The Equation of State of Dense Matter in the Multimessenger Era

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- Nuclear matter EOS and the symmetry energy (Esym)
- Dense nuclear matter from Nuclear experiments + Observed neutron star largest mass + Tidal deformability from GW170817
- Summary and outlook

“Xiamen-CUSTIPEN Workshop on the EOS of Dense Neutron-Rich Matter
in the Era of Gravitational Wave Astronomy”,
January 3-8, 2019, Xiamen, China

Outline

- **Nuclear matter EOS and the symmetry energy (Esym)**
- **Dense nuclear matter from Nuclear experiments + Observed neutron star largest mass +**
  Tidal deformability from GW170817
- **Summary and outlook**

The Symmetry Energy of Nuclear Matter

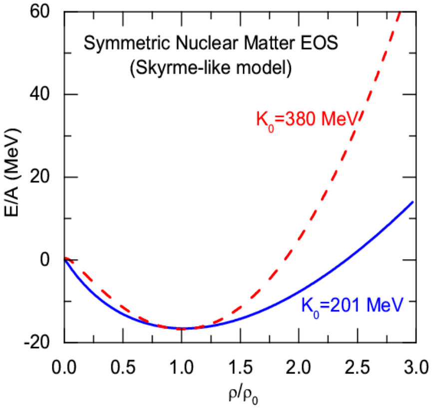
EOS of Isospin Asymmetric Nuclear Matter (Parabolic law)

$$E(\rho,\delta) = E(\rho,0) + E_{\rm sym}(\rho)\delta^2 + O(\delta^4), \quad \delta = (\rho_n - \rho_p)/\rho$$

Symmetric Nuclear Matter (relatively well-determined)

Isospin asymmetry

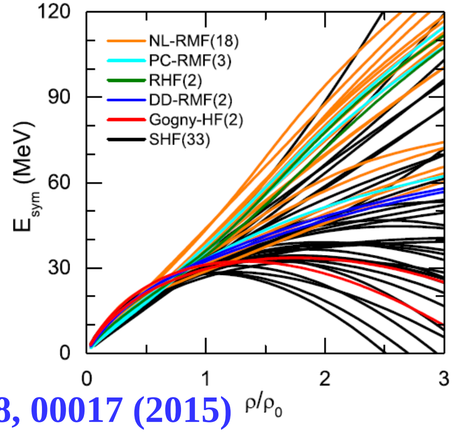
Symmetry energy term (poorly known)



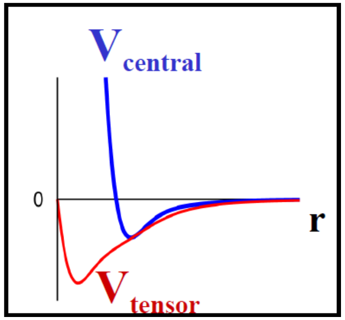
Nuclear Matter Symmetry Energy

$$E_{\rm sym}(\rho) \equiv \frac{1}{2}\frac{\partial^2 E(\rho,\delta)}{\partial \delta^2}$$

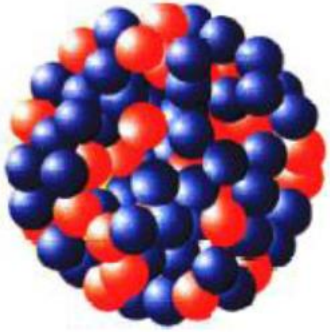
Saturation: $\rho_0 \approx 0.16\ \mathrm{fm}^{-3}$



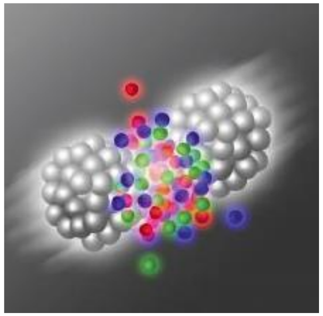
LWC, EPJ Web of Conf. 88, 00017 (2015)



Nature of the nuclear force?



Structure and stability of nuclei?



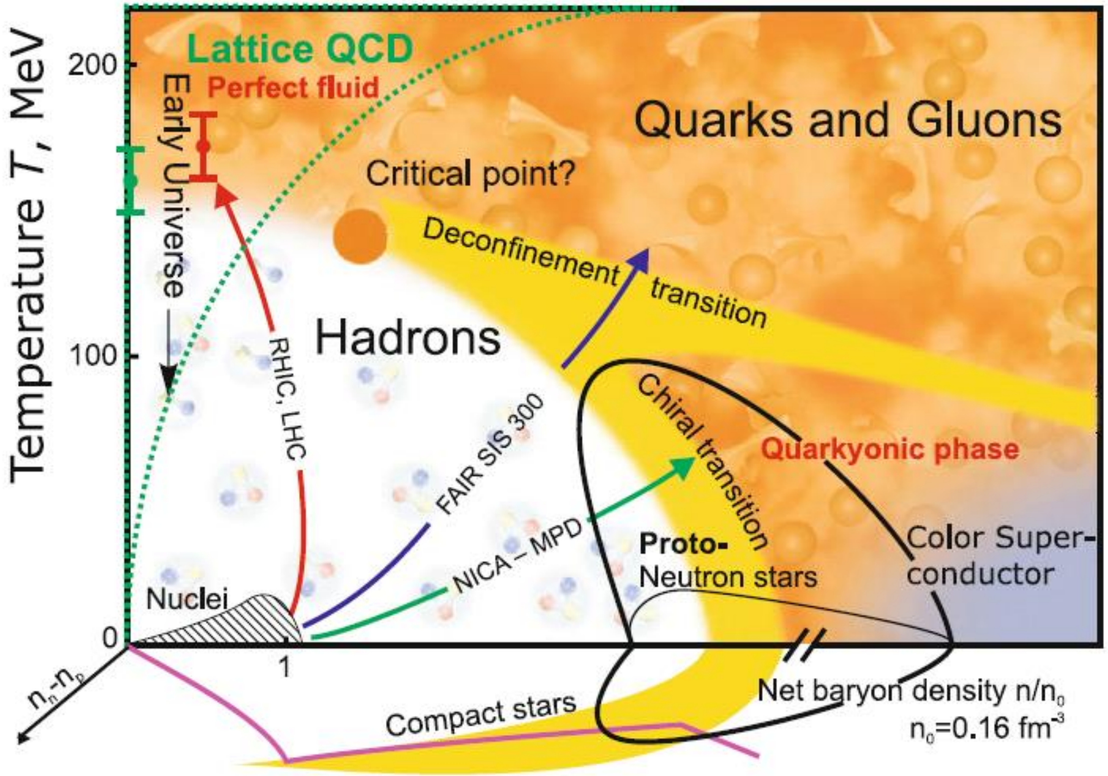
Dynamics of heavy ion collisions?

Nature of compact stars and dense nuclear matter?

Phase Diagram of Strong Interaction Matter

QCD Phase Diagram in 3D: density, temperature, and isospin

V.E. Fortov, Extreme States of Matter – on Earth and in the Cosmos, Springer-Verlag Berlin Heidelberg 2011



Esym: Important for understanding the EOS of strong interaction matter and QCD phase transitions at **extreme isospin conditions**

1. **Heavy Ion Collisions (Terrestrial Lab);**
2. **Compact Stars(In Heaven); …**

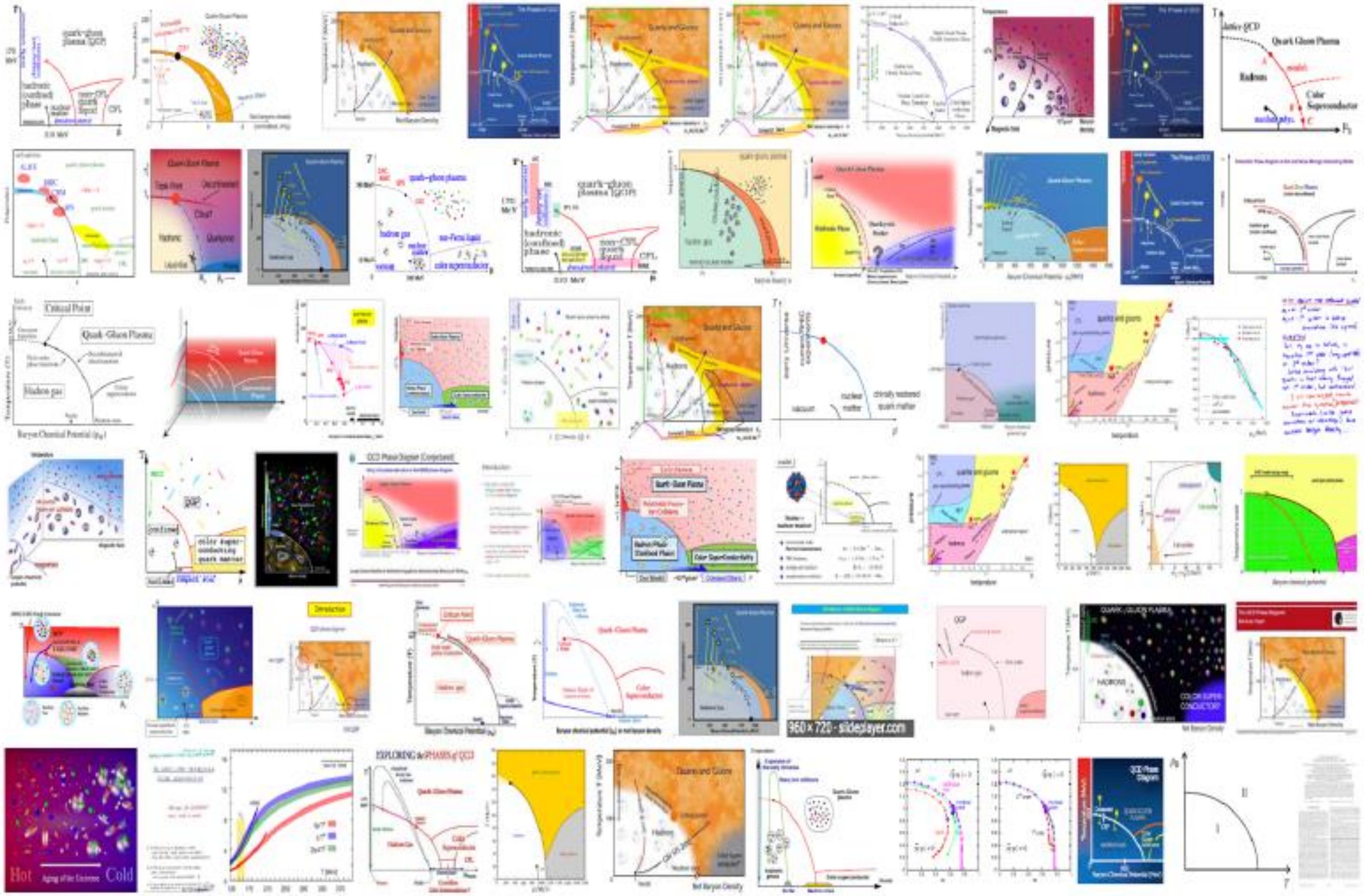
Quark Matter
Symmetry Energy ?

