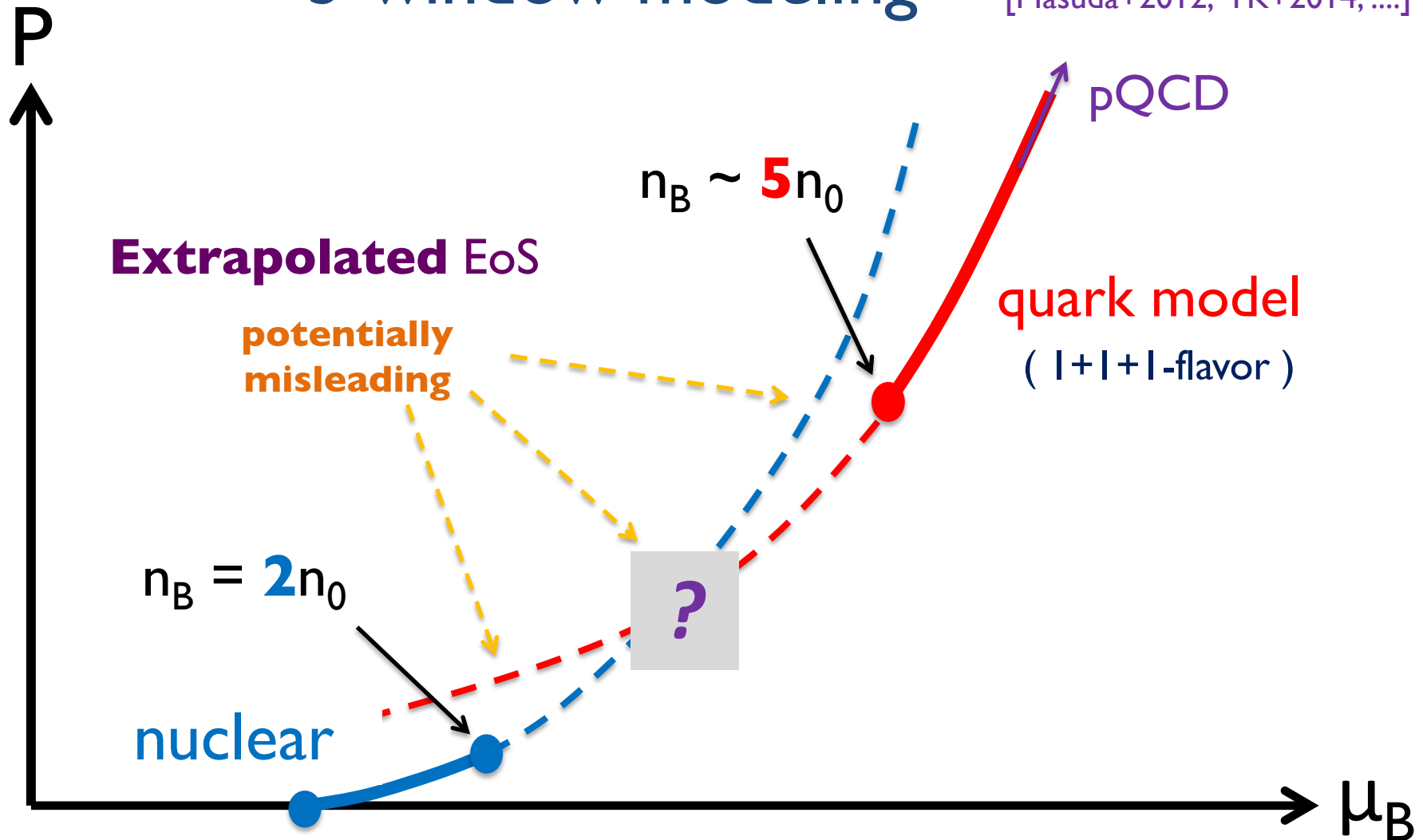
[Masuda+2012, TK+2014, ...]



[Akmal+1998, Togashi+2017,
Hebeler+2017, Gandolfi+, ...]

3-window modeling

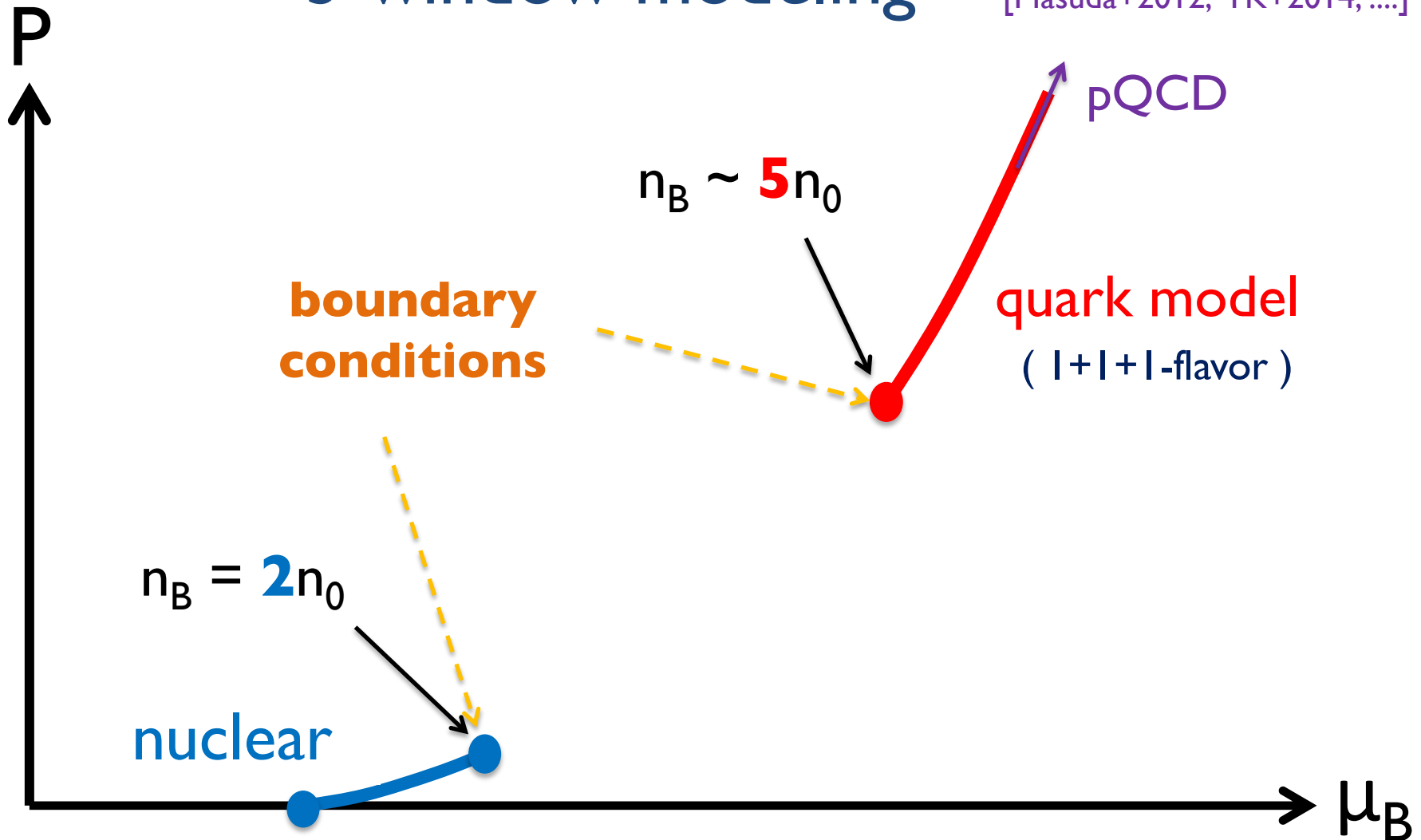
[Masuda+2012, TK+2014, ...]



[Akmal+1998, Togashi+2017,
Hebeler+2017, Gandolfi+, ...]

3-window modeling

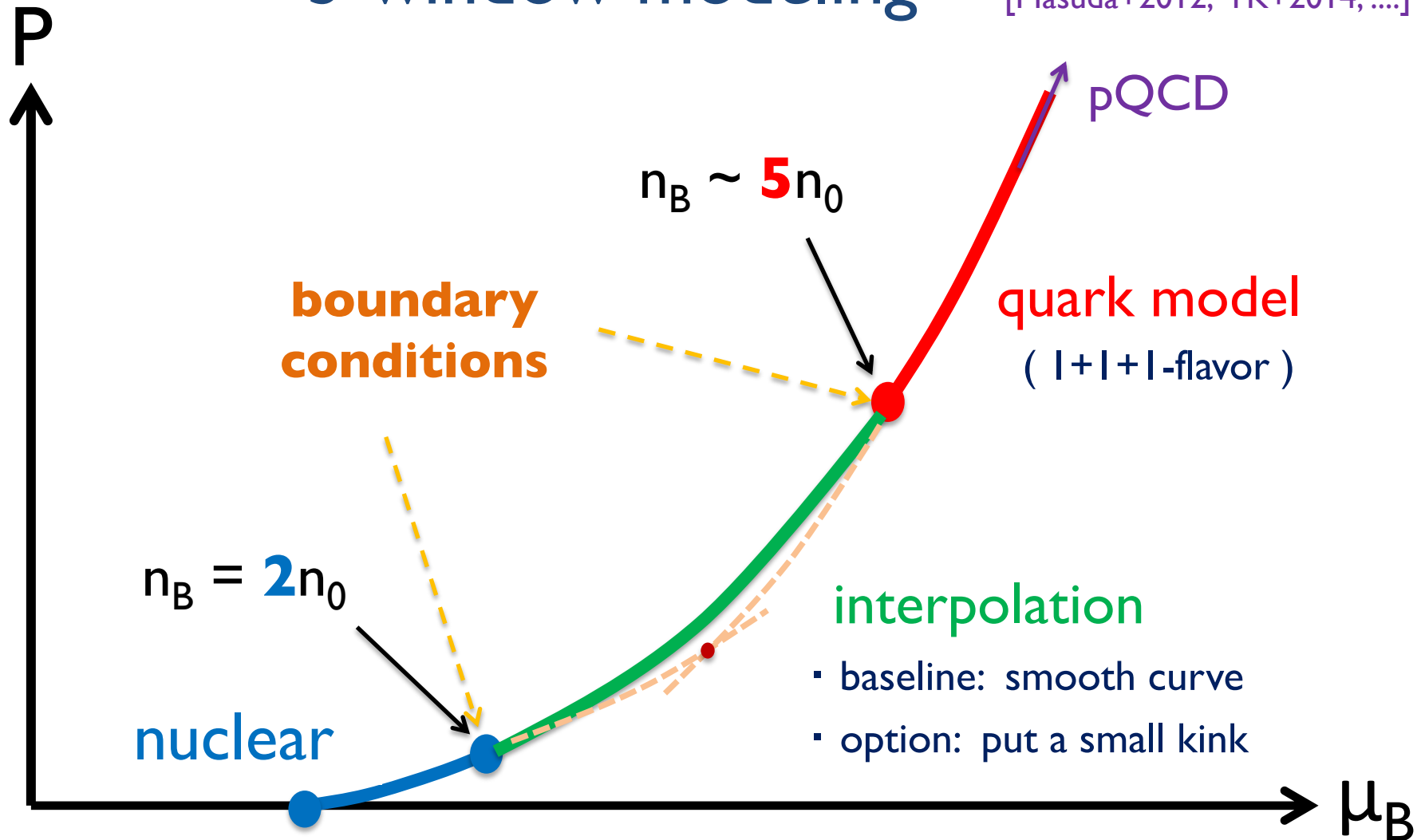
[Masuda+2012, TK+2014, ...]



