Role of the symmetry energy on neutron stars within the nuclear energy density functional theory

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in collaboration with
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Motivation

The energy per nucleon of nuclear matter at $T = 0$ around saturation density n_0 and for asymmetry $\eta = (n_n - n_p)/n$, is usually written as

$$e(n,\eta) = e_0(n) + S(n)\eta^2 + o\left(\eta^4\right) \text{ where}$$

$$e_0(n) = a_v + \frac{K_v}{18}\epsilon^2 - \frac{K'}{162}\epsilon^3 + o\left(\epsilon^4\right) \text{ with } \epsilon = (n - n_0)/n_0$$

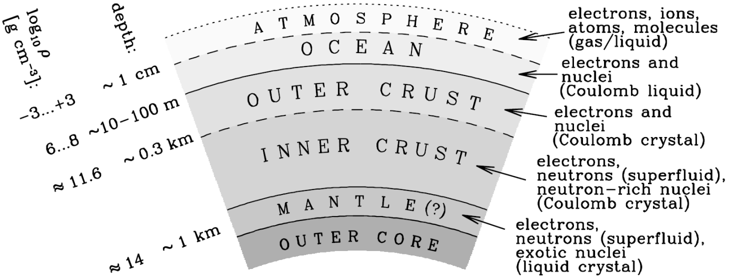
$$S(n) = J + \frac{L}{3}\epsilon + \frac{K_{sym}}{18}\epsilon^2 + o\left(\epsilon^3\right) \text{ is the } \textbf{symmetry energy}$$

The nuclear uncertainties are embedded in a_v, K_v, K', J, L, K_{sym}.

Main goal

Assess the role of the symmetry energy on the neutron-star properties using consistent models of dense matter.

Challenge



Haensel, Potekhin, Yakovlev, "Neutron Stars - 1. Equation of State and Structure" (Springer, 2007)

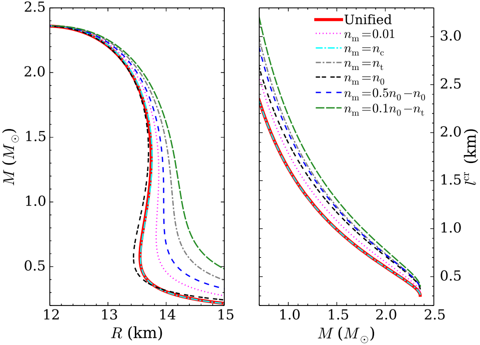
The interior of a neutron star exhibits

- very **different phases** (gas, liquid, solid, superfluid, etc.)
- over a very **wide range of densities**
- with possibly exotic particles (hyperons, quarks) in the inner core.

Blaschke&Chamel, contribution to the White Book of the COST Action MP1304, arXiv:1803.01836

Need for a unified treatment

- **Ad hoc matching** of different models of dense matter can lead to significant errors on the neutron-star structure & dynamics.



Fortin et al., Phys.Rev.C94, 035804 (2016)

- Combining inconsistent microscopic inputs leads to **multiple interpretations** of astrophysical phenomena (degeneracy).

This calls for a unified description of neutron-star interiors.

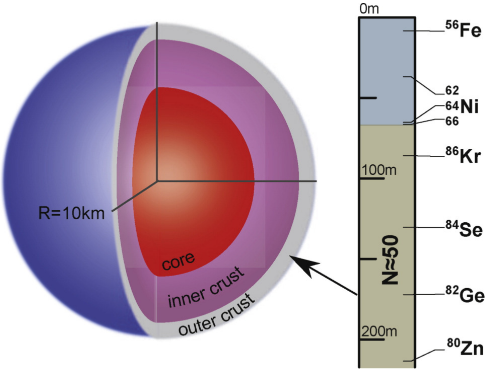
Description of the outer crust of a neutron star

Main assumptions:

- **cold "catalyzed" matter** (full thermodynamic equilibrium)
  Harrison, Wakano and Wheeler, Onzième Conseil de Physique Solvay (Stoops, Brussels, Belgium, 1958) pp 124-146
- the crust is stratified into **pure layers** made of nuclei ${}^{A}_{Z}\mathrm{X}$
- electrons are $\sim$ uniformly distributed and are highly degenerate
  $T < T_{\rm F} \approx 5.93 \times 10^9 (\gamma_r - 1)\ {\rm K}$
  $$\gamma_r \equiv \sqrt{1 + x_r^2}\,, \qquad x_r \equiv \frac{p_{\rm F}}{m_e c} \approx 1.00884 \left(\frac{\rho_6 Z}{A}\right)^{1/3}$$
- nuclei are arranged on a **perfect body-centered cubic lattice**
  $$T < T_m \approx 1.3 \times 10^5 Z^2 \left(\frac{\rho_6}{A}\right)^{1/3}\ {\rm K} \qquad \rho_6 \equiv \rho/10^6\ {\rm g\ cm^{-3}}$$
  Pearson et al.,Phys.Rev.C83, 065810 (2011)
  Chamel&Fantina,Phys.Rev.D93,063001 (2016)

Experimental “determination” of the outer crust

The composition of the crust is completely determined by experimental atomic masses down to about 200m for a $1.4 M_\odot$ neutron star with a 10 km radius



The physics governing the structure of atomic nuclei (magicity) leaves its imprint on the composition.

Due to β equilibrium and electric charge neutrality, Z is more tightly constrained than N: only a few layers with $Z = 28$.

Kreim, Hempel, Lunney, Schaffner-Bielich, Int.J.M.Spec.349-350,63(2013)

Deeper in the star, recourse must be made to theoretical models.

Theoretical challenge in the deeper regions

Models of dense matter should be:

- **versatile**: applicable to compute all properties
- **thermodynamically consistent**: avoid spurious instabilities
- **as microscopic as possible**: make reliable extrapolations
- **numerically tractable**: allow for systematic calculations

The nuclear **energy density functional theory** is the best suited.

Nucleons are treated as **independent quasiparticles in a self-consistent potential field** (Hartree-Fock-Bogolyubov method).

Dobaczewski&Nazarewicz, in "50 years of Nuclear BCS" (World Scientific Publishing, 2013), pp.40-60; Chamel,Goriely,Pearson, ibid., pp.284-296

+ This theory describes the many-body system **exactly** (Hohenberg-Kohn theorem).

− But the exact functional is unknown. In practice, phenomenological functionals are employed.

Brussels-Montreal Skyrme functionals (BSk)

For application to neutron stars, functionals should reproduce properties of both finite nuclei and infinite nuclear matter. We have developed a series of generalized Skyrme functionals (BSk).

Chamel et al., Acta Phys. Pol. B46, 349(2015)

Experimental data/constraints:

- $\sim$ 2300 nuclear masses from AME (rms $\sim 0.5 - 0.6$ MeV$/c^2$)
- $\sim$ 900 nuclear charge radii (rms $\sim$ 0.03 fm)
- symmetry energy $29 \leq J \leq 32$ MeV
- incompressibility $K_v = 240 \pm 10$ MeV

  Colò et al., Phys.Rev.C70, 024307 (2004).

Many-body ab initio calculations:

- equation of state of pure neutron matter
- $^1S_0$ pairing gaps in nuclear matter
- effective masses in nuclear matter
- stability against spin and spin-isospin fluctuations

Phenomenological corrections for atomic nuclei

For atomic nuclei, we add the following corrections to the HFB energy:

- Wigner energy

$$E_W = V_W \exp\left\{ -\lambda\left(\frac{N-Z}{A}\right)^2\right\} + V_W'|N-Z|\exp\left\{-\left(\frac{A}{A_0}\right)^2\right\}$$