M. Di Toro et al., NPA775 (2006); Pagliara/Schaffner-Bielich, PRD81, (2010); Shao et al., PRD85,(2012); Chu/Chen, ApJ780 (2014); H. Liu et al., PRD94 (2016); Xia/Xu/Zong, (2016); LWC, arXiv:1708.04433

At extremely high baryon density, the matter could be the deconfined **quark matter**, and there we should consider **quark matter symmetry energy** (isospin symmetry is still satisfied). The isopsin asymmetric quark matter could be produced/exist in **HIC/Compact Stars**

QCD Phase Diagram

A selection of representations of the QCD phase diagram in the (μ_B, T) plane

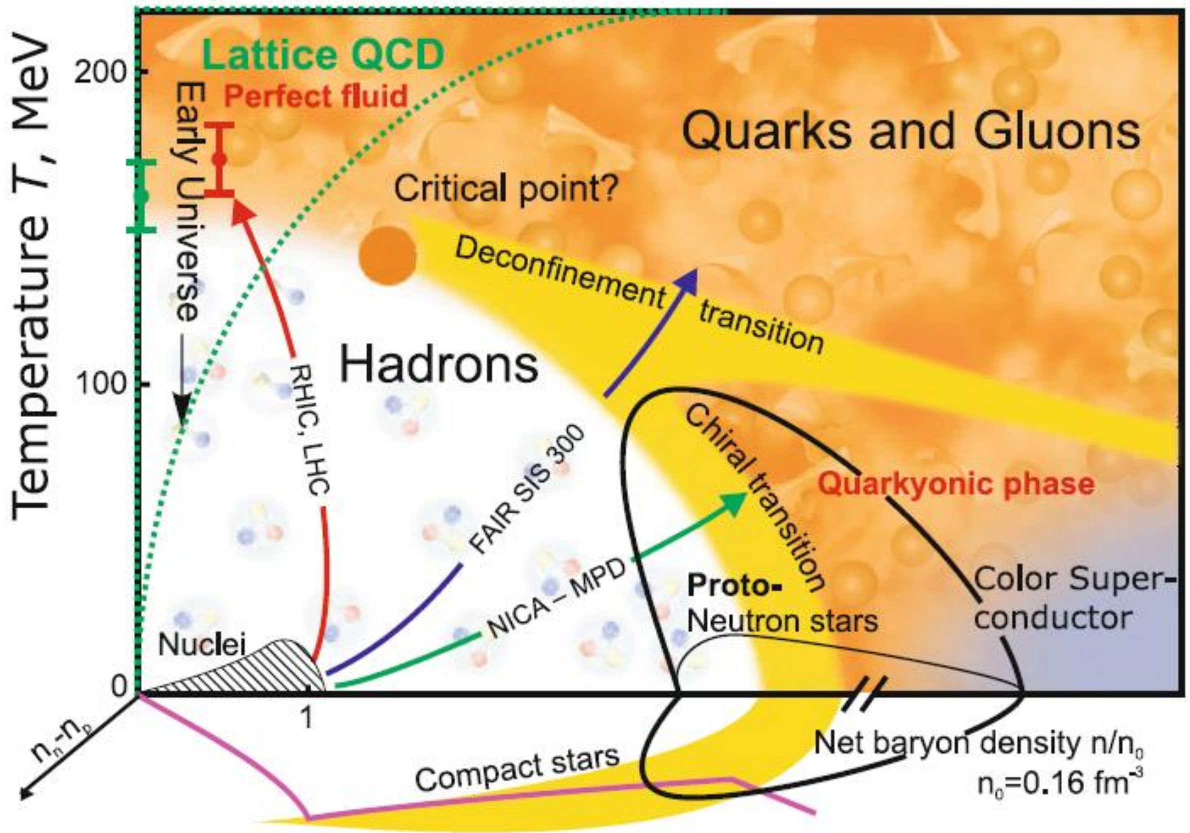


“All science is either physics or stamp collecting.” --- Ernest Rutherford

“The Way Forward – Closing Remarks at Quark Matter 2017”, W.A. Zajc, [arXiv:1707.01993]

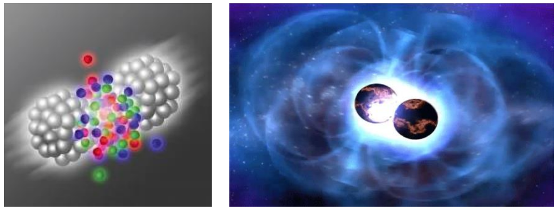
QCD Phase Diagram

V.E. Fortov, Extreme States of Matter – on Earth and in the Cosmos, Springer-Verlag, 2011



Probing QCD phase diagram in Heavy Ion Collisions in terrestrial labs

and in NStar-NStar Collisions (Merger) in heaven? (see, e.g., Andreas Bauswein's talk)

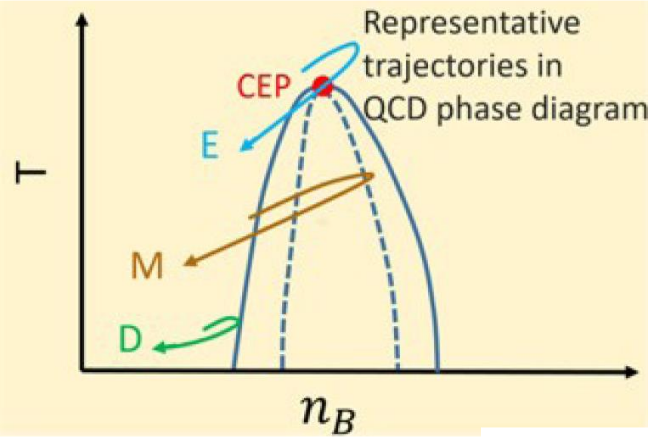


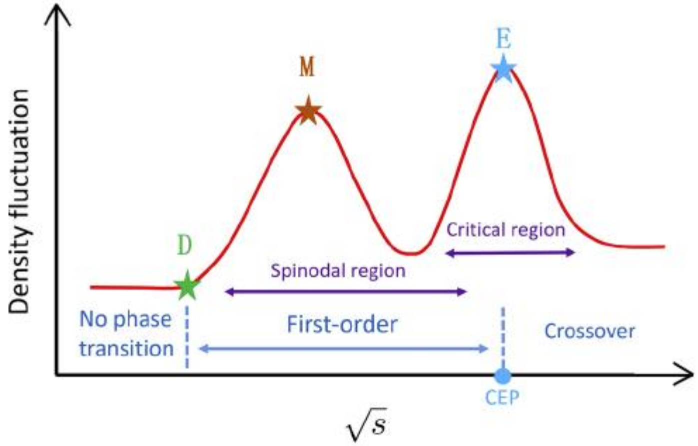
- Small baryon chemical potential: Smooth Crossover Transition
- Large baryon chemical potential: First-order Phase Transition
- QCD Critical Endpoint: where the first-order phase transition ends

QCD Phase Diagram

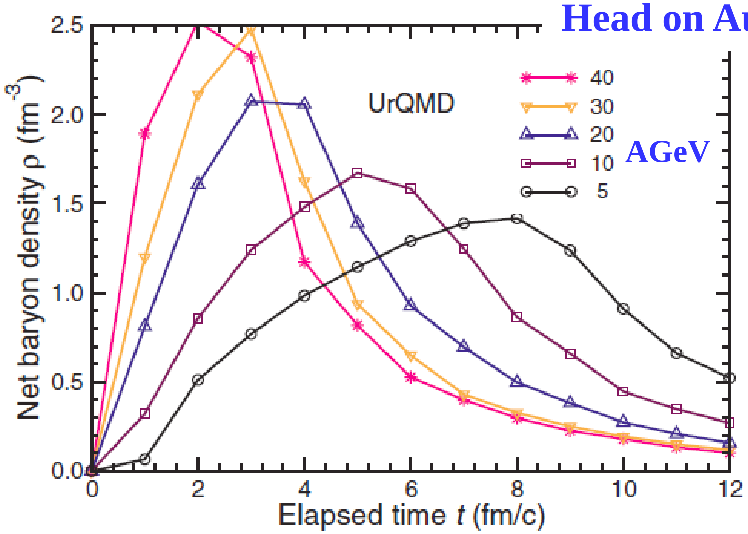
Light nuclei production as a probe of QCD phase diagram

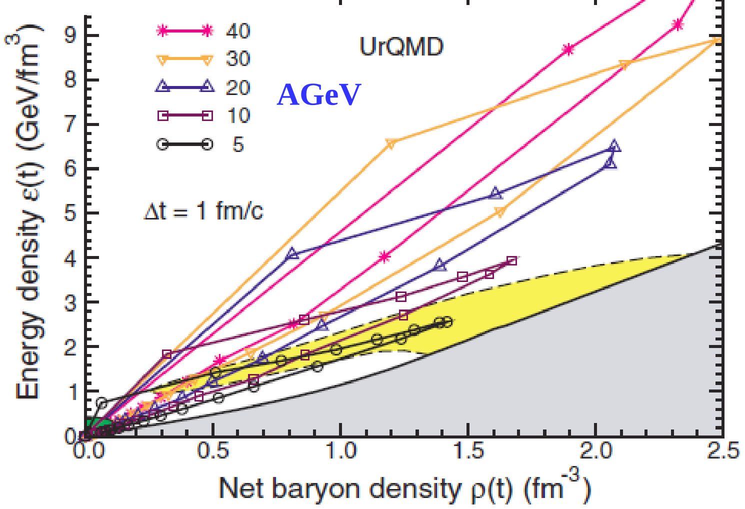
Sun/LWC/Ko,/Xu, PLB774, 103 (2017); Sun/LWC/Ko/Pu,/Xu, PLB781, 499 (2018).





Head on Au+Au collisions





I.C. Arsene et al., PRC75, 034902（2007）

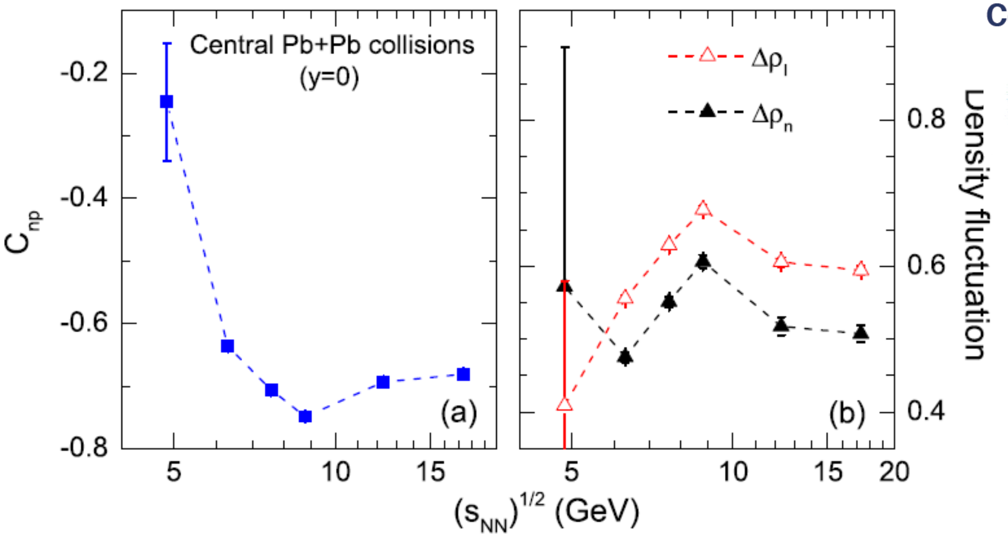
QCD Phase Diagram

Light nuclei production as a probe of QCD phase diagram

Sun/LWC/Ko,/Xu, PLB774, 103 (2017); Sun/LWC/Ko/Pu,/Xu, PLB781, 499 (2018).

$$C_{\rm np} \approx 0.0019\, R_{\rm np} V_{\rm ph} \underline{N_{\rm d}/N_{\rm p}^2} - 1,$$
$$\Delta\rho_n \approx 3.5\,(1 + C_{\rm np})^2 \underline{N_{\rm p} N_{^3\rm H}/N_{\rm d}^2} - 2C_{\rm np} - 1$$

$$\Delta\rho_I = \frac{\langle(\delta\rho_n - \delta\rho_p)^2\rangle}{(\langle\rho_n\rangle + \langle\rho_p\rangle)^2} = \frac{R_{\rm np}^2 \Delta\rho_p - 2R_{\rm np} C_{\rm np} + \Delta\rho_n}{(1 + R_{\rm np})^2}$$

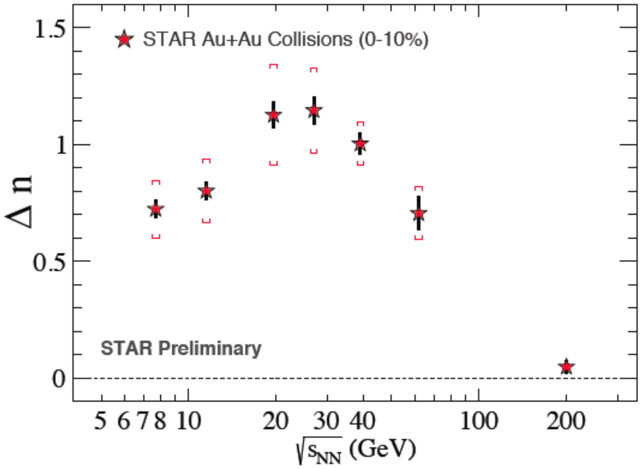


Double-peak structure for neutron density fluctuations inferred from light nuclei ratio is indeed observed in Pb+Pb collisions at SPS/AGS energies!