[Akmal+1998, Togashi+2017,
Hebeler+2017, Gandolfi+, ...]

3-window modeling

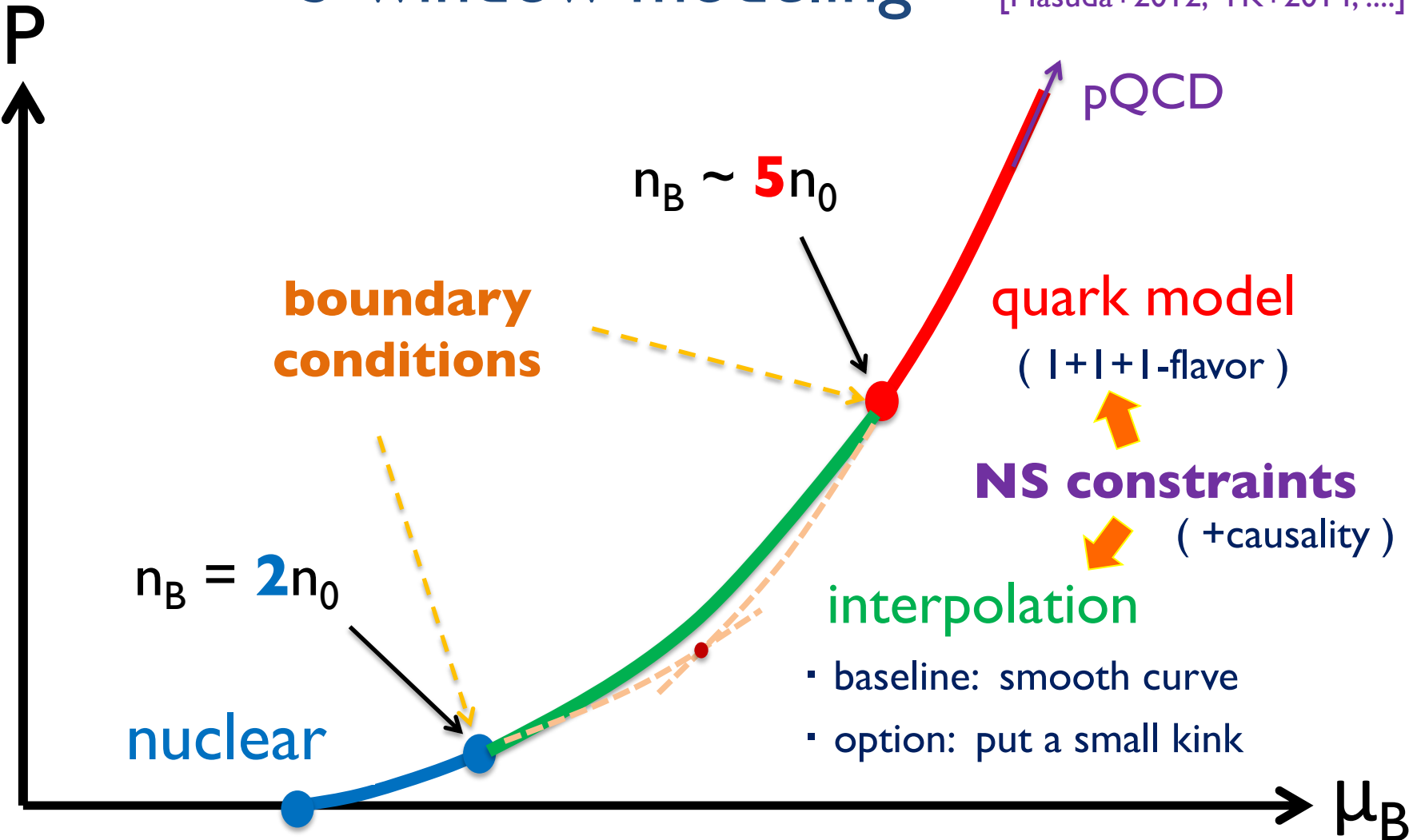
[Masuda+2012, TK+2014, ...]



[Akmal+1998, Togashi+2017,
Hebeler+2017, Gandolfi+, ...]

3-window modeling

[Masuda+2012, TK+2014, ...]



[Akmal+1998, Togashi+2017, Hebeler+2017, Gandolfi+, ...]

- baseline: smooth curve
- option: put a small kink

A quark model for $n_B > \sim 5n_0$

H-Q continuity : eff. Hamiltonian continuously evolves from hadron physics

"3-window" [Manohar-Georgi 1983, Weinberg 2010,...]

$Q < \sim 0.2 \text{ GeV}$

very long-range ($> 1\text{fm}$)

confinement

$0.2 \text{ GeV} < Q < 1\text{-}2 \text{ GeV}$

constituent quarks + OGE
(quasi-particles)

chiral SB & color-mag. int.
& **baryon-baryon. int.**

$\sim 2 \text{ GeV} < Q$

short range

pQCD

A template)

chiral

color-mag.

nB-nB int.

$$\mathcal{H} = \mathcal{H}_{\text{NJL}} - \underline{H} \sum_A (q\Gamma_A q)(\bar{q}\Gamma_A \bar{q}) + \underline{g_V} (\bar{q}\gamma_0 q)^2$$

[Masuda+2015, TK+2014, Blaschke+....]

(for the moment phenomenological,
should be derived from color-mag. int.)

Color-mag. int. : multi-facet aspects

1) **Coupling** \propto **velocity** \sim **p/E**

more important in relativistic regime & high density

2) **Pairing** : smaller color- & flavor- reps are favored

(\rightarrow anti-symmetric w.f.) [cf DeRujula+ (1975), Isgur-Karl (1978), ...]

hadron mass ordering: N- Δ splitting, etc.

source of color-super-conductivity [Alford, Wilczek, Rajagopal, Schafer, ... 1998-]

3) **Baryon-Baryon int.** : short-range correlation

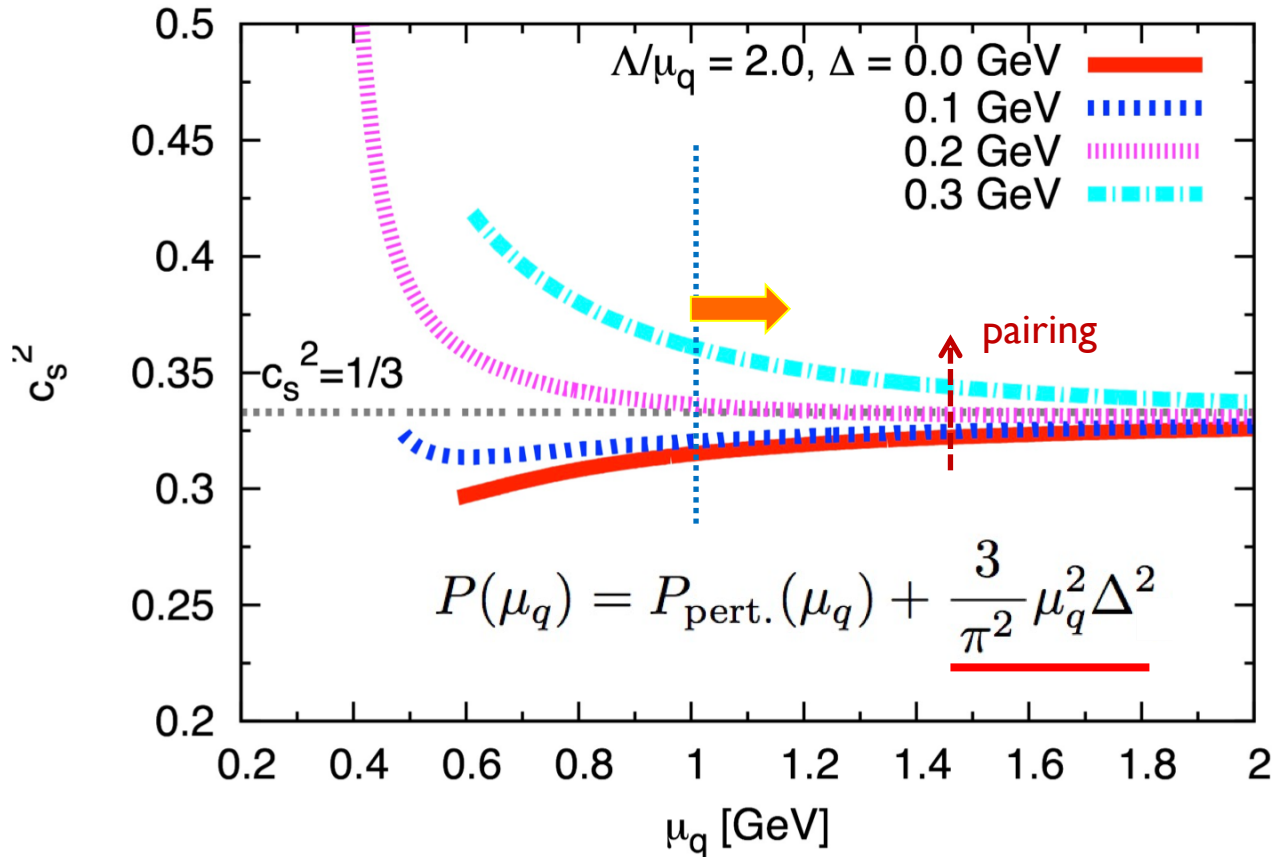
(**Pauli + color-mag.**) [Oka-Yazaki (1980), ...]

channel dependence \rightarrow **non-universal** hard core (some are attractive!)

mass dependence \rightarrow weaker repulsion for **strangeness**

\rightarrow remarkably **consistent with the lattice QCD** [HAL-collaboration]

Remark) pQCD + pairing vs speed of sound



[pQCD: Kurkela + (2009)]

For $\Delta > \sim 0.2 \text{ GeV}$

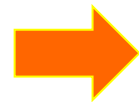
$$(\Delta / \mu_q)^2 \sim 4 \%$$

nevertheless,

c_s^2 approach $1/3$
from **above**

$$P \sim \mu^4 \rightarrow c_s^2 = 1/3$$

$$P \sim \mu^2 \rightarrow c_s^2 = 1$$



If μ^2 dominates $\rightarrow c_s^2 > 1/3$

(perhaps Fermi surface effects)