$V_W \sim -2$ MeV, $V_W' \sim 1$ MeV, $\lambda \sim 300$ MeV, $A_0 \sim 20$

- rotational and vibrational spurious collective energy

$$E_{\rm coll} = E_{\rm rot}^{\rm crank}\Big\{b\ \tanh(c|\beta_2|) + d|\beta_2|\ \exp\{-l(|\beta_2| - \beta_2^0)^2\}\Big\}$$

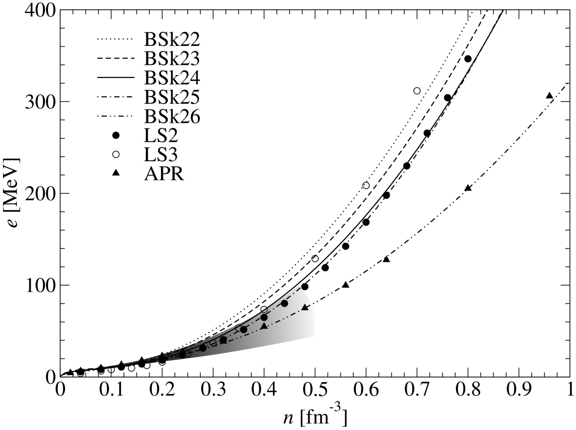
This latter correction was shown to be in good agreement with calculations using 5D collective Hamiltonian.

Goriely, Chamel, Pearson, Phys.Rev.C82,035804(2010).

In this way, these collective effects do not contaminate the parameters (≤ 20) of the functional.

Neutron-matter constraint

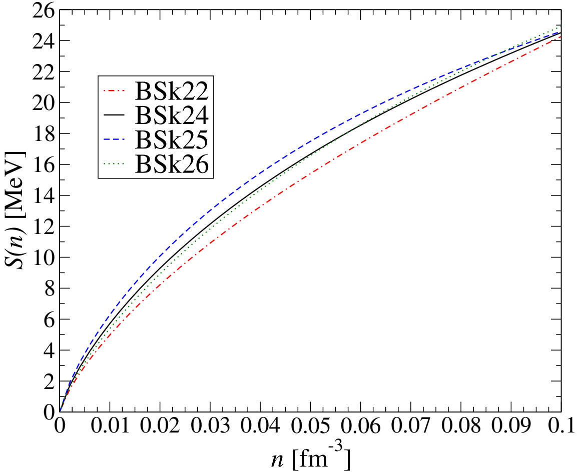
BSk22-26 were simultaneously fitted to realistic neutron-matter equations of state in addition to nuclear masses:



Goriely, Chamel, Pearson, Phys.Rev.C 88, 024308 (2013).

Symmetry-energy constraint

BSk22-26 were adjusted to different values of J. The symmetry energy function $S(n)$ was completely determined by the fit:



Note that all curves cross at $n \sim (2/3)n_0$ from the mass fit.

Goriely, Chamel, Pearson, Phys.Rev.C 88, 024308 (2013).

Nuclear-matter parameters

	BSk22	BSk23	BSk24	BSk25	BSk26
a_v [MeV]	-16.088	-16.068	-16.048	-16.032	-16.064
n_0 [fm^{-3}]	0.1578	0.1578	0.1578	0.1587	0.1589
J [MeV]	32.0	31.0	30.0	29.0	30.0
L [MeV]	68.5	57.8	46.4	36.9	37.5
K_{sym}[MeV]	13.0	-11.3	-37.6	-28.5	-135.6
K_v [MeV]	245.9	245.7	245.5	236.0	240.8
K' [MeV]	275.5	275.0	274.5	316.5	282.9
M_s^*/M	0.80	0.80	0.80	0.80	0.80
M_v^*/M	0.71	0.71	0.71	0.74	0.65

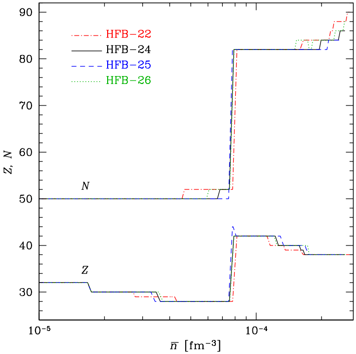
Lower and higher values of J were considered but yielded substantially worse fits to masses.