Collision energy and centrality dependence of light nuclei(triton) production at STAR

Dingwei Zhang, *for the STAR Collaboration*

Central China Normal University



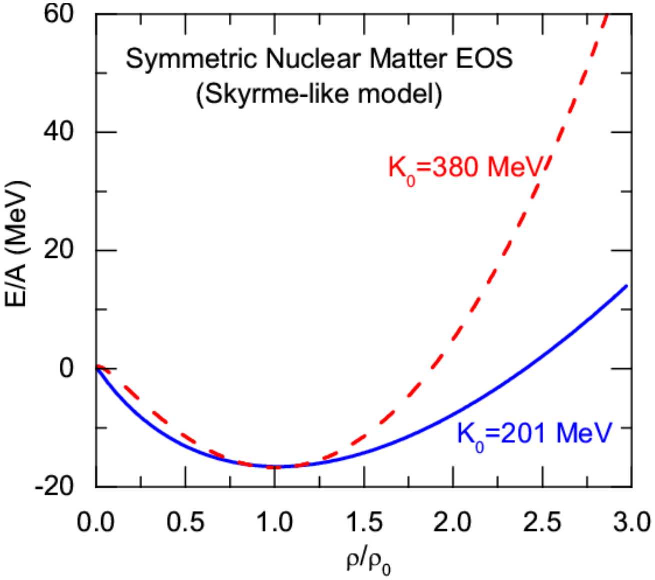
Δn shows a non-monotonic energy dependence with a peak around 20 – 27 GeV. Proton [4] and deuteron [5] measured by STAR.

STAR-BES seems also to suggest a peak!

EOS of Symmetric Nuclear Matter

(1) EOS of symmetric matter around the saturation density ρ_0

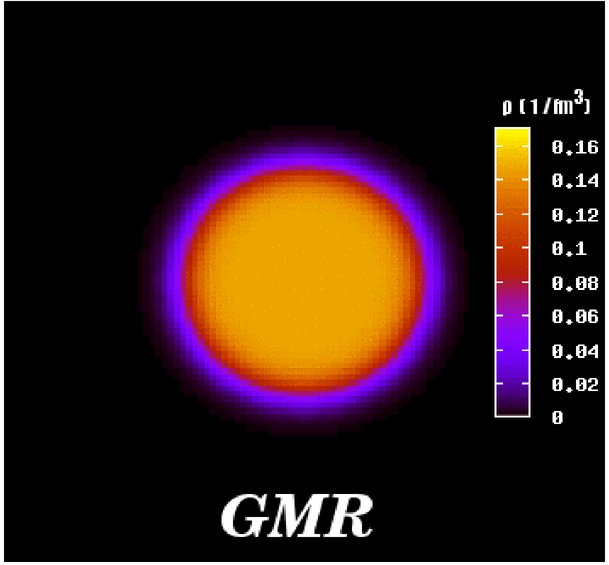
$$E_0(\rho) = E_0(\rho_0) + \frac{K_0}{2!}\chi^2 + \frac{J_0}{3!}\chi^3 + \mathcal{O}(\chi^4) \quad \chi = \frac{\rho-\rho_0}{3\rho_0}$$



Incompressibility:

$$K_0 = 9\rho_0^2 \left(\frac{d^2E}{d\rho^2}\right)_{\rho_0}$$

Giant Monopole Resonance



$$\text{Frequency } f_{GMR} \propto \sqrt{K_0}$$

K_0=231±5 MeV
Youngblood/Clark/Lui, PRL82, 691 (1999)

Recent results:

K_0=230±20 MeV

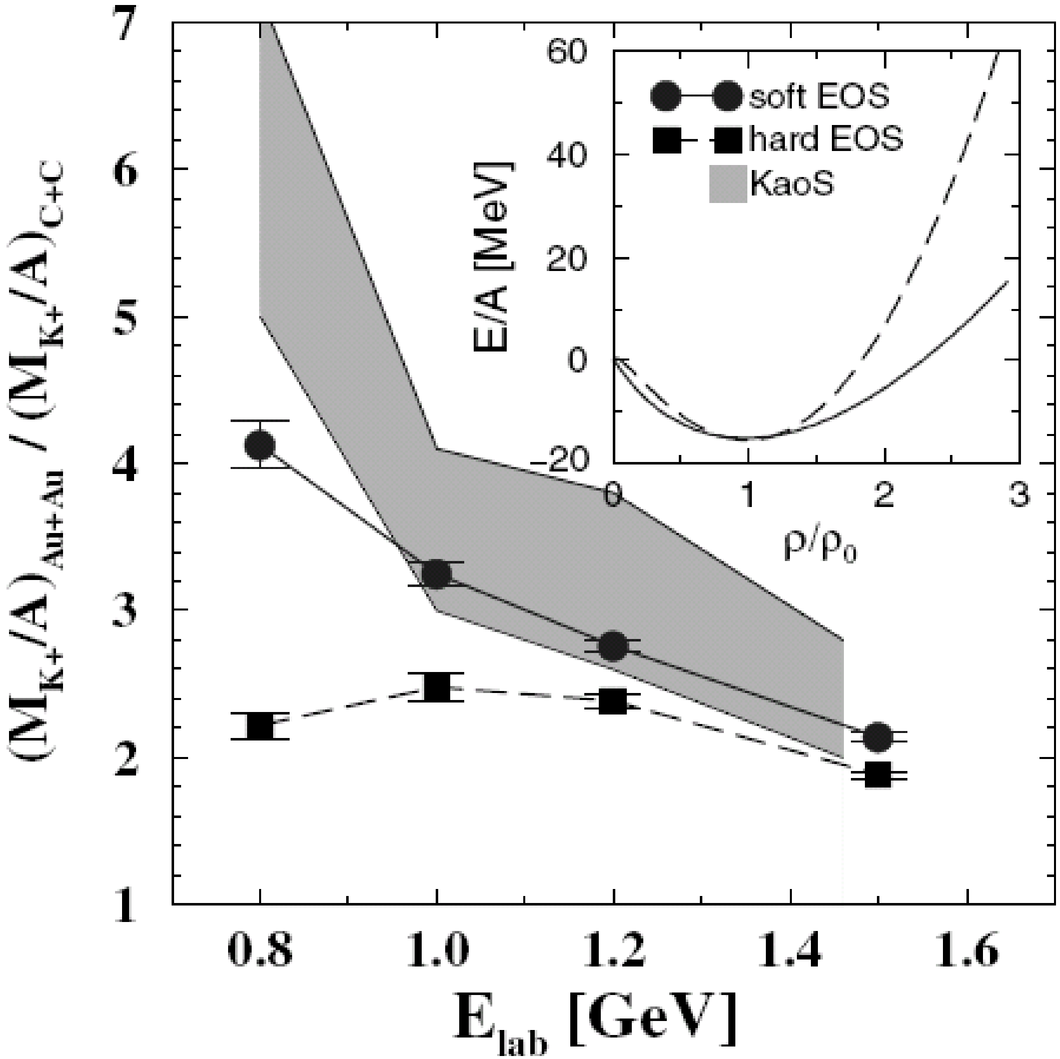
G. Colo, U. Garg,
J. Margueron,
J. Piekarewicz,
H. Sagawa, S. Shlomo et al.

Uncertainty of the extracted K_0 is mainly due to the uncertainty of *L* (slope parameter of the symmetry energy) and m^*_0 (isoscalar nucleon effective mass)

(See, e.g., LWC/J.Z. Gu, JPG39, 035104(2012))

EOS of Symmetric Nuclear Matter

(2) EOS of symmetric matter for $1\rho_0 < \rho < 3\rho_0$ from K^+ production in HIC's



J. Aichelin and C.M. Ko,
PRL55, (1985) 2661

C. Fuchs,
Prog. Part. Nucl. Phys. 56, (2006) 1

C. Fuchs et al,
PRL86, (2001) 1974

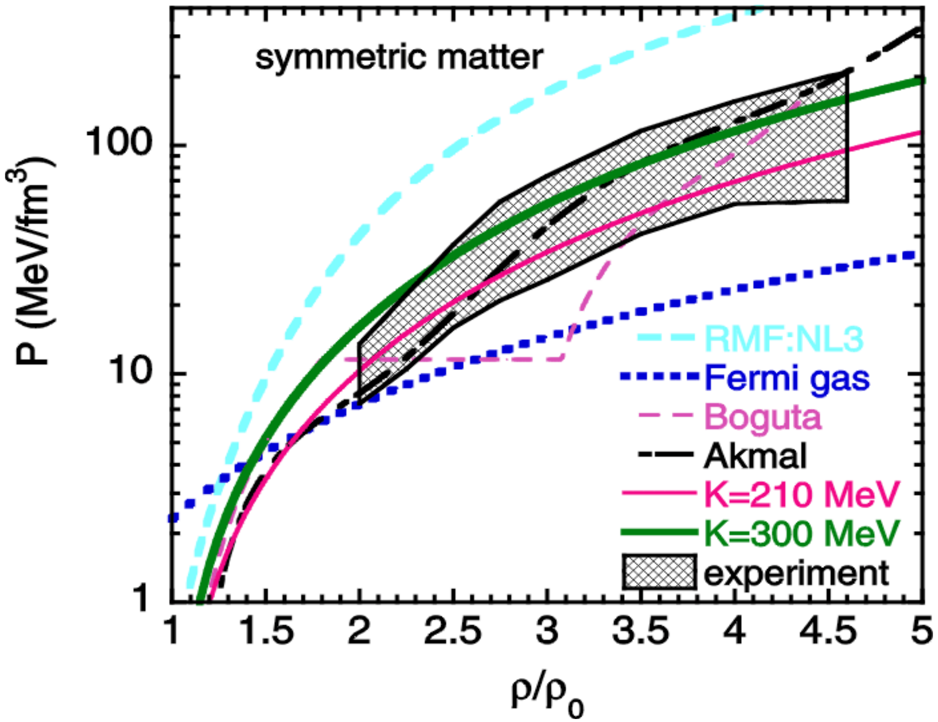
Transport calculations indicate that "results for the K^+ excitation function in Au + Au over C + C reactions as measured by the KaoS Collaboration strongly support the scenario with a soft EOS."

See also: C. Hartnack, H. Oeschler, and J. Aichelin,
PRL96, 012302 (2006)

EOS of Symmetric Nuclear Matter

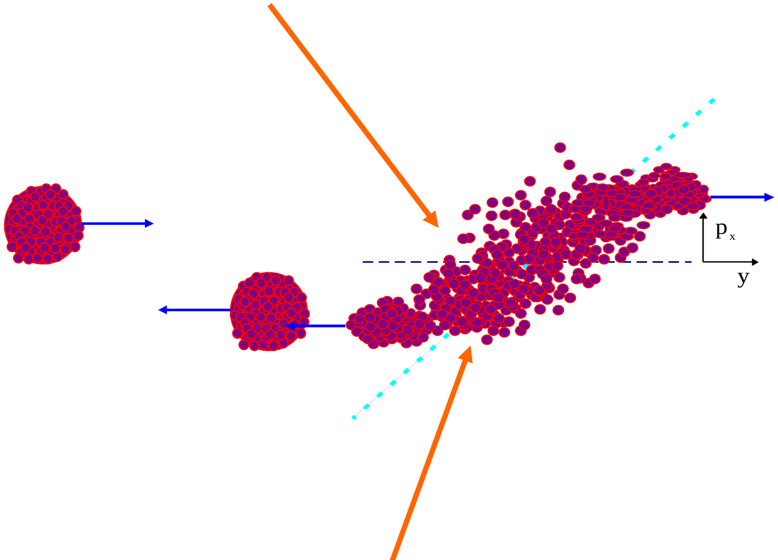
(3) Constraints on the EOS of symmetric nuclear matter for $2\rho_0 < \rho < 5\rho_0$ using flow data from BEVALAC, SIS/GSI and AGS

P. Danielewicz, R. Lacey and W.G. Lynch, Science 298, 1592 (2002)



- Use constrained mean fields to predict the EOS for symmetric matter
  - Width of pressure domain reflects uncertainties in comparison and of assumed momentum dependence.