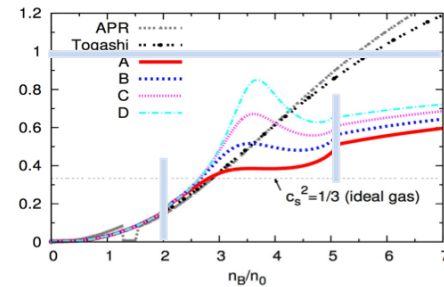
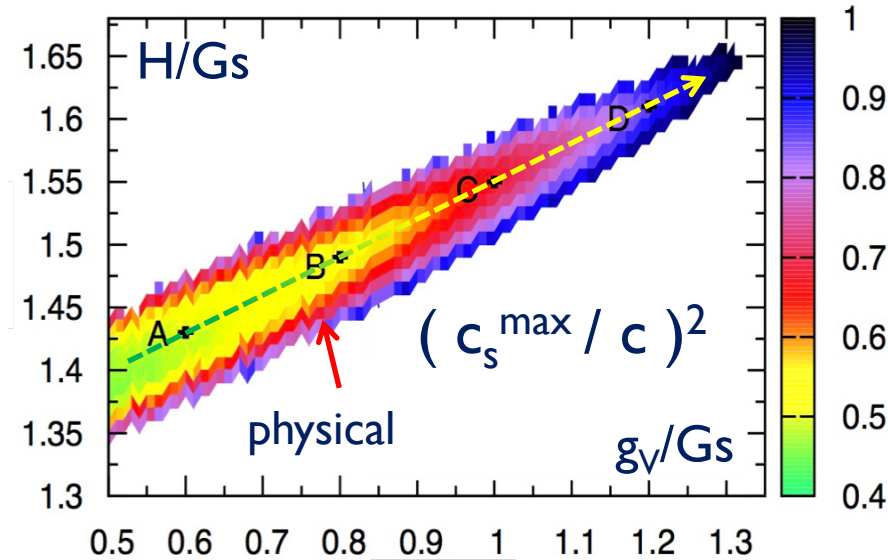
[see also McLerran-Reddy (2018)]

An exercise: survey for (g_V, H) @5n0

Step 1) Prepare **realistic** nuclear EoS for **0.5-2n0**

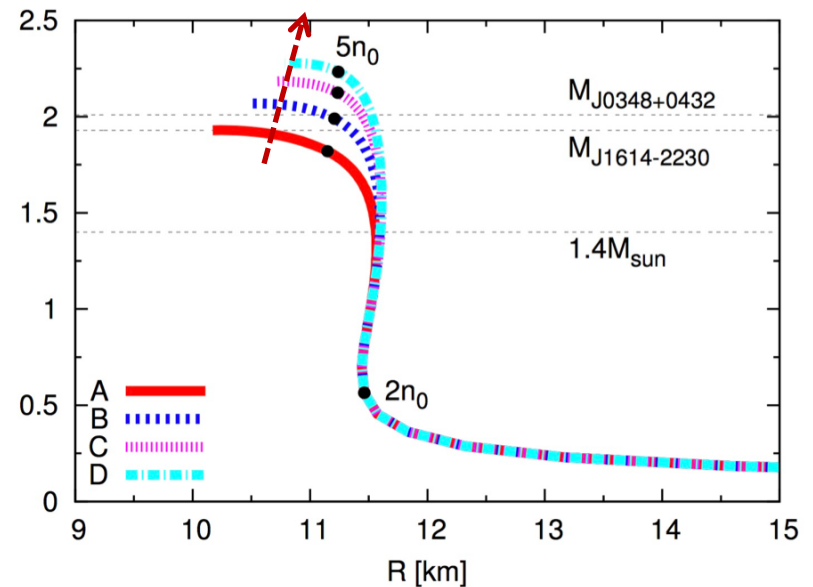
[e.g. Akmal+1998, Togashi+2017, ChEFT, ...] \rightarrow **radius constraint OK**

Step 2) Survey the range of (g_V, H) compatible with **causality**



Step 3) Impose **2M_{sun}** constraint

found: $M_{\max} < 2.35 M_{\text{sun}}$



Trends found in this exercise

for quark EoS consistent with all constraints

- **Strangeness** seems **unavoidable** for **soft-stiff EoS** :

$$n_s \sim n_u \sim n_d \quad \text{for } n_B > 5n_0 \quad (\mu_B > \sim 1.5 \text{ GeV} \sim 3M_s)$$
- **Pairing**; in MF, color-flavor-locked condensates (color-super)
stiffening at **high** n_B
- **Repulsive n_B - n_B int.**
tempers the growth of $n_B \rightarrow$ smooth chiral restoration
- **not acceptable** for conventional hybrid construction
in this sense, a **new class** of quark EoS

Summary

Coherent studies essential (Astro-QCD-condensed matter)

- Hints for **soft-stiff** EoS & **causality** → H-Q continuity
- Quark descriptions important :
 - **B.C. for EoS** & **BB-int.** & **hyperon problems**
- Pairing important :
 - Hard to prevent theoretically.
 - Temper softening associated with strangeness.
- Quark descriptions at **5 - 100** n_0 can & should be improved.
(work in progress)

Back Up

"3-window": constituent quarks for hadrons

cf) Manohar-Georgi (1983), Weinberg (2010)

> **1-2 GeV**

(< 0.2 fm)

~ **1 GeV**

(~ 0.2-1 fm)

$$\Lambda_\chi \sim 4 \pi f_\pi$$

< ~ **0.2 GeV**

(> ~ 1 fm)

Perturbative

weakly coupled quarks & gluons

Chiral + OGE

(one-gluon-exchange)

ChSB -> constituent quark mass ~300 MeV

OGE -> int. b.t.w **constituent** quarks

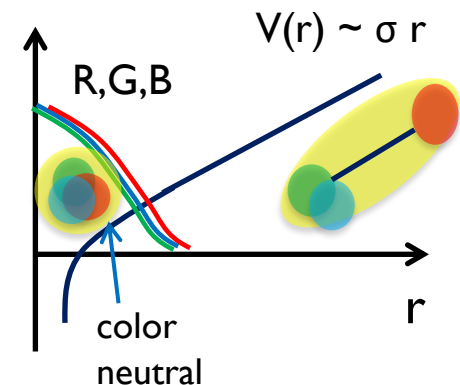
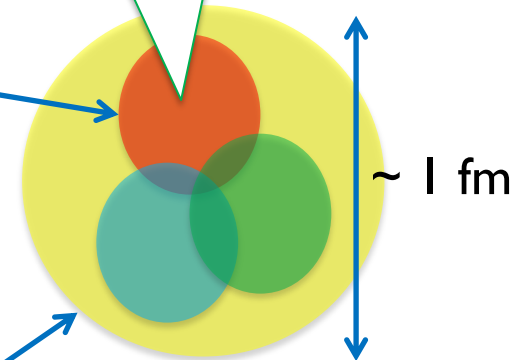
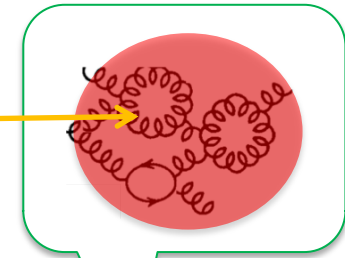
based on **quasi-particle** picture

Confinement

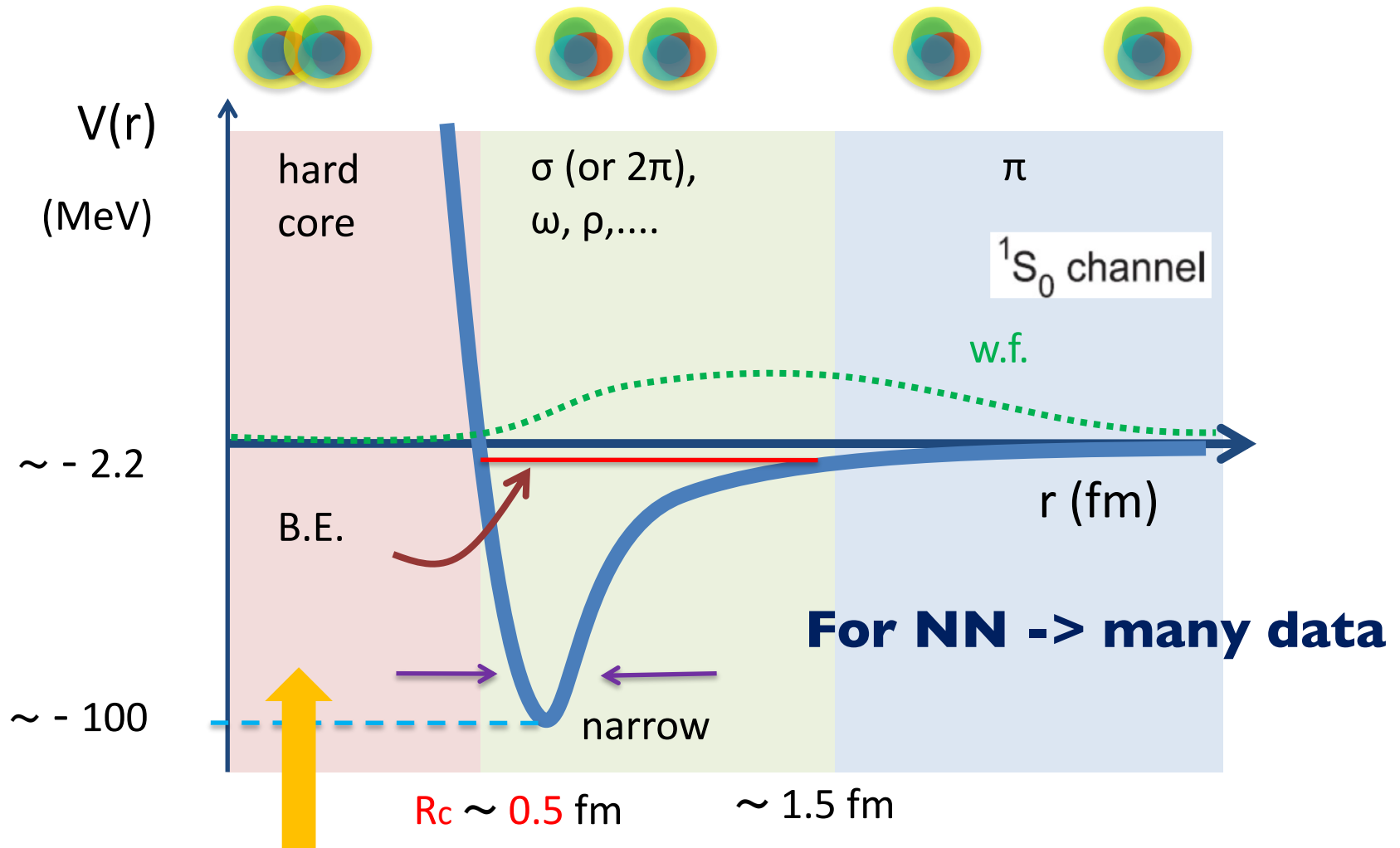
trap quarks to keep color white

quasi-particle gluons

→ **unlikely** generate confining forces



"3-window": N-N interactions



How about other channels, NY, YY, N Δ , ??