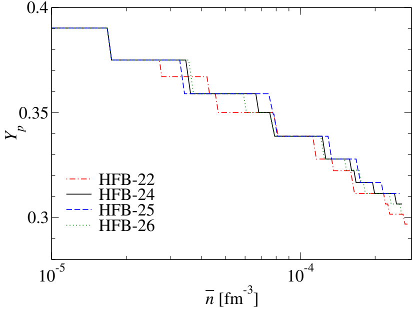
Goriely, Chamel, Pearson, Phys.Rev.C 88, 024308 (2013).

Composition of the outer crust

The structure of the outer crust is only slightly influenced by the density dependence of the symmetry energy $S(n)$.

The proton fraction varies roughly as $Y_p = \dfrac{Z}{A} \sim \dfrac{1}{2} - \dfrac{(12\pi^2(\hbar c)^3 P)^{1/4}}{8S}$

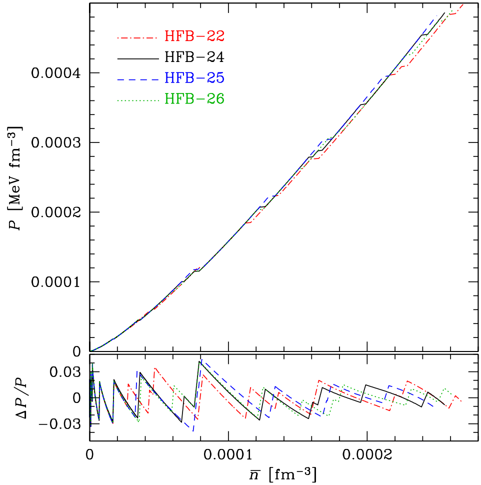




Pearson et al.,MNRAS 481, 2994 (2018)

Equation of state of the outer crust

The pressure, determined by electrons, is almost independent of the composition. **Analytical fits**: `http://www.ioffe.ru/astro/NSG/BSk/`



Neutron-star crust and nuclear masses

The composition of the outer crust is completely determined by **nuclear masses** $M'(A,Z)$.

Essentially **exact analytical expressions** valid for any degree of relativity of the electron gas and including electrostatic correction:
Chamel&Fantina,Phys.Rev.C94,065802(2016)

In the limit of an ultrarelativistic electron Fermi gas:

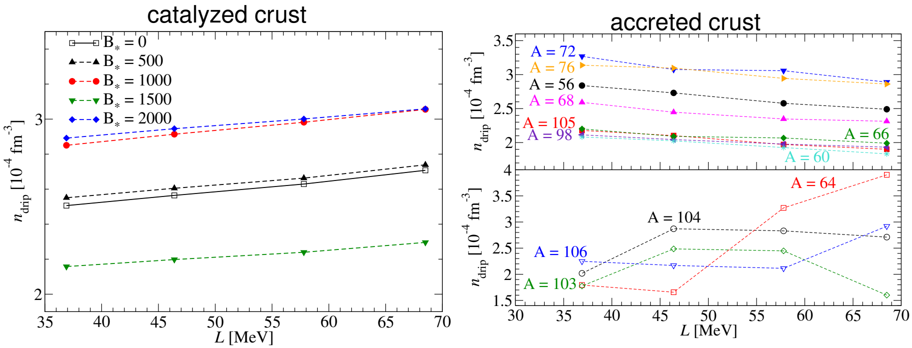
$$P_{1\to2} \approx \frac{(\mu_e^{1\to2})^4}{12\pi^2(\hbar c)^3}, \qquad \bar{n}_1^{\rm max} \approx \frac{A_1}{Z_1}\frac{(\mu_e^{1\to2})^3}{3\pi^2(\hbar c)^3}, \qquad \bar{n}_2^{\rm min} \approx \frac{A_2}{Z_2}\frac{Z_1}{A_1}\bar{n}_1^{\rm max}$$