The highest pressure recorded under laboratory controlled conditions in nucleus-nucleus collisions



High density nuclear matter 2 to $5\rho_0$

$$\text{Pressure}\ \ P(\rho)=\rho^2\left(\frac{\partial E}{\partial \rho}\right)_s$$

Esym: Experimental Probes

Promising Probes of the $E_{sym}(\rho)$
(an incomplete list !)

At sub-saturation densities (亚饱和密度行为)

- Sizes of n-skins of unstable nuclei from total reaction cross sections
- Proton-nucleus elastic scattering in inverse kinematics
- Parity violating electron scattering studies of the n-skin in ^{208}Pb
- n/p ratio of FAST, pre-equilibrium nucleons
- Isospin fractionation and isoscaling in nuclear multifragmentation
- Isospin diffusion/transport
- Neutron-proton differential flow
- Neutron-proton correlation functions at low relative momenta
- t/^{3}He ratio
- Hard photon production
- Pigmy/Giant resonances
- Nucleon optical potential

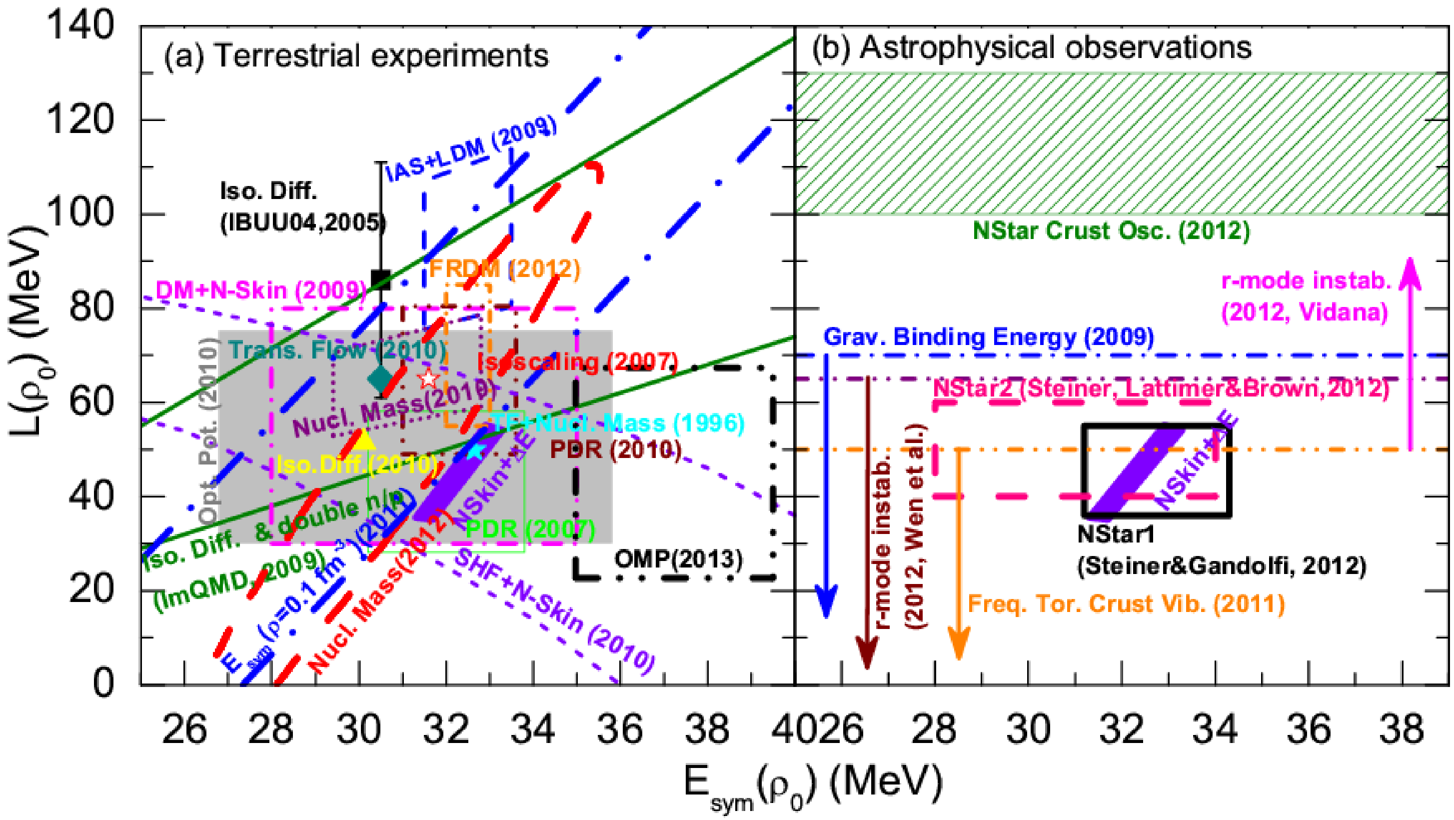
Towards high densities reachable at CSR/Lanzhou, FAIR/GSI, RIKEN, GANIL and, FRIB/MSU (高密度行为)

- π^-/π^+ ratio, K^+/K^0 ratio?
- Neutron-proton differential transverse flow
- n/p ratio at mid-rapidity
- Nucleon elliptical flow at high transverse momenta
- n/p ratio of squeeze-out emission

B.A. Li, L.W. Chen, C.M. Ko
Phys. Rep. 464, 113(2008)

E_{sym}: Around saturation density

Current constraints (An incomplete list) on E_{sym} (ρ_0) and L from terrestrial experiments and astrophysical observations



L and $E_{sym}(\rho_0)$ are usually correlated because the observables are usually probing the Esym NOT at saturation density !!!

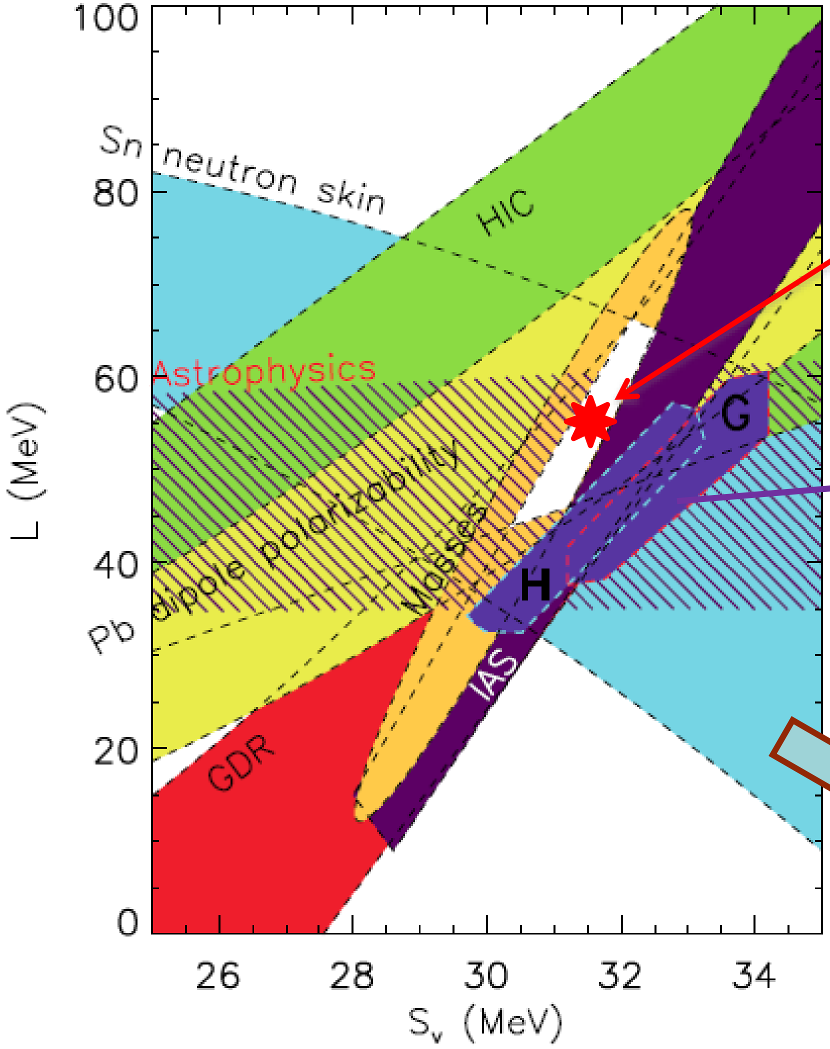
$E_{sym}(\rho_0)$ = 32.5±2.5 MeV, *L* = 55±25 MeV

L.W. Chen, Nucl. Phys. Rev. (原子核物理评论) 31, 273 (2014) [arXiv:1212.0284]
B.A. Li, L.W. Chen, F.J. Fattoyev, W.G. Newton, and C. Xu, arXiv:1212.1178

SHANGHAI JIAO TONG UNIVERSITY

E_{sym}: Around saturation density

Jim Lattimer and Andrew Steiner using 6 out of approximately 30 available constraints



The centroid is around
S_v=32MeV and L=55 MeV

Microscopic calculations (H/G) do not include higher-order contribution? EOS of SNM?

Chen/Ko/Li/Xu, PRC82, 024321 (2010)

Why? Zhang/Chen, PLB726, 234 (2013)

Neutron skin is actually determined by L(0.11 fm^{-3}) rather than L(0.16 fm^{-3})

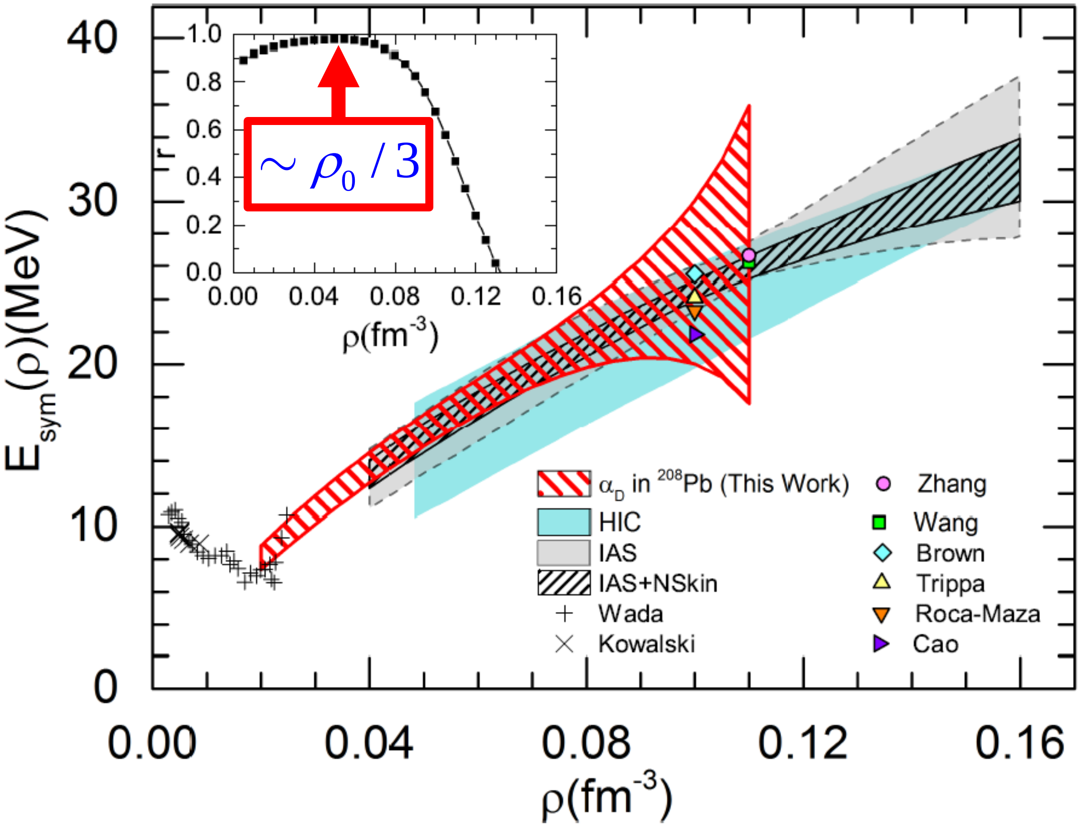
0.11 fm^{-3} ~ Average density of Heavy Nuclei