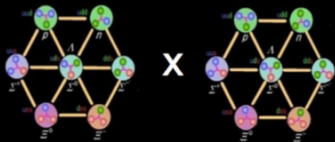
Baryon-Baryon int. on a lattice ($SU_f(3)$ limit)

[Hatsuda's talk at NFQCD2018]

8x8 BB interactions with flavor basis: $V_C(r)$

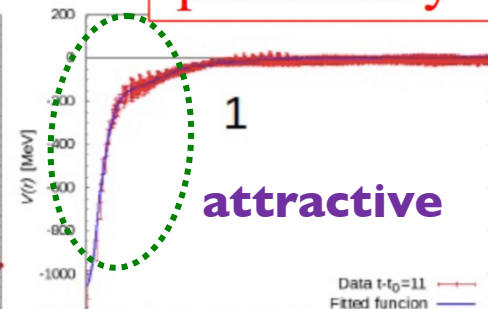
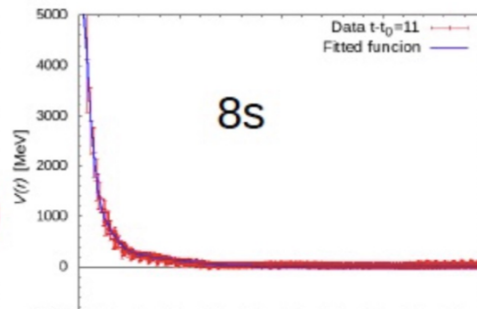
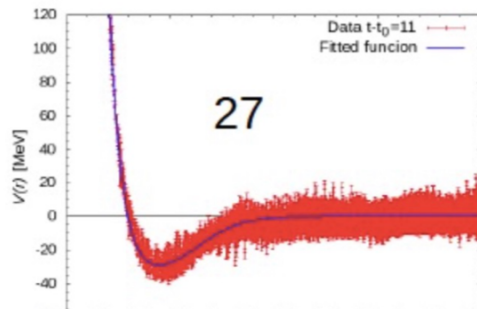
K. Sasaki+ HAL QCD Coll.

T. Inoue+ HAL QCD Coll.

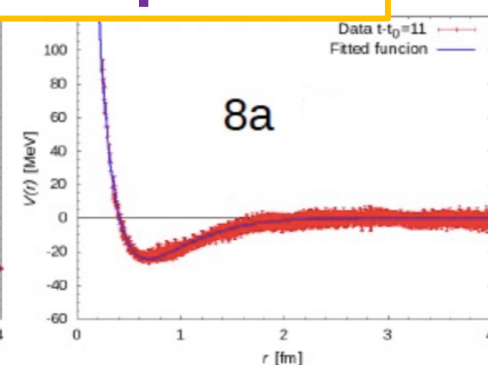
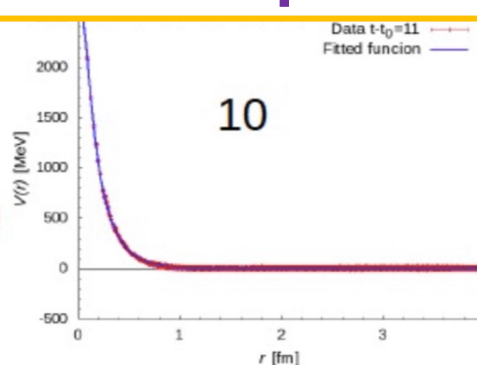
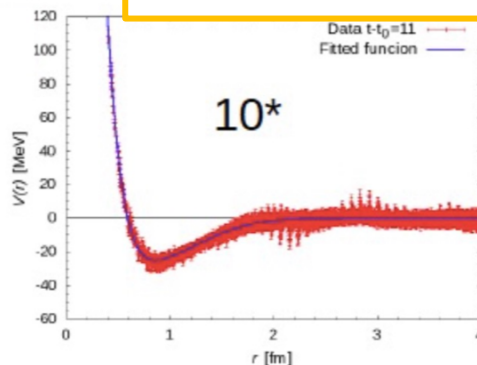


$$8 \times 8 = \frac{27 + 8s + 1}{^1S_0} + \frac{10^* + 10 + 8a}{^3S_1, ^3D_1}$$

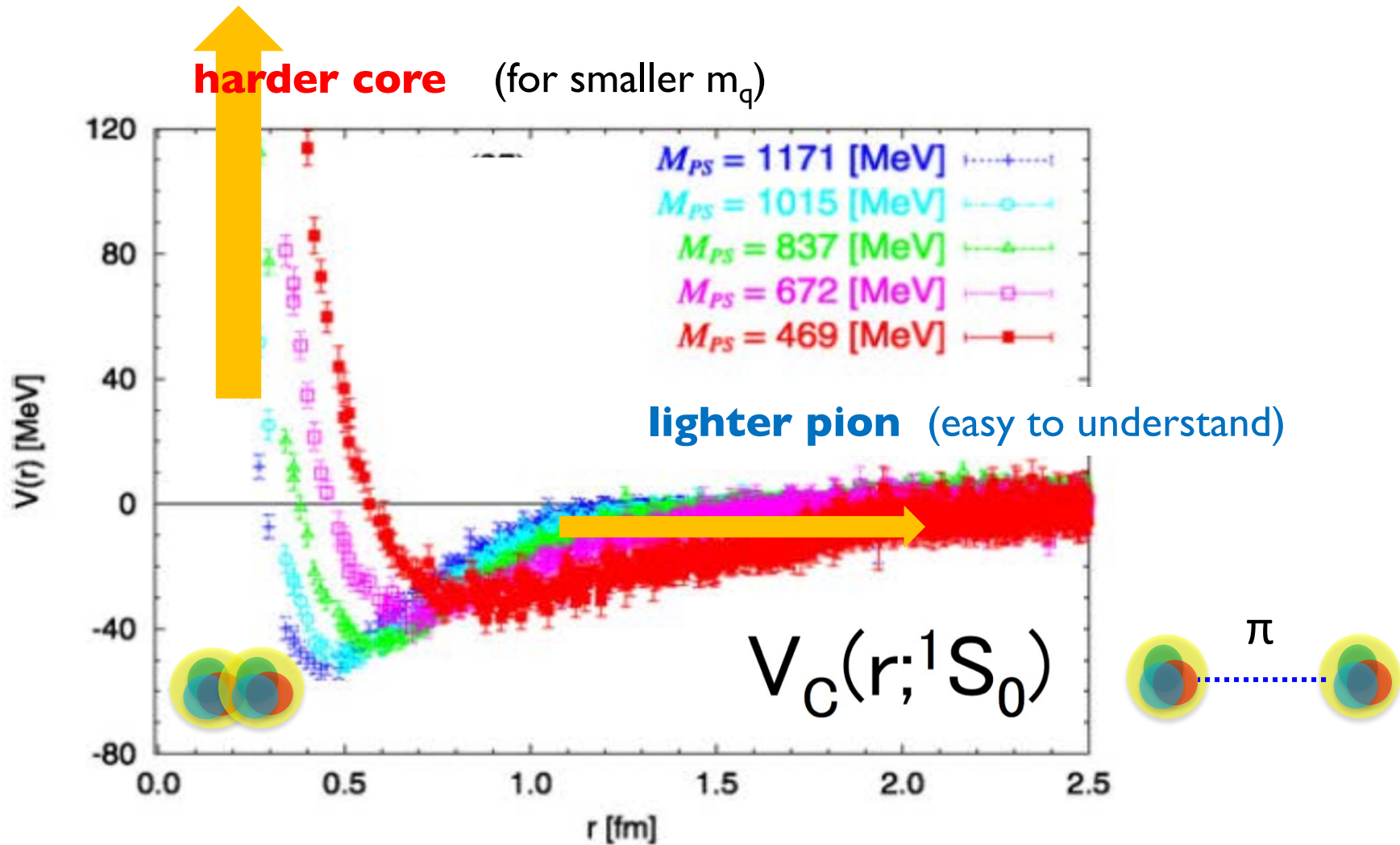
preliminary



hard core: channel dependence important



Mass dependence of NN interactions



Hard core → due to some **relativistic effects?**

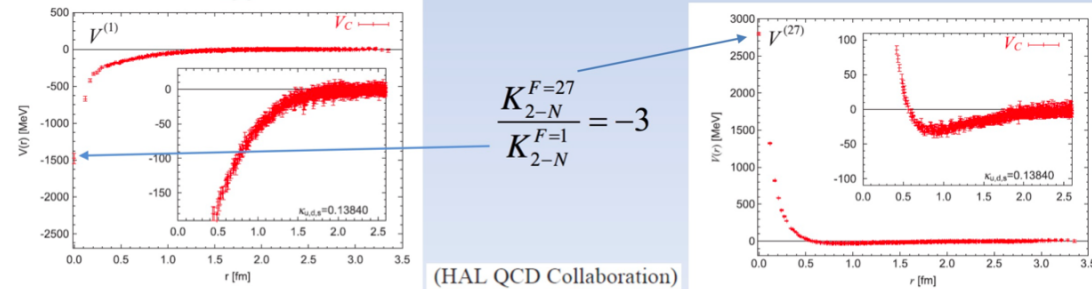
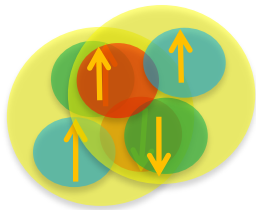
Recent quark model studies for hard cores

cf) A.Park-W.Park-SuHoungLee (2016),...

evaluate matrix for **color-mag. int.** for overlapped baryons

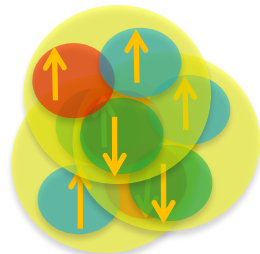
→ **Channel dep.** of the **height of the hard core**

2-body)



→ semi-quantitative agreement with lattice

3-body)

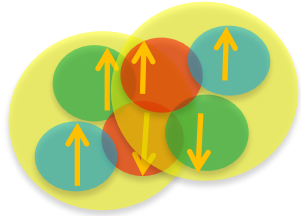


NNN, NN \bar{Y} are partially investigated

→ overall **repulsion**, though not universal

See, Su Houng Lee's talk in NFQCD2018 (3rd week), Kyoto

Quark descriptions for the hard core



cf) Oka-Yazaki (1980),...