$$\mu_e^{1\to2} \equiv \left[\frac{M'(A_2,Z_2)c^2}{A_2} - \frac{M'(A_1,Z_1)c^2}{A_1}\right]\left(\frac{Z_1}{A_1} - \frac{Z_2}{A_2}\right)^{-1} + m_e c^2$$

Since $\bar{n}_2^{\rm min} > \bar{n}_1^{\rm max}$ in hydrostatic equilibrium, nuclei become more **neutron rich** ($Z_2/A_2 < Z_1/A_1$) and **less bound** with increasing depth.

Neutron-drip transition: role of the symmetry energy

The lack of knowledge of the symmetry energy translates into uncertainties in the neutron-drip density:

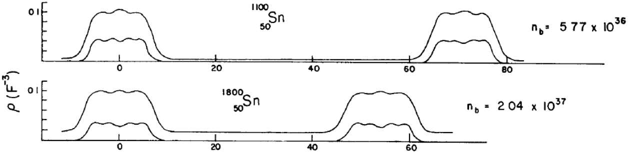


! In accreted crusts, the neutron-drip transition may be more sensitive to nuclear-structure effects than the symmetry energy **!**

Fantina et al.,Phys.Rev.C93,015801(2016).

Description of the inner crust of a neutron star

At densities $\sim 4.4 \times 10^{11}$ g cm^{-3}, neutrons drip out of nuclei thus marking the transition to the inner crust.



Negele&Vautherin,Nucl.Phys.A207,298(1973)

- The **neutron-saturated clusters** owe their stability to the presence of a highly degenerate surrounding neutron liquid.
- Unbound neutrons are expected to be **superfluid** at $T \leq T_c$ by forming Cooper pairs analogously to electrons in conventional superconductors.

The conditions prevailing in the inner crust of a neutron star cannot be reproduced in terrestrial laboratories.

Fast numerical implementation of HFB equations

We use the **4th order Extended Thomas-Fermi+Strutinsky Integral** method with the *same* functional as in the outer crust:

- **semiclassical expansion in powers of $\hbar^2$**: the energy becomes a functional of $n_n(\boldsymbol{r})$ and $n_p(\boldsymbol{r})$ and their gradients only.
- **proton shell effects** are added perturbatively (neutron shell effects are much smaller and therefore neglected).

In order to further speed-up the calculations, clusters are supposed to be spherical (no pastas) and $n_n(\boldsymbol{r}), n_p(\boldsymbol{r})$ are parametrized.

Pearson,Chamel,Pastore,Goriely,Phys.Rev.C91, 018801 (2015).

Pearson,Chamel,Goriely,Ducoin,Phys.Rev.C85,065803(2012).

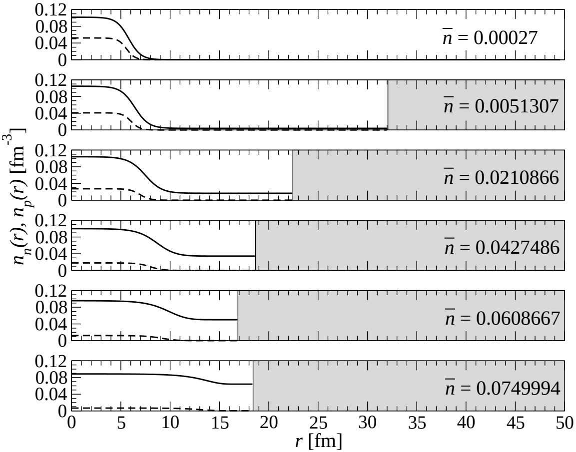
Onsi,Dutta,Chatri,Goriely,Chamel,Pearson, Phys.Rev.C77,065805 (2008).