J.M. Lattimer and A.W. Steiner, EPJA50, (2014) 40

E_{sym}: Subsaturation densities

Z. Zhang and LWC, PRC92, 031301(R) (2015)

- $\Delta E(A\sim 208)\propto E_{sym}(\rho_{A=208}),\ \rho_{A=208}\approx 2/3\rho_0$
- $1/\alpha_D(A=208)\propto E_{sym}(\rho_{A=45}),\ \rho_{A=45}\approx 1/3\rho_0$



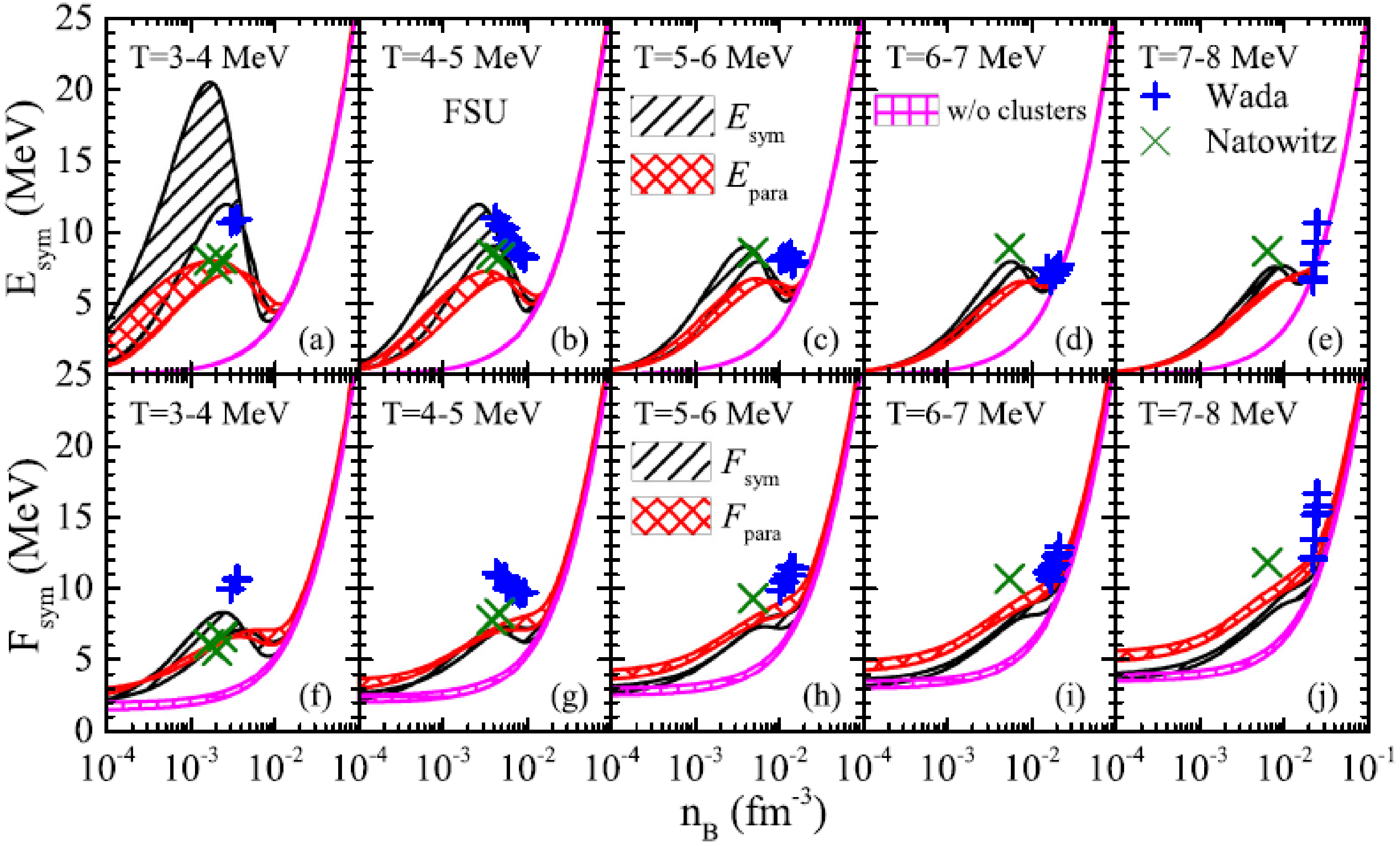
- HIC: Sn+Sn
  M.B. Tsang *et al.*, Phys. Rev. Lett.**102**, 122701(2009)
- IAS and IAS+NSkin
  P. Danielewicz and J. Lee, Nucl. Phys. **A922**, 1 (2014)
- Zhang: Isotope binding energy difference
  Z. Zhang and L.W. Chen, Phys. Lett. **B726**, 234 (2013)
- Wang: Fermi energy difference
  N. Wang *et al.*, Phys. Rev. C **87**, 034327 (2013)
- Brown: Doubly magic nuclei
  B.A. Brown, Phys. Rev. Lett. **11**, 232502 (2013)
- Trippa: Giant dipole resonance
  L. Trippa *et al.*, Phys. Rev. C **77**
- Roca-Maza: Giant quadrupole resonance
  X. Roca-Maza *et al.*, Phys. Rev. C **87**, 034301 (2013)
- Cao: Pygmy dipole resonance
  L.G. Cao and Z.Y. Ma, Chin. Phys. Lett. **25**, 1625 (2008)

Wada and Kowalski: experimental results of the symmetry energies at densities below $0.2\rho_0$ and temperatures in the range 3 ~11 MeV from the analysis of cluster formation in heavy ion collisions.

Wada et al., Phys. Rev. C85, (2012) 064618; Kowlski et al., Phys. Rev. C75, (2007) 014601.
Natowitz et al., Phys. Rev. Lett. 104, (2010) 202501.

E_{sym}: Subsaturation densities

Clustering effects on Esym within NL-RMF for n, p, t, h, α matter

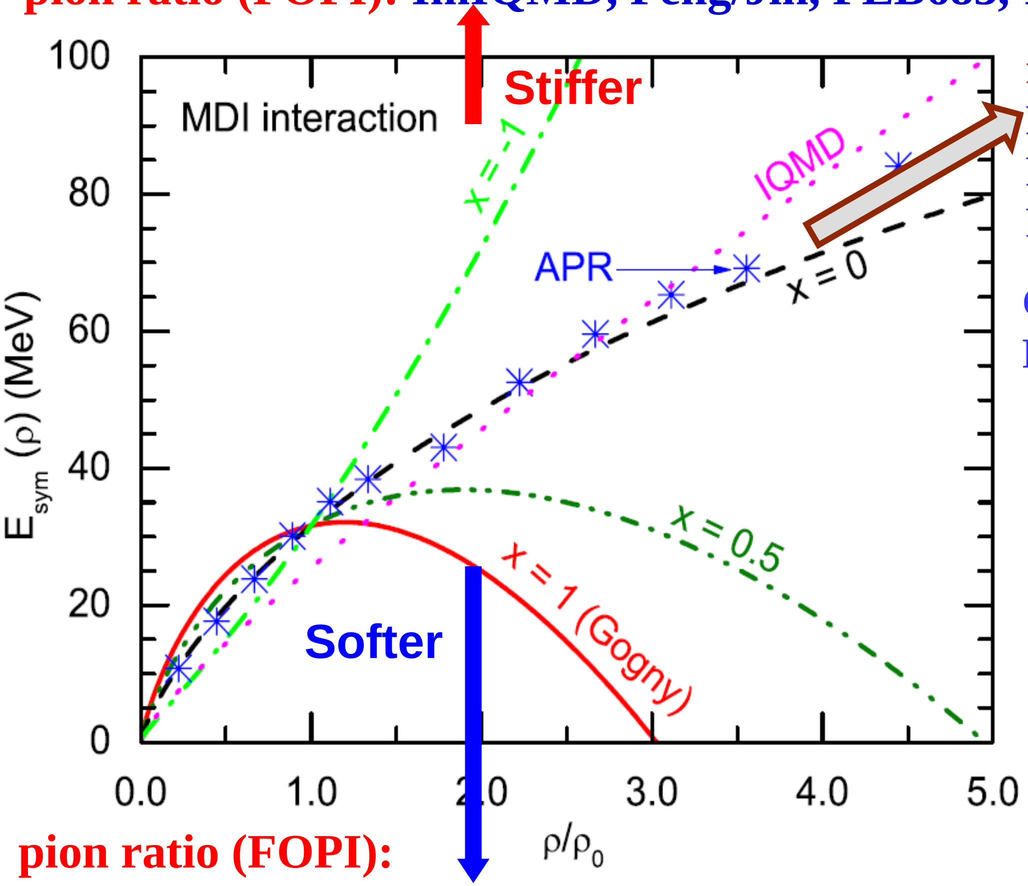


Zhao-Wen Zhang and LWC, PRC95, 064330 (2017)

E_{sym}: Supra-saturation density

A Soft or Stiff Esym at supra-saturation densities ???

pion ratio (FOPI): ImIQMD, Feng/Jin, PLB683, 140(2010)



pion ratio (FOPI):
IBUU04, Xiao/Li/Chen/Yong/Zhang, PRL102,062502(2009)
ImIBLE, Xie/Su/Zhu/Zhang, PLB718,1510(2013)

n/p v2 (FOPI): $\boxed{(\rho/\rho_0)^\gamma \text{ with } \gamma = 0.9 \pm 0.4}$
Russotto/Trautmann/Li et al.,
PLB697, 471(2011) (UrQMD)
PRC94, 034608 (2016) $\boxed{\gamma = 0.72 \pm 0.19}$

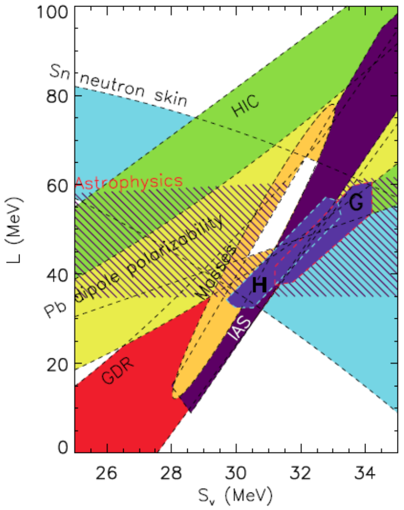
Cozma/Trautmann/Li et al.,
PRC88, 044912 (2013) (Tubingen QMD - MDI)

Pion Medium Effects?
Threshold effects?
Δ resonances?
……
Xu/Ko/Oh
PRC81, 024910(2010)
Xu/Chen/Ko/Li/Ma
PRC87, 067601(2013)
Hong/Danielewicz,
PRC90, 024605 (2014)
Song/Ko, PRC91, 014901 (2015)

E_{sym}: Current Status

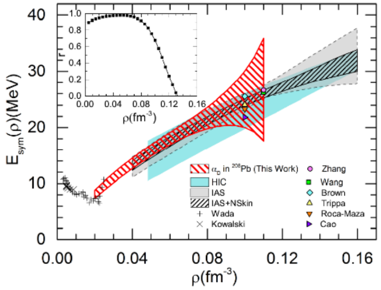
- There are MANY constraints on E_{sym} (ρ_0) and L, essentially all the constraints seem to agree with:

$$E_{sym}(\rho_0) = 32.5 \pm 2.5 \text{ MeV}$$
$$L = 55 \pm 25 \text{ MeV}$$



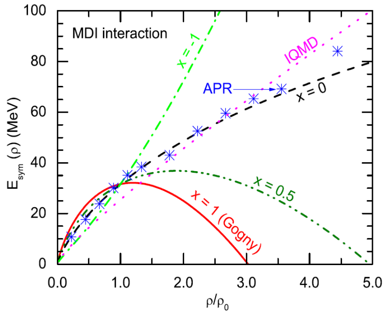
Lattimer/Steiner, EPJA50, 40 (2014)