6q problem in constituent quark models

Resonating group method (RGM) [Wheeler 1937, Hill-Wheeler 1953]

$$\int \phi_A^+(\xi_A) \phi_B^+(\xi_B) (H - E) \Psi(\xi_A, \xi_B, \mathbf{R}_{AB}) d\xi_A d\xi_B = 0$$

$$\Psi(\xi_A, \xi_B, \mathbf{R}_{AB}) = \mathcal{A}[\phi_A(\xi_A) \phi_B(\xi_B) \chi(\mathbf{R}_{AB})]$$

scattering problems \rightarrow phase shift

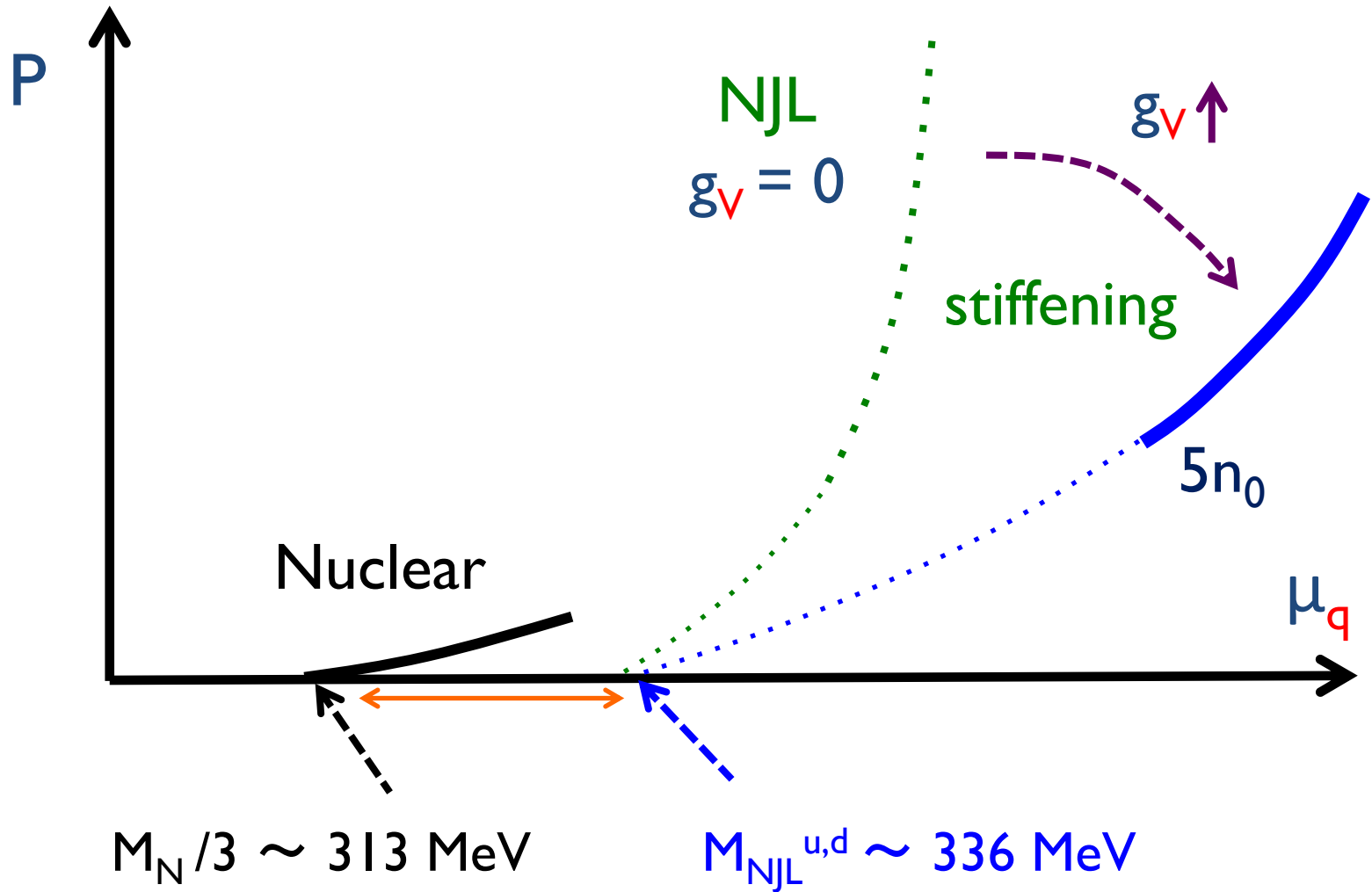
Findings

- 1, Quark Pauli blocking : **NOT enough** for the hard core
- 2, **Color-magnetic** interaction is crucial (enhanced at small mass)
- 3, Hard cores are **not universal**: attractive for some channels

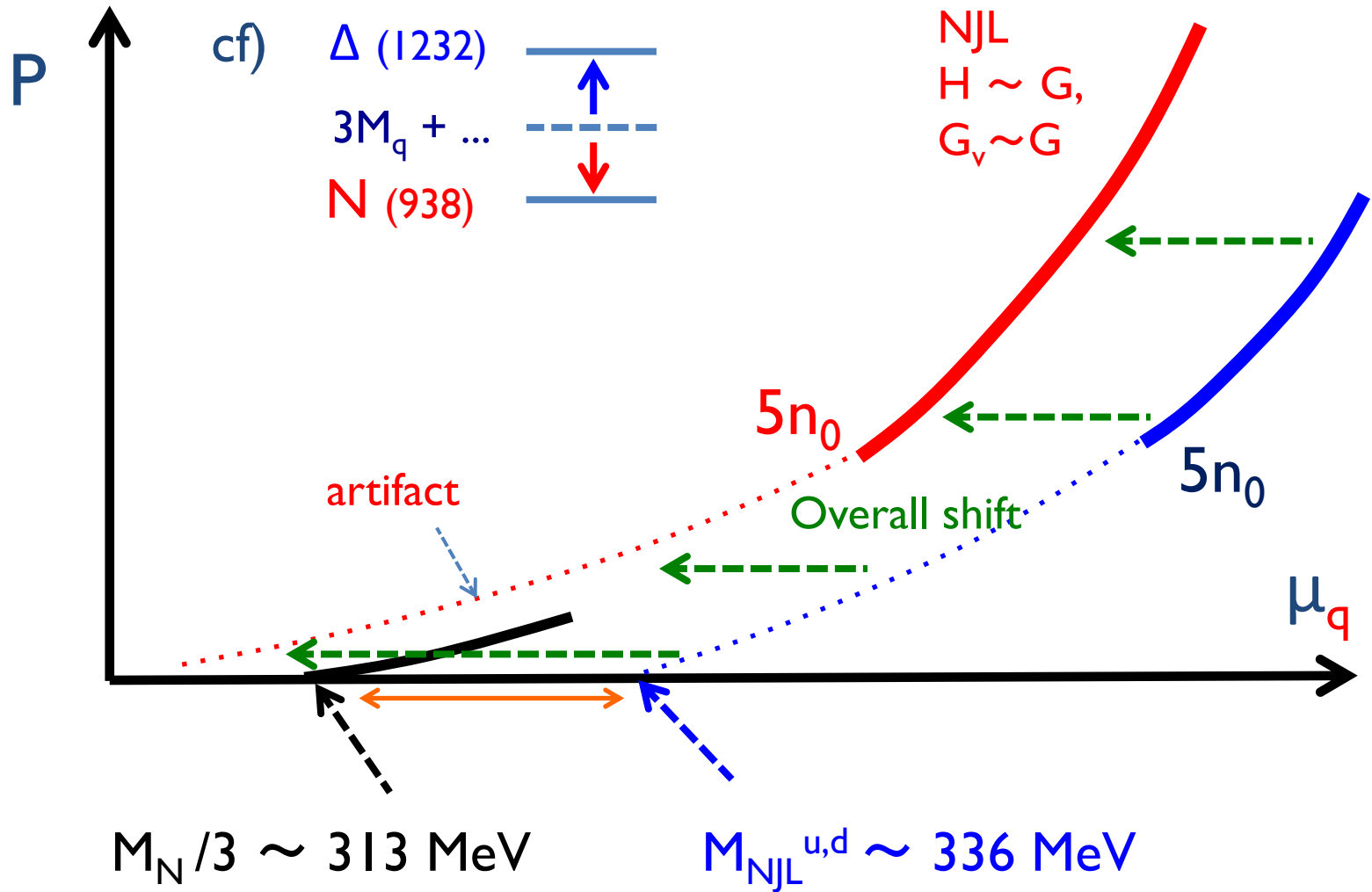
The picture so far is consistent with the lattice's !

see also A.Park-W-Park-SuHoungLee (2016)

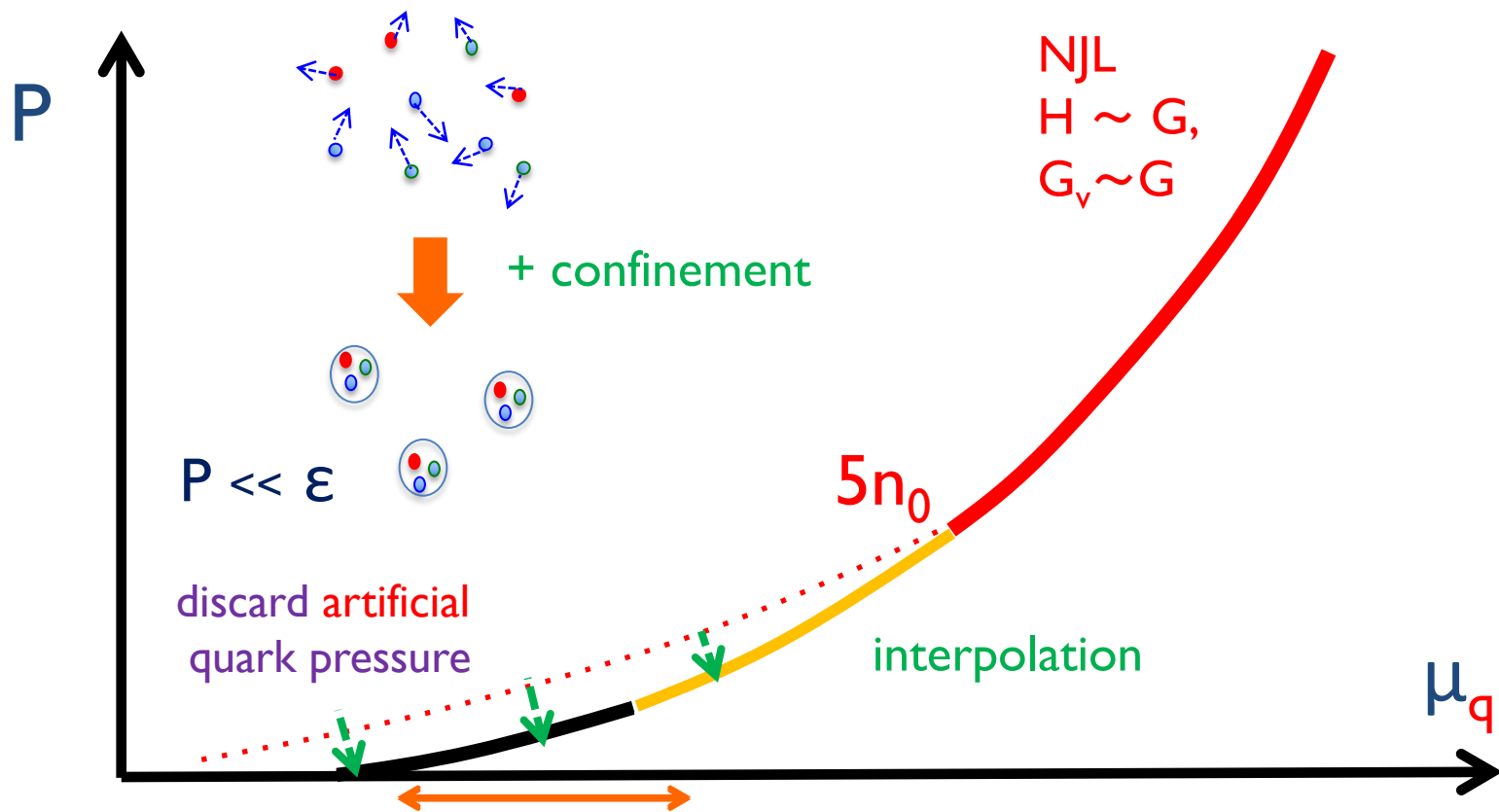
minimal + **vector** int.