Advantages of this ETFSI method:

- very fast approximation to the full HFB equations
- avoids the pitfalls related to continuum states

Structure of the inner crust of neutron star

Example of ETFSI calculations with BSk24:

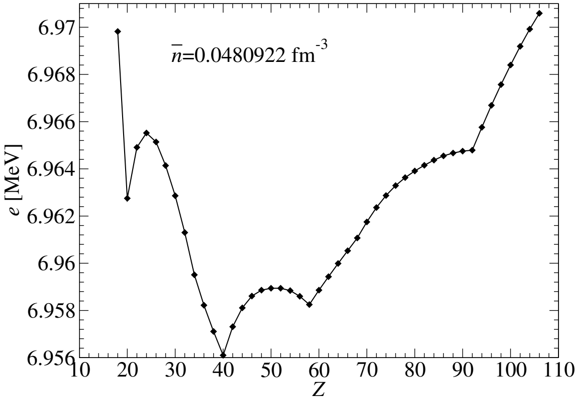


The crust-core transition density $\bar{n}_{cc} \sim 0.07 - 0.09$ fm^{-3} is found to be anticorrelated to L.

Proton shell effects in stellar environments

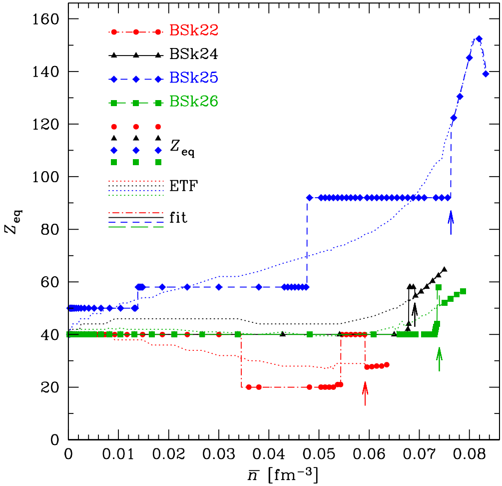
The ordinary nuclear shell structure is altered in dense matter:
$Z = 28, 82$ disappear, while $40, 58, 92$ appear (quenched spin-orbit).

Energy per nucleon obtained with BSk24:



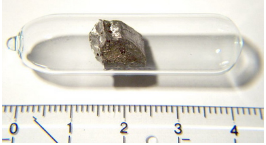
Role of shell effects and symmetry energy

The composition of the inner crust is strongly influenced by proton shell effects and the symmetry energy:



Terrestrial abundances:

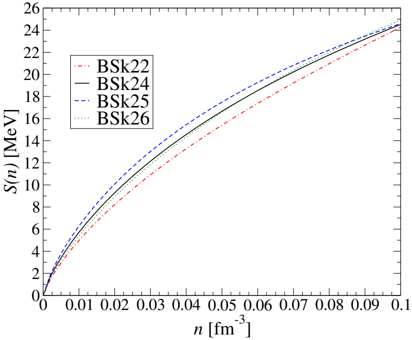
Zirconium ($Z = 40$): 0.02%



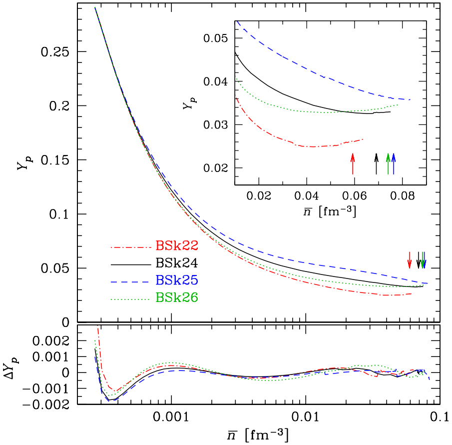
Cerium ($Z = 58$): 0.007%

Symmetry energy and proton fraction

The proton fraction Y_p of the inner crust is governed by the density dependence of the symmetry energy $S(n)$: the lower S the lower Y_p.
Analytical fits: `http://www.ioffe.ru/astro/NSG/BSk/`