- The symmetry energy at subsaturation densities have been relatively well-constrained



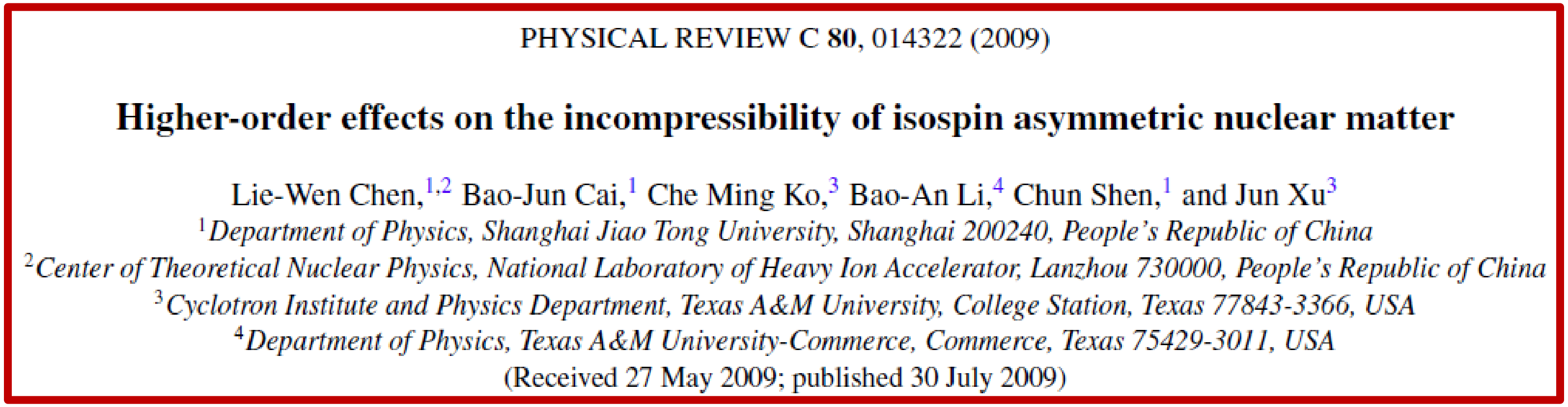
Z. Zhang/LWC, PLB726, 234 (2013); PRC92, 031301(R)(2015)

- The constraints on the high density Esym are still elusive and controversial for the moment !!!



Xiao/Li/Chen/Yong/Zhang, PRL102, 062502 (2009)

Characteristic Parameters of NM EOS



PHYSICAL REVIEW C **80**, 014322 (2009)

Higher-order effects on the incompressibility of isospin asymmetric nuclear matter

Lie-Wen Chen,[1,2] Bao-Jun Cai,[1] Che Ming Ko,[3] Bao-An Li,[4] Chun Shen,[1] and Jun Xu[3]
[1] *Department of Physics, Shanghai Jiao Tong University, Shanghai 200240, People's Republic of China*
[2] *Center of Theoretical Nuclear Physics, National Laboratory of Heavy Ion Accelerator, Lanzhou 730000, People's Republic of China*
[3] *Cyclotron Institute and Physics Department, Texas A&M University, College Station, Texas 77843-3366, USA*
[4] *Department of Physics, Texas A&M University-Commerce, Commerce, Texas 75429-3011, USA*

(Received 27 May 2009; published 30 July 2009)

$$\boxed{E(\rho,\delta) = E_0(\rho) + E_{\text{sym}}(\rho)\delta^2 + E_{\text{sym},4}(\rho)\delta^4 + O(\delta^6)}$$

$$E_0(\rho) = E_0(\rho_0) + \frac{K_0}{2!}\chi^2 + \frac{J_0}{3!}\chi^3 + \frac{I_0}{4!}\chi^4 + O(\chi^5)$$

$$\boxed{\chi = \frac{\rho - \rho_0}{3\rho_0}}$$

$$E_{\text{sym}}(\rho) = E_{\text{sym}}(\rho_0) + L\chi + \frac{K_{\text{sym}}}{2!}\chi^2 + \frac{J_{\text{sym}}}{3!}\chi^3 + \frac{I_{\text{sym}}}{4!}\chi^4 + O(\chi^5)$$

$$E_{\text{sym},4}(\rho) = E_{\text{sym},4}(\rho_0) + L_{\text{sym},4}\chi + \frac{K_{\text{sym},4}}{2}\chi^2 + \frac{J_{\text{sym},4}}{3!}\chi^3 + \frac{I_{\text{sym},4}}{4!}\chi^4 + O(\chi^5)$$

Order of the characteristic parameters according to the expansion with χ and δ:

Order-0: $E_0(\rho_0)$; Order-2: K_0, $E_{sym}(\rho_0)$;

Order-3: J_0, L; Order-4: $K_{sym}(\rho_0)$, I_0, $E_{sym,4}(\rho_0)$

Characteristic Parameters of NM EOS

Order of the characteristic parameters according to the expansion with χ and δ:

Order-0: $E_0(\rho_0)$; Order-2: K_0, $E_{sym}(\rho_0)$;
Order-3: J_0, L; Order-4: $K_{sym}(\rho_0)$, I_0, $E_{sym,4}(\rho_0)$

Order-0 → $E_0(\rho_0)$ = -16±1 MeV

Order-2 → K_0 = 230±20 MeV, $E_{sym}(\rho_0)$ = 32.5±2.5 MeV

Order-3 → L = 55±25 MeV, J_0=???

Order-4 → $K_{sym}(\rho_0)$=???, I_0=???, $E_{sym,4}(\rho_0)$ =???

Order-5 → ??????

……

- **$J_0 \approx$ -408.5 +/- 66.5 MeV and $K_{sym} \approx$ -118.5 +/- 84.5 MeV:**
  Data of finite nuclei + Flow Data in HIC + Observed NStar Largest Mass + Tidal Deformability of Neutron Star (from recent GW170817 signal) analyzed simultaneously within the same EDF – extended SHF

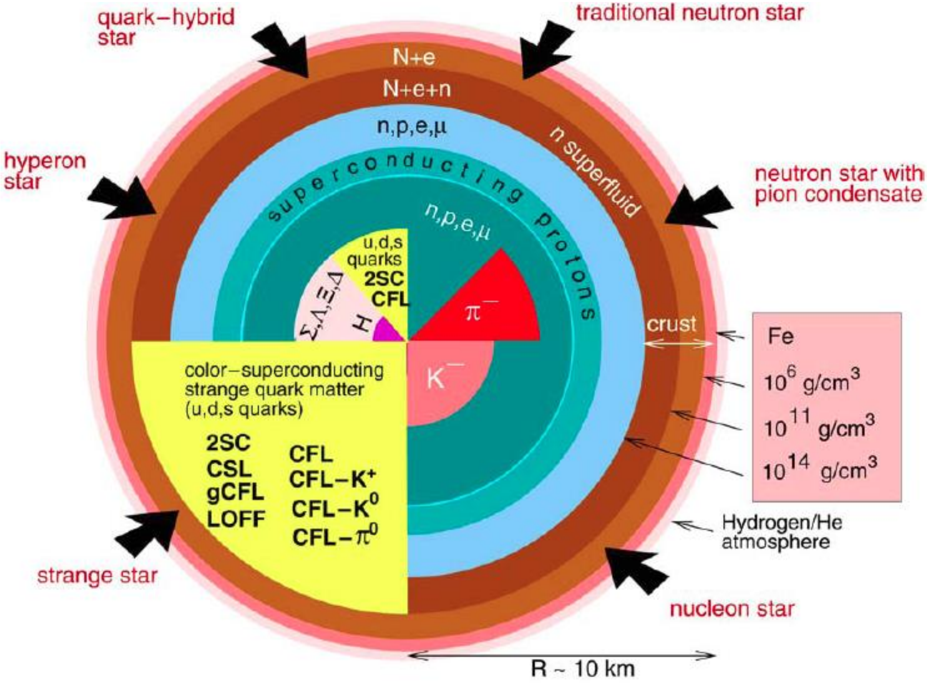
Outline

- Nuclear matter EOS and the symmetry energy (Esym)
- Dense nuclear matter from Nuclear experiments + Observed neutron star largest mass + Tidal deformability from GW170817
- Summary and outlook

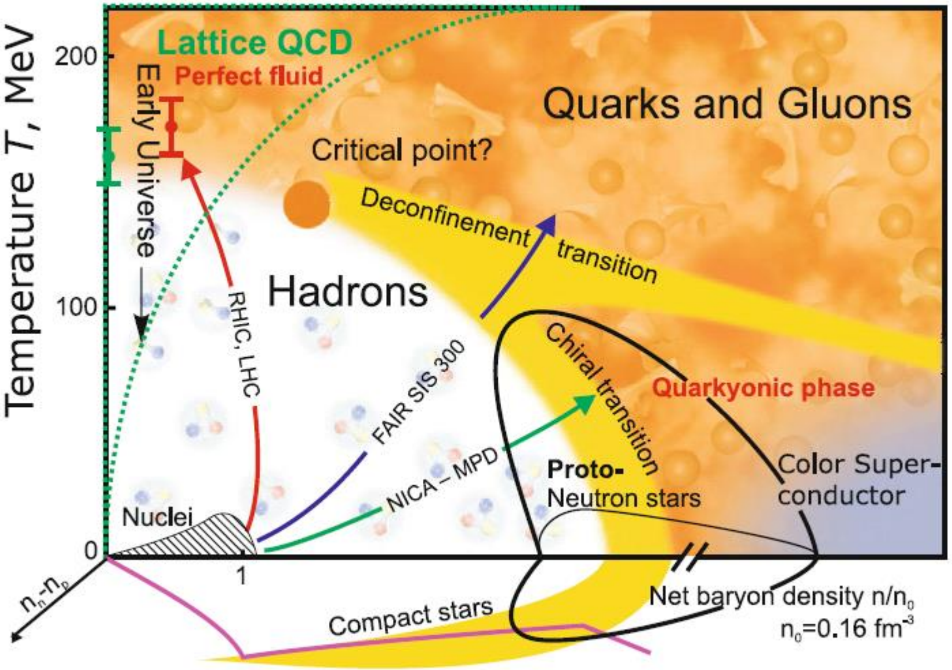
Composition of Neutron Star Matter

Pulsars: Neutron Stars? Quark Stars? Hybrid Stars? Others?

F. Weber, PPNP54, 193 (2005)



V.E. Fortov, Extreme States of Matter – on Earth and in the Cosmos, Springer-Verlag Berlin Heidelberg 2011



Mass: ~ 1.4 $M_{\odot}$, Radius: ~ 10 km
Extremely neutron-rich matter
Density at the center: ~ $6\rho_0$
Average density: ~ $2.5\rho_0$

Assuming there are no phase transitions and no strangeness in NStar or they play unimportant role (for inspiral). The NStar is conventional star composed of npeμ.