+ attractive color-magnetic int.



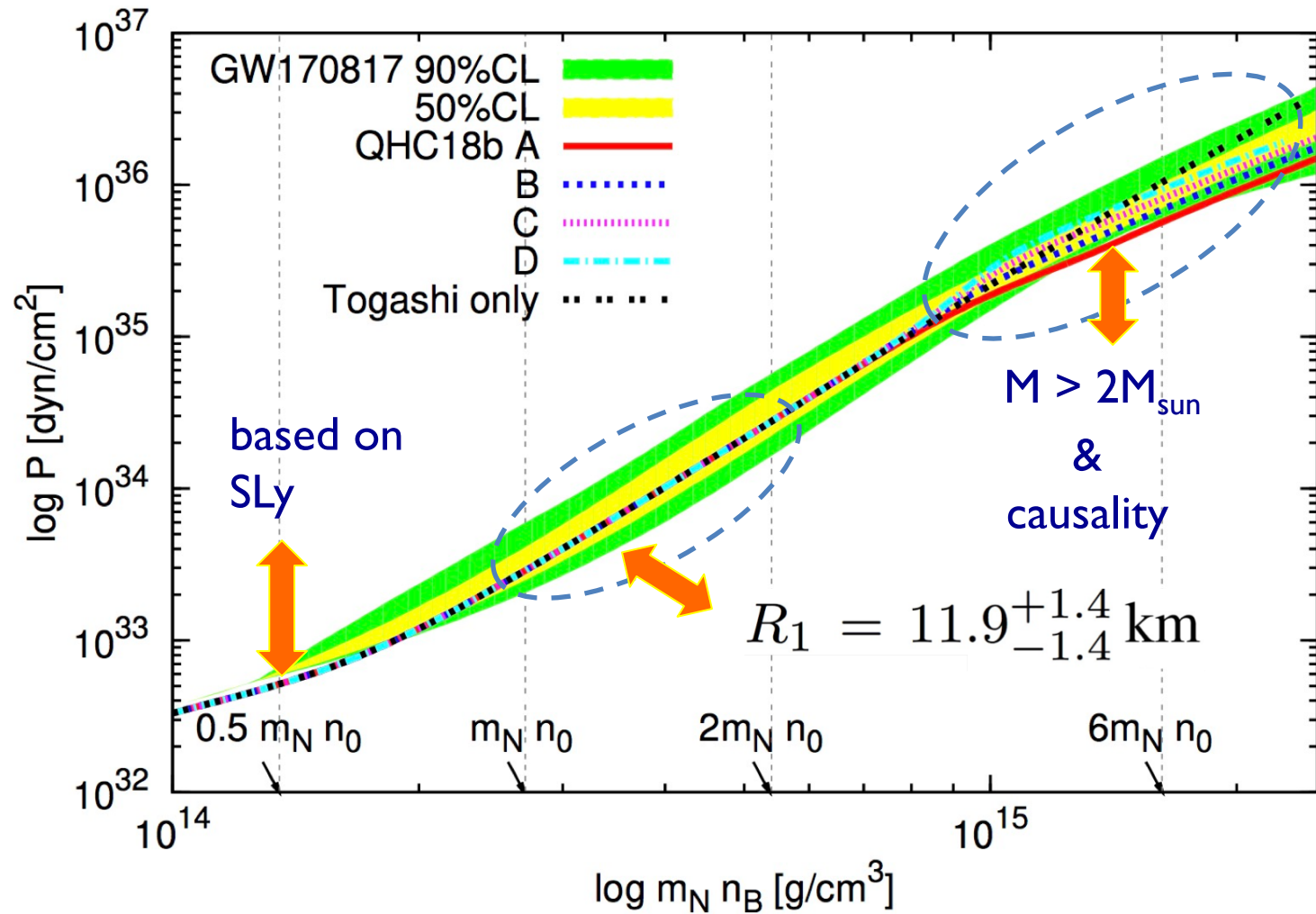
+ interpolation



In this picture: **confinement** → **softening**
or **deconfinement** → **stiffening**
(**opposite** to conventional discussions for $T=0$)

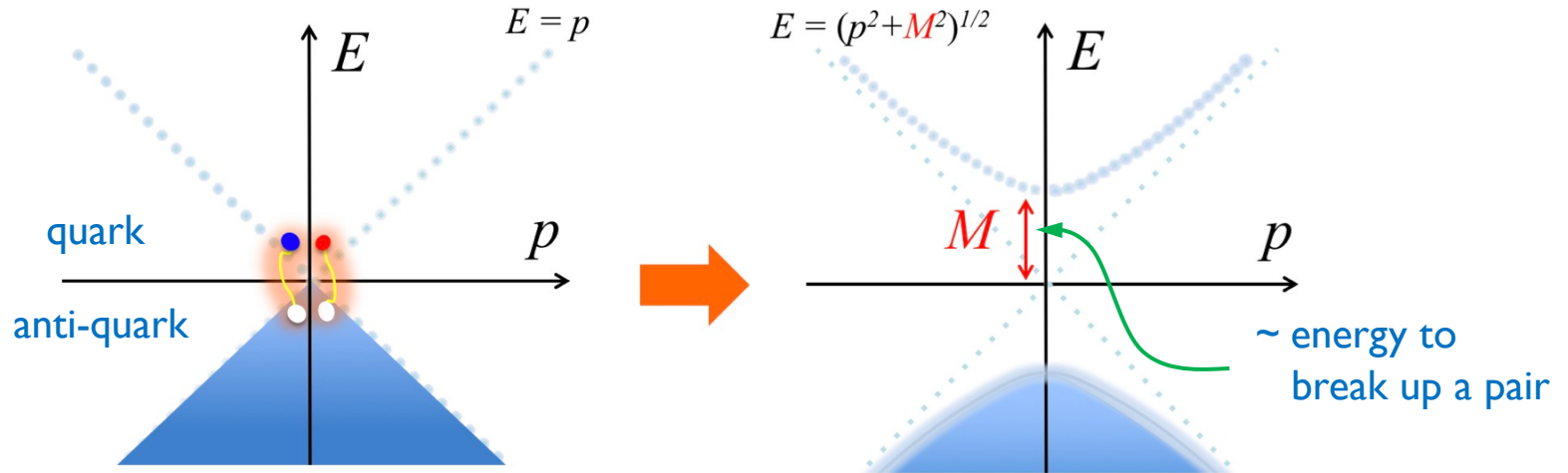
EoS from aLIGO vs QHC18b

aLIGO & Virgo new analyses for GW170817 arXiv: 1805.11581 [gr-qc]