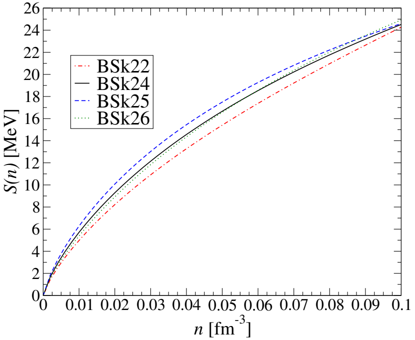


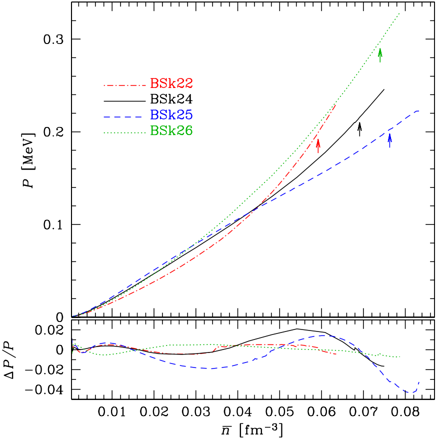
Pearson et al.,MNRAS 481, 2994 (2018)



Equation of state of the inner crust

The pressure in the inner crust is related to the slope of the symmetry energy $P \sim n^2 S'(n)$
Analytical fits: `http://www.ioffe.ru/astro/NSG/BSk/`

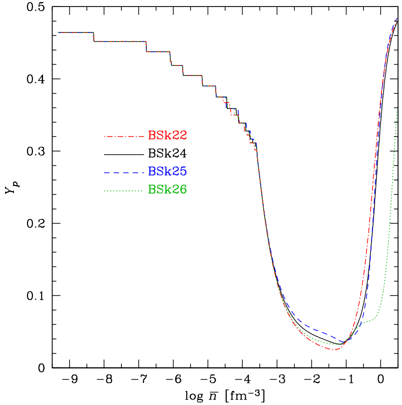


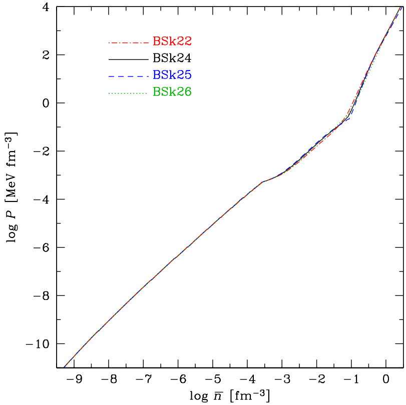


Pearson et al.,MNRAS 481, 2994 (2018)

Unified equations of state of neutron stars

The same functionals used in the crust can be also used in the core (n, p, e^-, μ^-) thus providing a **unified and thermodynamically consistent description of all regions of neutron stars**.