EOS of Neutron Star Matter

- Core of the neutron stars consist of infinite β-equilibrium npeμ matter with charge neutrality. Its EoS is determined by the extended Skyrme-Hartree-Fock(eSHF)

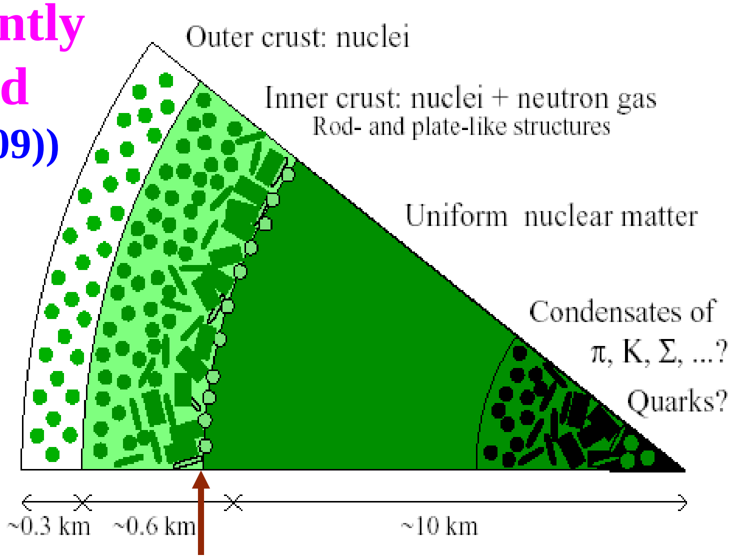
- The inner crust $2.46\times10^{-4}\,\mathrm{fm}^{-3} = n_{\mathrm{out}} < n < n_t$

n_t is determined self-consistently by using dynamical method

(Xu/LWC/Li/Ma, ApJ697,1549(2009))

$$P = a + b\epsilon^{4/3}$$

$$a = \frac{P_{\mathrm{out}}\epsilon_{\mathrm{t}}^{4/3} - P_{\mathrm{t}}\epsilon_{\mathrm{out}}^{4/3}}{\epsilon_{\mathrm{t}}^{4/3} - \epsilon_{\mathrm{out}}^{4/3}} \qquad b = \frac{P_{\mathrm{t}} - P_{\mathrm{out}}}{\epsilon_{\mathrm{t}}^{4/3} - \epsilon_{\mathrm{out}}^{4/3}}$$



Core-Crust edge

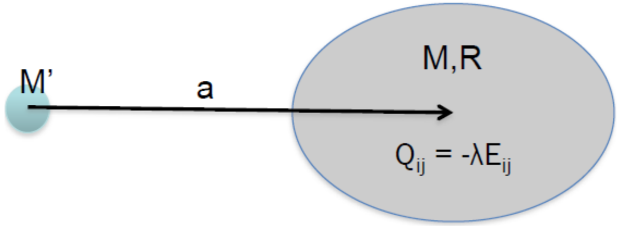
- The outer crust

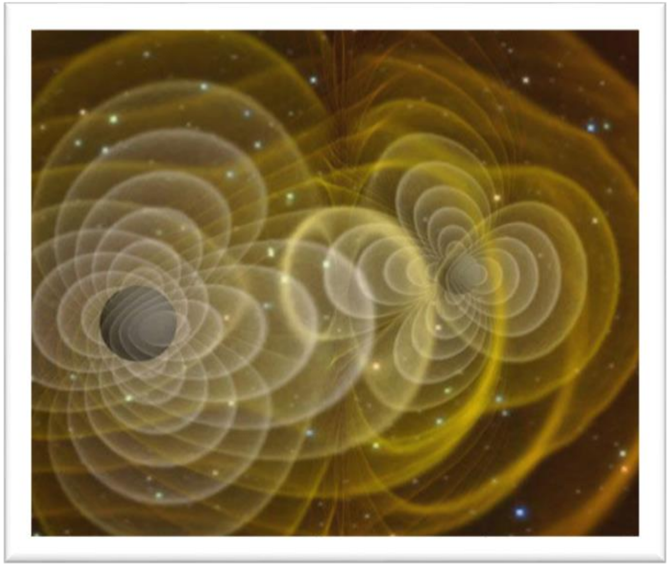
$6.93\times10^{-13}\,\mathrm{fm}^{-3} < n < n_{\mathrm{out}}$ (EOS of BPS)

$4.73\times10^{-15}\,\mathrm{fm}^{-3} < n < 6.93\times10^{-13}\,\mathrm{fm}^{-3}$ (EOS of Feynman-Metropolis-Teller)

Tidal Deformability

Tidal Deformability (Polarizability) (oscillation response coefficient λ)





$$Q_{ij} = \lambda \varepsilon_{ij}$$

Q_{ij}: Quadrupole moment

ε_{ij}: Tidal field of companion

$$\boxed{\lambda = \frac{2}{3} k_2 R^5}$$

k_2: Love number
R: Radius
M: Mass

Dimensionless Tidal Deformability

$$\Lambda = \frac{2}{3} k_2 (R / M)^5$$

Éanna É. Flanagan and Tanja Hinderer, Phys.Rev.D 77, 021502(R) (2008)
F.J. Fattoyev, J. Carvajal, W.G. Newton, and Bao-An Li, Phys. Rev. C 87, 015806 (2013)

The extended SHF Energy Density Functional

Extended Skyrme Interaction:

$$
\begin{aligned}
v_{i,j} = {} & t_0(1+x_0P_\sigma)\delta(\boldsymbol{r}) \\
& + \frac{1}{2}t_1(1+x_1P_\sigma)[\boldsymbol{K}'^2\delta(\boldsymbol{r})+\delta(\boldsymbol{r})\boldsymbol{K}^2] \\
& + t_2(1+x_2P_\sigma)\boldsymbol{K}'\cdot\delta(\boldsymbol{r})\boldsymbol{K} \\
& + \frac{1}{6}t_3(1+x_3P_\sigma)n(\boldsymbol{R})^\alpha\delta(\boldsymbol{r}) \\
& + iW_0(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\boldsymbol{K}'\cdot\delta(\boldsymbol{r})\boldsymbol{K} \\
& + \frac{1}{2}t_4(1+x_4P_\sigma)[\boldsymbol{K}'^2n(\boldsymbol{R})^\beta\delta(\boldsymbol{r})+\delta(\boldsymbol{r})n(\boldsymbol{R})^\beta\boldsymbol{K}^2] \\
& + t_5(1+x_5P_\sigma)\boldsymbol{K}'\cdot n(\boldsymbol{R})^\gamma\delta(\boldsymbol{r})\boldsymbol{K}
\end{aligned}
$$

N. Chamel, S. Goriely, and J.M. Pearson, PRC80, 065804 (2009)

Z. Zhang/LWC, PRC94, 064326 (2016)

Momentum-dependence of many-body forces

LWC/Ko/Li/Xu, PRC82, 024321(2010)

13 Skyrme parameters: $\alpha,\ t_0 \sim t_5,\ x_0 \sim x_5$

$$
\begin{aligned}
\mathcal{H} = {} & \mathcal{K}+\mathcal{H}_0+\mathcal{H}_3+\mathcal{H}_{\rm eff}+\frac{G_S}{2}(\nabla\rho)^2-\frac{G_V}{2}(\nabla\rho_1)^2 \\
& -\frac{G_{SV}}{2}\delta\nabla\rho\nabla\rho_1+\mathcal{H}_{\rm Coul}+\mathcal{H}_{\rm SO}+\mathcal{H}_{\rm sg},
\end{aligned}
$$

13 macroscopic nuclear properties:

$n_0,\ E_0,\ K_0,\ J_0,\ E_{\rm sym},\ L,\ K_{\rm sym},\ m^*_{s,0},\ m^*_{v,0},\ G_S,\ G_V,\ G_{SV},\ G'_0$

Why extended SHF EDF?

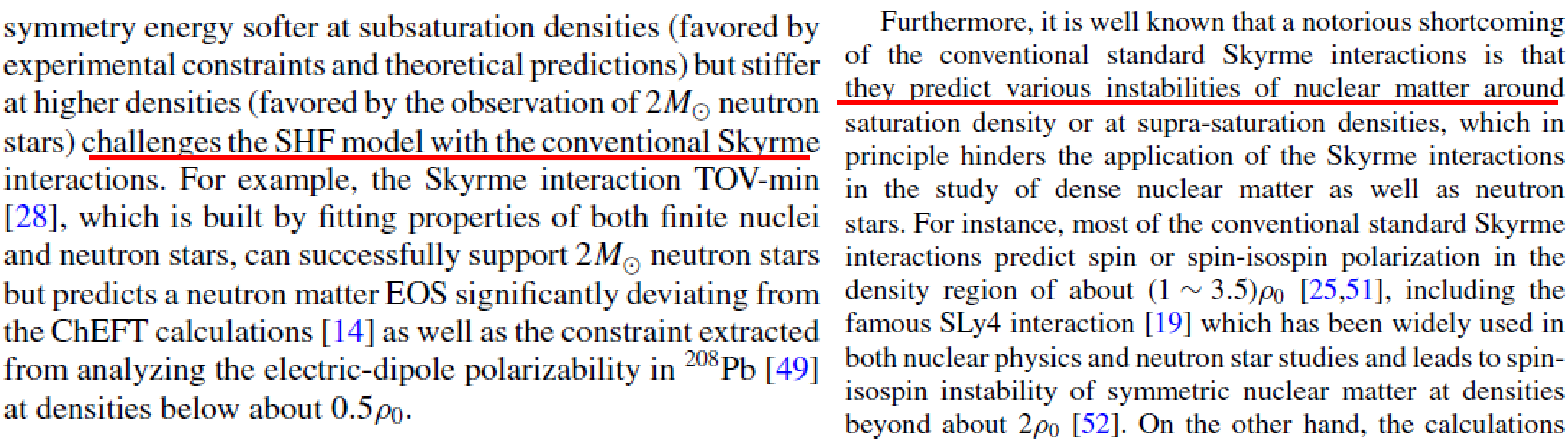
PHYSICAL REVIEW C **94**, 064326 (2016)

Extended Skyrme interactions for nuclear matter, finite nuclei, and neutron stars

Zhen Zhang[1] and Lie-Wen Chen[1,2,*]

[1] *Department of Physics and Astronomy and Shanghai Key Laboratory for Particle Physics and Cosmology, Shanghai Jiao Tong University, Shanghai 200240, China*

[2] *Center of Theoretical Nuclear Physics, National Laboratory of Heavy Ion Accelerator, Lanzhou 730000, China*



symmetry energy softer at subsaturation densities (favored by experimental constraints and theoretical predictions) but stiffer at higher densities (favored by the observation of $2M_\odot$ neutron stars) challenges the SHF model with the conventional Skyrme interactions. For example, the Skyrme interaction TOV-min [28], which is built by fitting properties of both finite nuclei and neutron stars, can successfully support $2M_\odot$ neutron stars but predicts a neutron matter EOS significantly deviating from the ChEFT calculations [14] as well as the constraint extracted from analyzing the electric-dipole polarizability in ^{208}Pb [49] at densities below about $0.5\rho_0$.