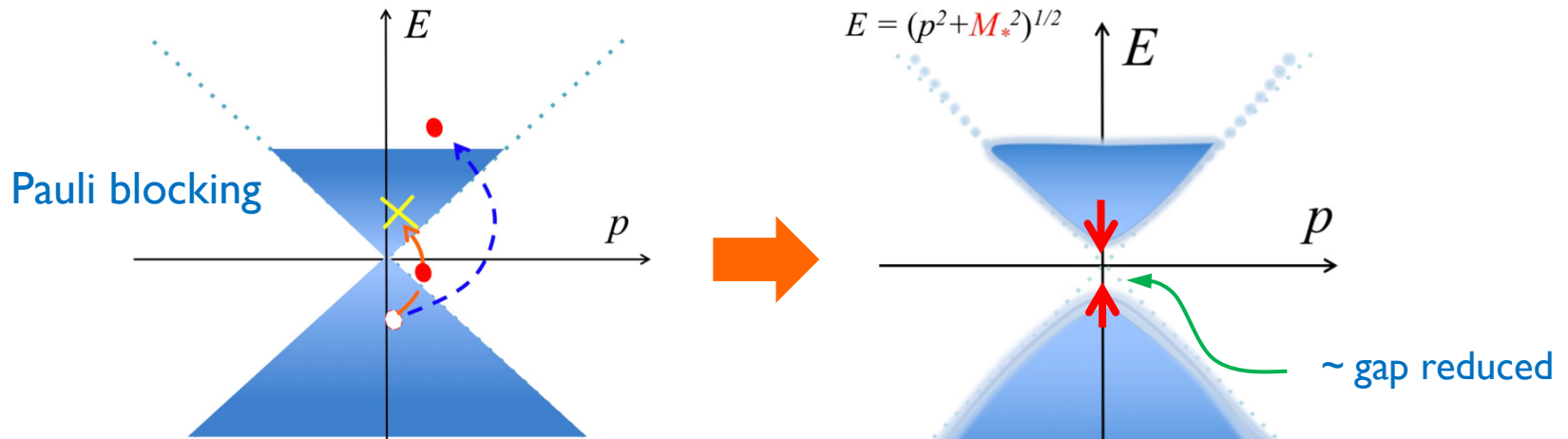


Chiral sym. breaking & restoration

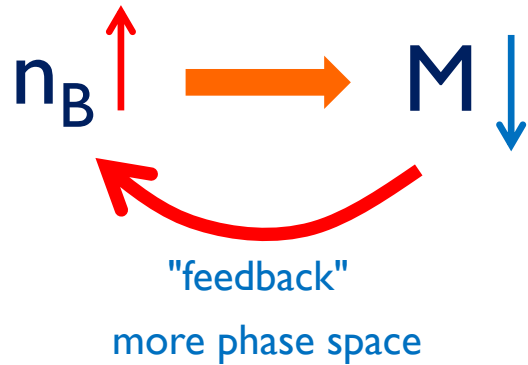
vac



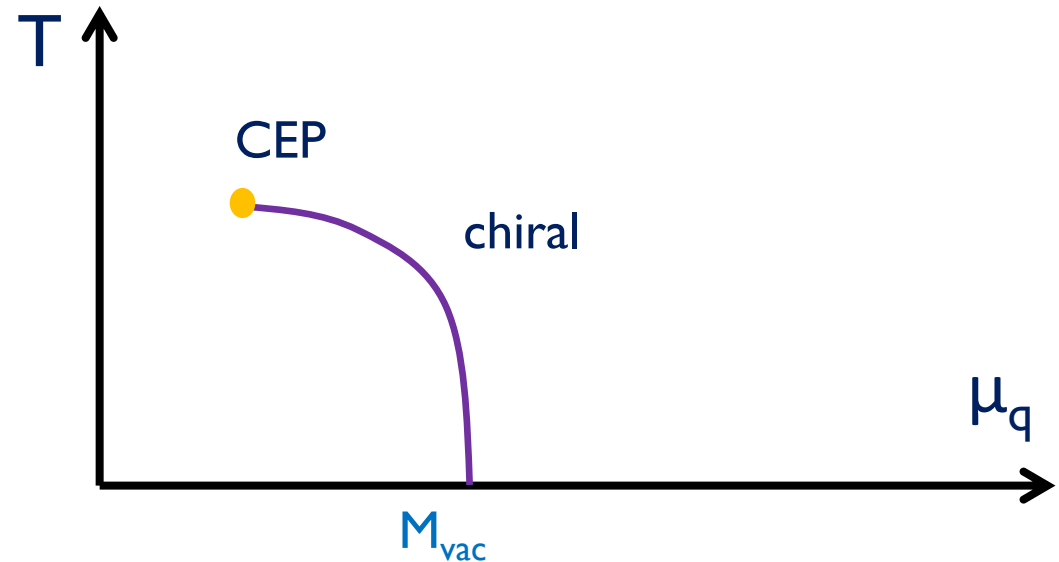
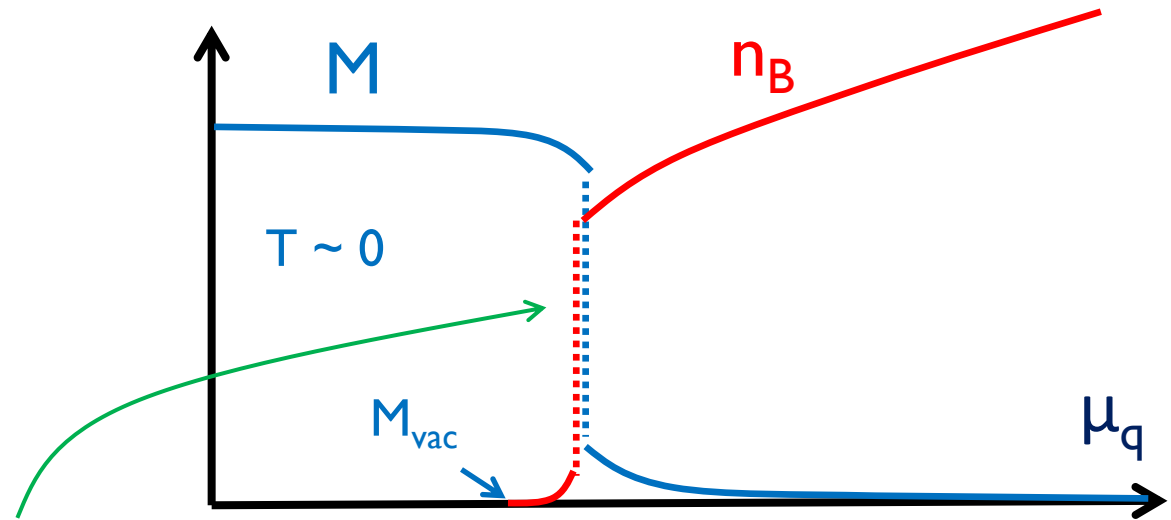
finite density



1st order chiral transition (typical quark **models**)



→ radical changes in n_B & M



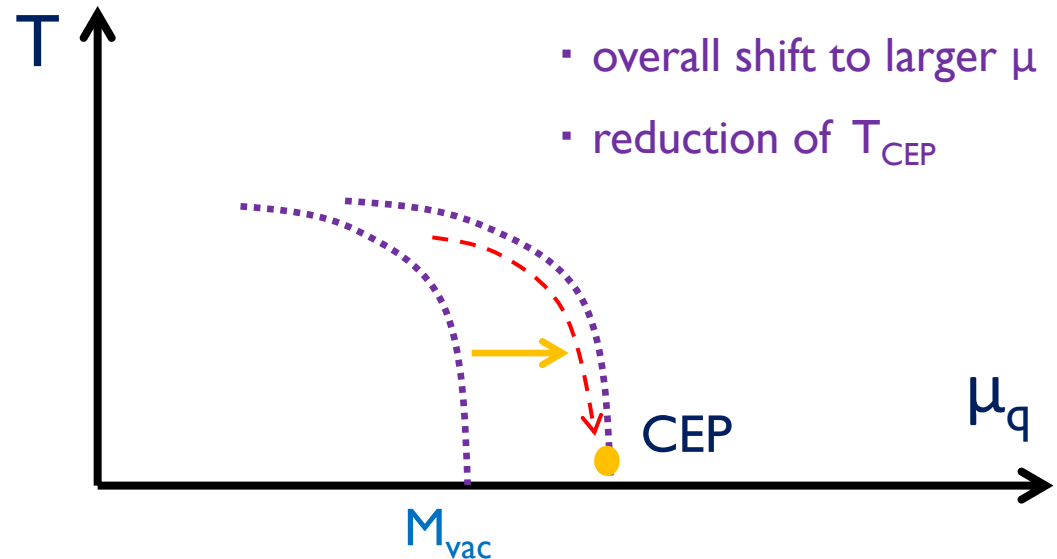
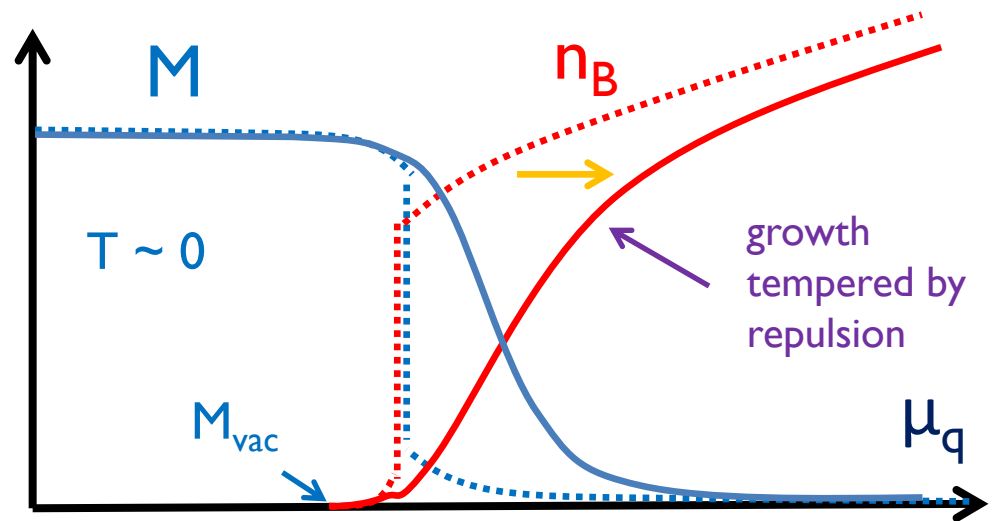
Braking density evolution: 1st → crossover

Now add
density-density repulsion

$$\Delta H \sim g_V (n_B)^2$$

braking the evolution of n_B
→ milder changes in M

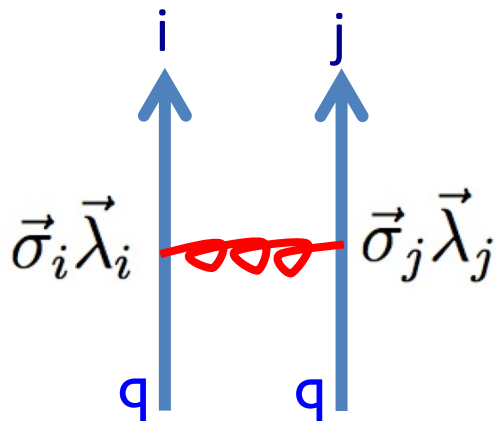
Details of int. are crucial



Constituent quark models for hadrons

cf) DeRujula-Georgi-Glashow (1975), Isgur-Karl (1978), ...

Color-**magnetic** interactions : responsible for **level splitting**



non-rela.

color-color spin-spin

$$\alpha_s \lambda_i \frac{\vec{\sigma}_i \cdot \vec{q}}{m_i} \frac{1}{q^2} \lambda_j \frac{\vec{\sigma}_j \cdot \vec{q}}{m_j} \simeq \frac{\alpha_s}{\underbrace{m_i m_j}} (\lambda_i \cdot \lambda_j) (\vec{\sigma}_i \cdot \vec{\sigma}_j) \delta(\vec{r}_{ij})$$

channel dependence

& Fermi statistics → **flavor-flavor correlation**

mag. int. is **enhanced** in **relativistic** regimes

non-rela.

coupling \propto **velocity** \sim **p/E** (\rightarrow **p/M** \ll 1.)

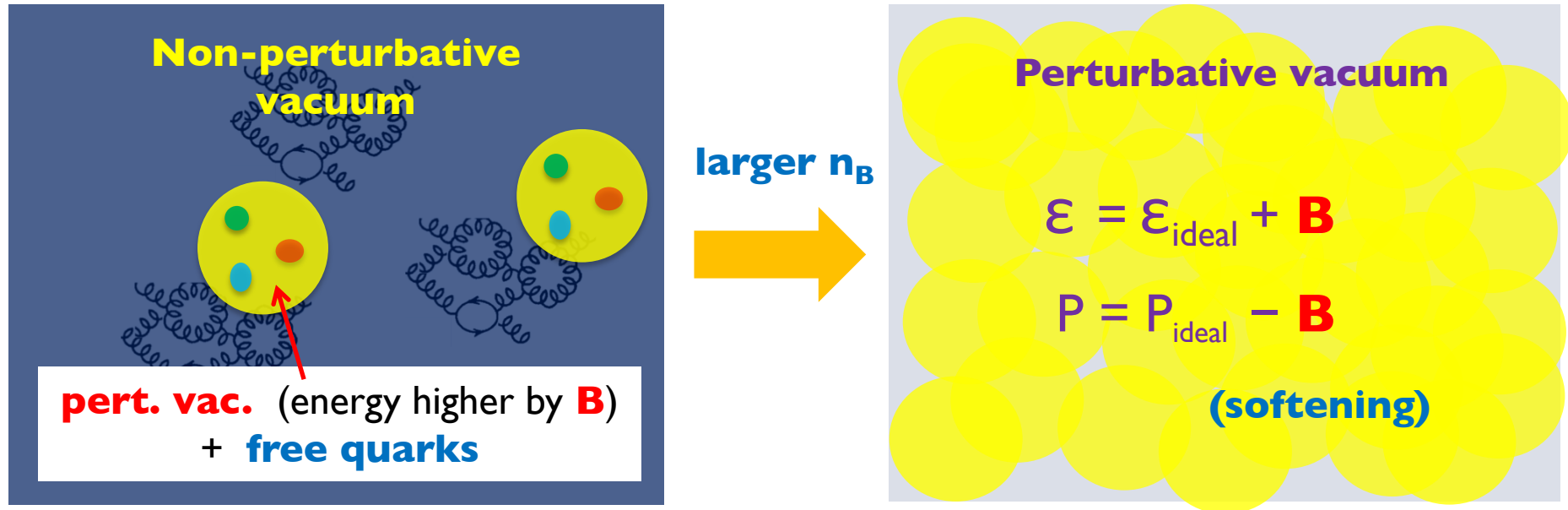
(sensitive to the quark mass)

Capture the **gross properties of (S-wave) baryons**

(~10% accuracy)

A quark model traditionally used in astro-EoS

cf) Chodos et al (1974), MIT bag model



Over-simplifications

- 1, Inside of hadrons is NOT like free media: **ChSB** & constituent quark mass
- 2, Even in the quark matter regime, interactions are **critically important**

($n_B \sim 100 n_0$ is not enough for free gas picture)

Implications for dense matter

If one accepts the quark description for hard cores, it follows

1, Hard core repulsions are weaker **for YN & YY** than for NN

$$\text{color-mag.} \sim 1/M_i M_j \quad M_{u,d}/M_s \sim 3/5$$

2, Short-range int. **can be attractive** (though relatively rare)

e.g.) H-dibaryon (uds-uds); double Ω (sss-sss),....