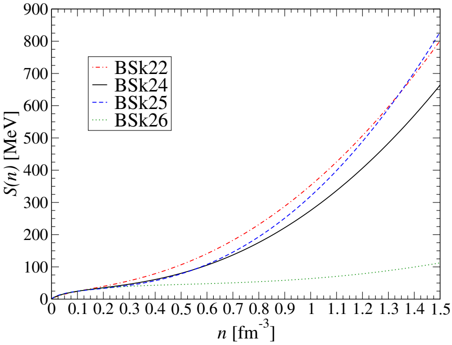


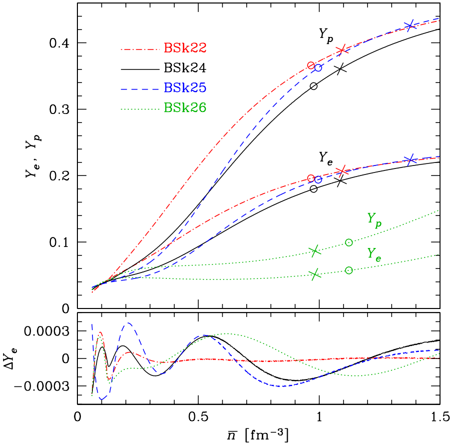


Analytical fits: `http://www.ioffe.ru/astro/NSG/BSk/`

Symmetry energy and proton fraction

The proton fraction Y_p of the core is governed by the density dependence of the symmetry energy $S(n)$: the lower S the lower Y_p.
Analytical fits: `http://www.ioffe.ru/astro/NSG/BSk/`





Note that the proton fraction can reach $Y_p \sim 40\%$ in the core.

Symmetry energy and direct Urca

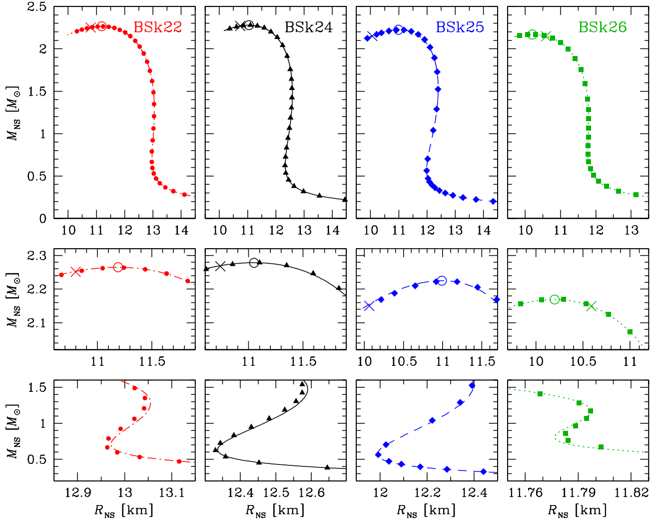
EoS	$n_{\rm DU}$ (fm^{-3})	$\rho_{\rm DU}$ (g cm^{-3})	$M_{\rm DU}/M_\odot$
BSk22	0.333	5.88×10^{14}	1.151
BSk24	0.453	8.25×10^{14}	1.595
BSk25	0.469	8.56×10^{14}	1.612

The direct Urca cooling process is required to explain

- the thermal luminosities of some accreting neutron stars in quiescence (e.g. SAX J1808.4−3658)
- the thermal relaxation of some transiently accreting neutron stars (e.g. MXB 1659−29).

- The dUrca process is allowed in all models but BSk26.
- The low value for $M_{\rm DU}$ predicted by BSk22 implies that dUrca would operate in most neutron stars, at variance with observations.

Gross properties of nonrotating neutron stars



Maximum masses and radii are consistent with constraints inferred from GW170817.

Conclusions & Perspectives

We have developed a set of **unified equations of state for neutron stars** using **accurately calibrated nuclear-energy density functionals** varying the neutron-matter stiffness & symmetry energy.

Analytical fits: `http://www.ioffe.ru/astro/NSG/BSk/`

- The inner crust (core) composition is found to be very sensitive to the symmetry energy at densities below (above) saturation.
- Varying the symmetry energy from $J = 29$ to 32 MeV leads to radii between $R = 11.8$ and 13.1 km for a $1.4 M_{\odot}$ neutron star
- Considering dUrca restricts R to lie within 12.3 and 12.6 km.

Perspectives:

- Allowance for nuclear “pasta” mantle (if any) beneath the crust,
- Accounting for magnetic fields (magnetars) and accretion,
- Extension to finite temperatures (neutron-star mergers).