Can we block strangeness to $n_B \sim 5n_0$??

3, $M_{u,d,s}$ reduction \rightarrow **overall enhancement** of hard core repulsion

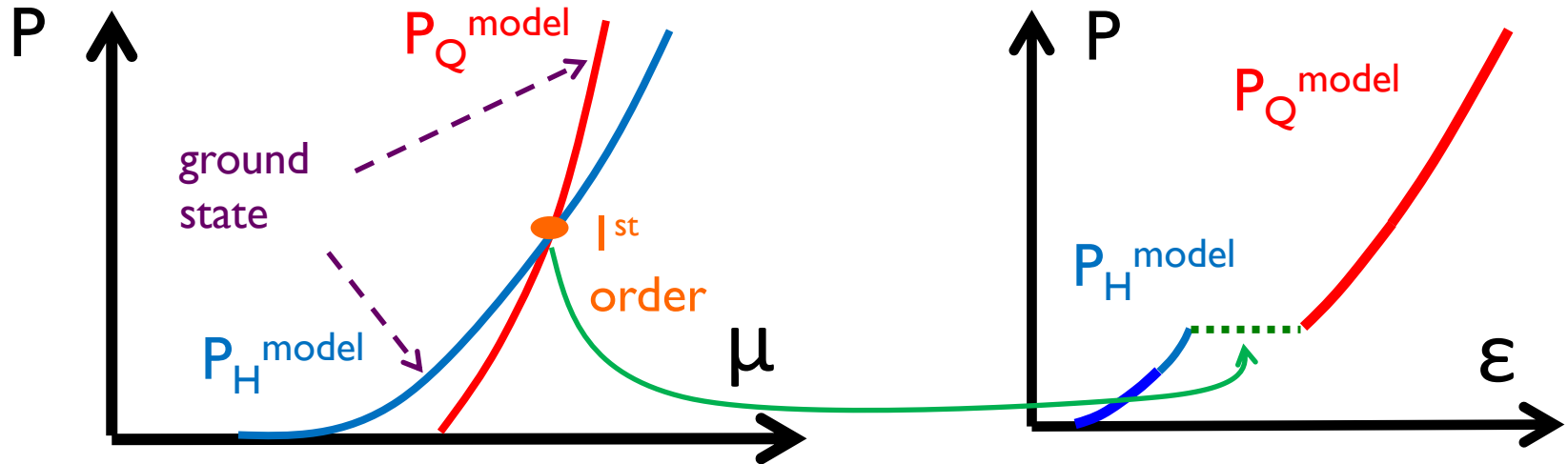
chiral restoration is delayed by the repulsion?

These features are hard to infer from purely hadronic models...

Quark-Hadron continuity (some history)

- 1, Percolation picture Baym-Chin 1978; Satz-Karsch 1979,...
- 2, In the context of color-superconductivity (CSC) Schafer-Wilczek 1998
 symmetry: **hadron super fluidity** \sim **color-flavor-locked (CFL)** phases
 same order parameters : $\langle BB \rangle \sim \langle (qqq)^2 \rangle$
 color singlet, but break $U(1)_B$; chiral sym. is also broken
 confinement-Higgs complementarity Fradkin-Shenkar 1979
 dynamics: the interplay between chiral & diquark
 proposal of **double CEP** Kitazawa+ 2002; Hatsuda+2006; Zhang+ 2009, ...
- 3, Inferred from the NS constraints (for $2n_0 - 5n_0$) Masuda+2012, Kojo+2014, ...
soft-stiff EoS & causality \rightarrow **crossover** or **weak** 1st order

Traditional hybrid construction



- Key (implicit) **assumptions** :
 - 1) Hadronic & quark phases are **distinct** (e.g. by order parameters)
 - 2) Both P_H and P_Q are **reliable in the overlap region**
- by construction, Q-EoS must be much softer than H-EoS
(unless fine tuning worked out)

Constituent quark models for hadrons I

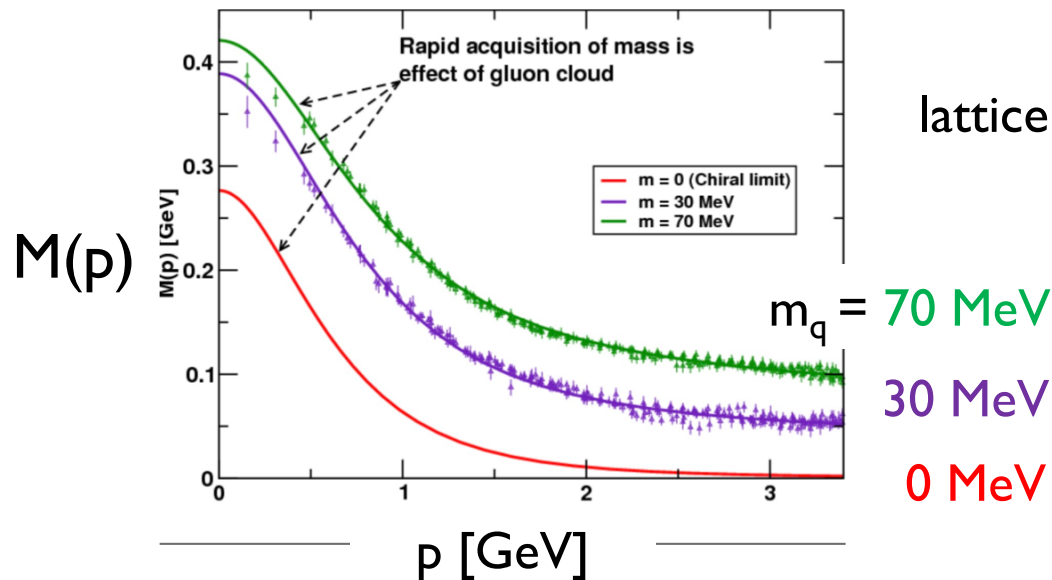
cf) DeRujula-Georgi-Glashow (1975), Isgur-Karl (1978), ...

1, Confining potential put by hand

Even now, no satisfactory analytic derivation... Main info from lattice

2, Constituent quarks assumed: $M_u \sim 350$ MeV, $M_s \sim 540$ MeV

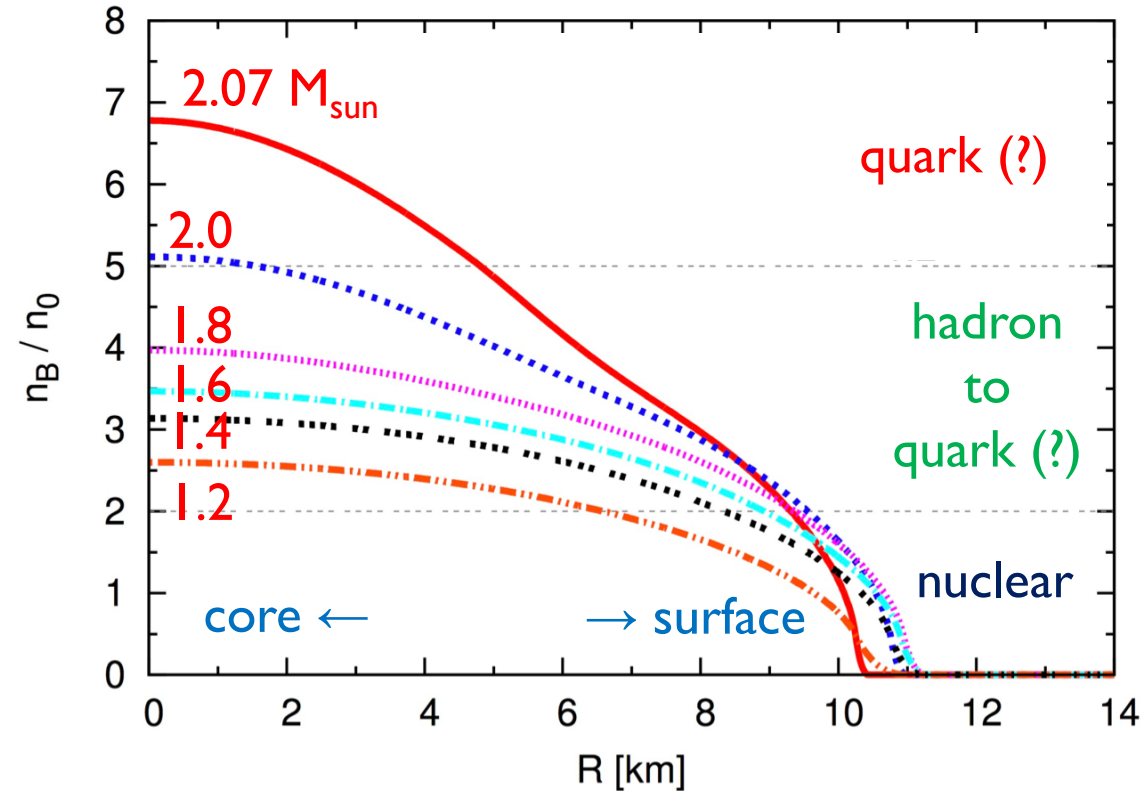
In modern language, produced by dynamical chiral sym. breaking



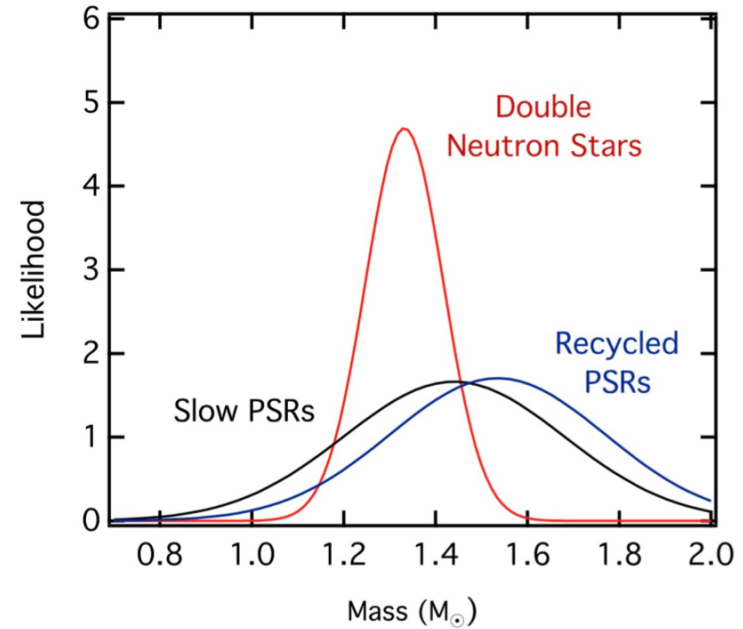
3, OGE -> **semi-short** range **color-electric** & **magnetic** int.

Baryon density in a neutron star (our EoS)

Distribution of n_B for various NSs



Distribution of NSs



based on observations

For typically observed NSs ($M > 1.2M_{\text{sun}}$) :

large fraction ($> 50\%$) of matter has $n_B > 2n_0$

→ beyond the nuclear regime is crucial for most of NSs