Furthermore, it is well known that a notorious shortcoming of the conventional standard Skyrme interactions is that they predict various instabilities of nuclear matter around saturation density or at supra-saturation densities, which in principle hinders the application of the Skyrme interactions in the study of dense nuclear matter as well as neutron stars. For instance, most of the conventional standard Skyrme interactions predict spin or spin-isospin polarization in the density region of about $(1 \sim 3.5)\rho_0$ [25,51], including the famous SLy4 interaction [19] which has been widely used in both nuclear physics and neutron star studies and leads to spin-isospin instability of symmetric nuclear matter at densities beyond about $2\rho_0$ [52]. On the other hand, the calculations

- ☐ **The eSHF provides a nice approach that can describe simultaneously nuclear matter, finite nuclei, and neutron stars!**
- ☐ **The eSHF EDF is very flexible to mimic various density behaviors for EOS (13 parameters)**

Extended Skyrme forces with fixed J_0 and K_{syn}

$$\boxed{n_0,\ E_0,\ K_0,\ J_0,\ E_{\mathrm{sym}},\ L,\ K_{\mathrm{sym}},\ m^*_{s,0},\ m^*_{v,0},\ G_S,\ G_V,\ G_{SV},\ G'_0}$$

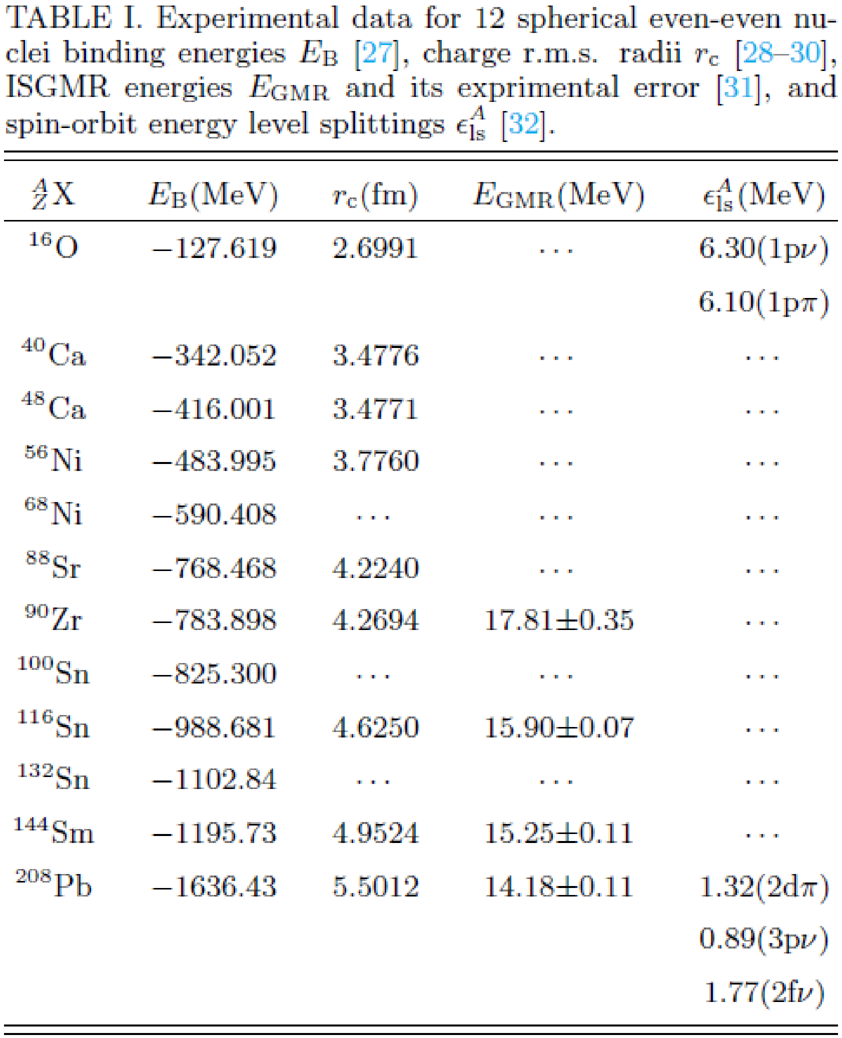


TABLE I. Experimental data for 12 spherical even-even nuclei binding energies E_{B} [27], charge r.m.s. radii r_{c} [28–30], ISGMR energies E_{GMR} and its exprimental error [31], and spin-orbit energy level splittings ϵ^A_{ls} [32].

${}^A_Z\mathrm{X}$	E_{B}(MeV)	r_{c}(fm)	E_{GMR}(MeV)	ϵ^A_{ls}(MeV)
^{16}O	−127.619	2.6991	⋯	6.30(1pν) 6.10(1pπ)
^{40}Ca	−342.052	3.4776	⋯	⋯
^{48}Ca	−416.001	3.4771	⋯	⋯
^{56}Ni	−483.995	3.7760	⋯	⋯
^{68}Ni	−590.408	⋯	⋯	⋯
^{88}Sr	−768.468	4.2240	⋯	⋯
^{90}Zr	−783.898	4.2694	17.81±0.35	⋯
^{100}Sn	−825.300	⋯	⋯	⋯
^{116}Sn	−988.681	4.6250	15.90±0.07	⋯
^{132}Sn	−1102.84	⋯	⋯	⋯
^{144}Sm	−1195.73	4.9524	15.25±0.11	⋯
^{208}Pb	−1636.43	5.5012	14.18±0.11	1.32(2dπ) 0.89(3pν) 1.77(2fν)

Our Strategy:

- ☐ Higher-order J0 and Ksym are fixed at various values
- ☐ $E_{\mathrm{sym}}(\rho_c)$ and $L(\rho_c)$ at $\rho_c = 0.11\ \mathrm{fm}^{-3}$ are fixed at $E_{\mathrm{sym}}(\rho_c) = 26.65$ MeV and $L(\rho_c) = 47.3 +/- 7.8$ MeV using heavy isotope binding energy difference and α_D of ^{208}Pb (Z. Zhang/LWC, PLB726, 234(2013); PRC90, 064317(2014))
- ☐ Other 9 lower-order parameters and W_0 are calibrated to fit data of finite nuclei
- ☐ Causality

Minimizing the Chi-square $\chi^2(p)$:

$$\chi^2(P) = \sum_{n=1}^{N}\left(\frac{\mathcal{O}_n^{(\mathrm{th})}(P) - \mathcal{O}_n^{(\mathrm{exp})}}{\Delta\mathcal{O}_n}\right)^2$$

Observed Nstar max. mass and Tidal def.

28 OCTOBER 2010 | VOL 467 | NATURE | 1081

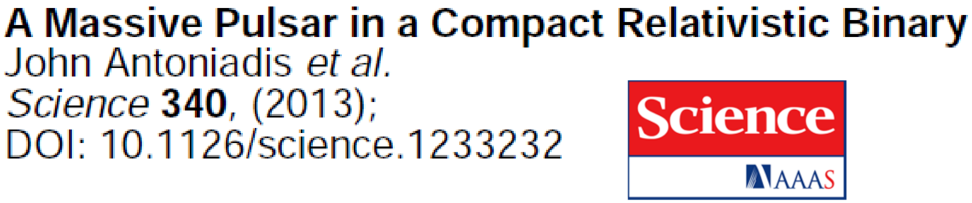
LETTER

doi:10.1038/nature09466

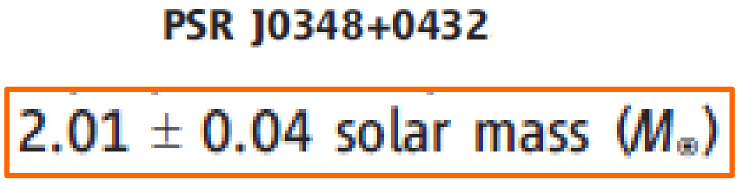
A two-solar-mass neutron star measured using Shapiro delay

P. B. Demorest[1], T. Pennucci[2], S. M. Ransom[1], M. S. E. Roberts[3] & J. W. T. Hessels[4,5]

Observed heaviest Nstar so far:

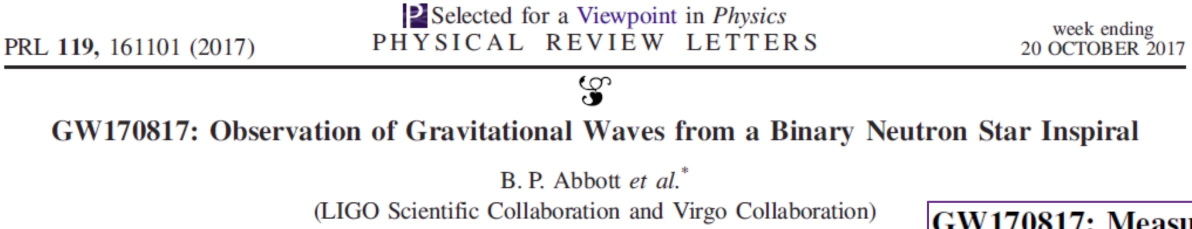


A Massive Pulsar in a Compact Relativistic Binary
John Antoniadis *et al.*
Science **340**, (2013);
DOI: 10.1126/science.1233232



PSR J0348+0432

2.01 ± 0.04 solar mass ($M_{\odot}$)



Selected for a Viewpoint in *Physics*

PRL **119,** 161101 (2017) PHYSICAL REVIEW LETTERS week ending 20 OCTOBER 2017

GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral

B. P. Abbott *et al.**

(LIGO Scientific Collaboration and Virgo Collaboration)

PRL121, 161101 (2018)

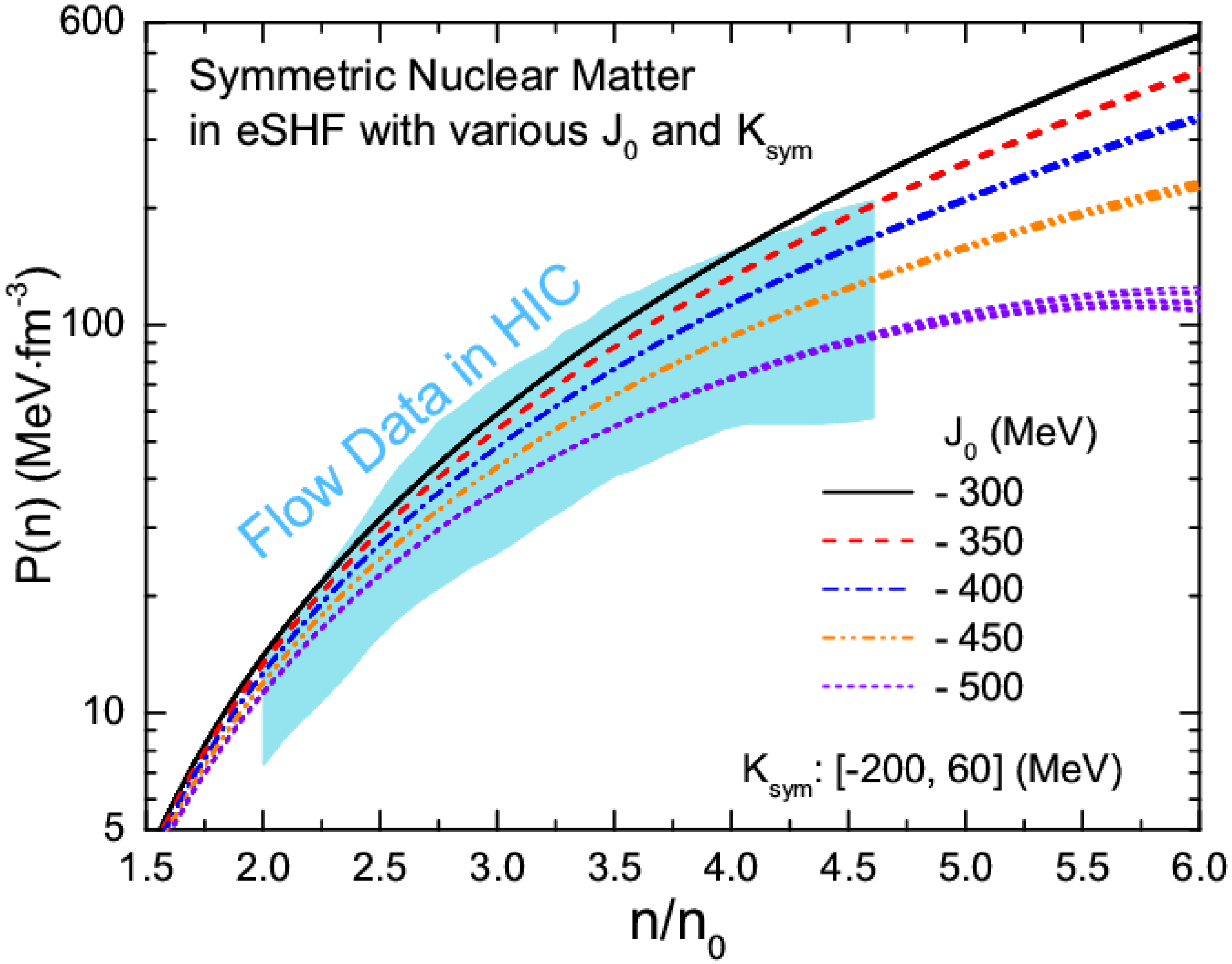
GW170817: Measurements of neutron star radii and equation of state

The LIGO Scientific Collaboration and The Virgo Collaboration
(compiled 30 May 2018)

GW170817 (LIGO/Virgo):
$\Lambda_{1.4} < 580$

J0: Flow data in HIC's

Y. Zhou/LWC/Z. Zhang, to be submitted



For various J0 and Ksym:[-200, 60] MeV

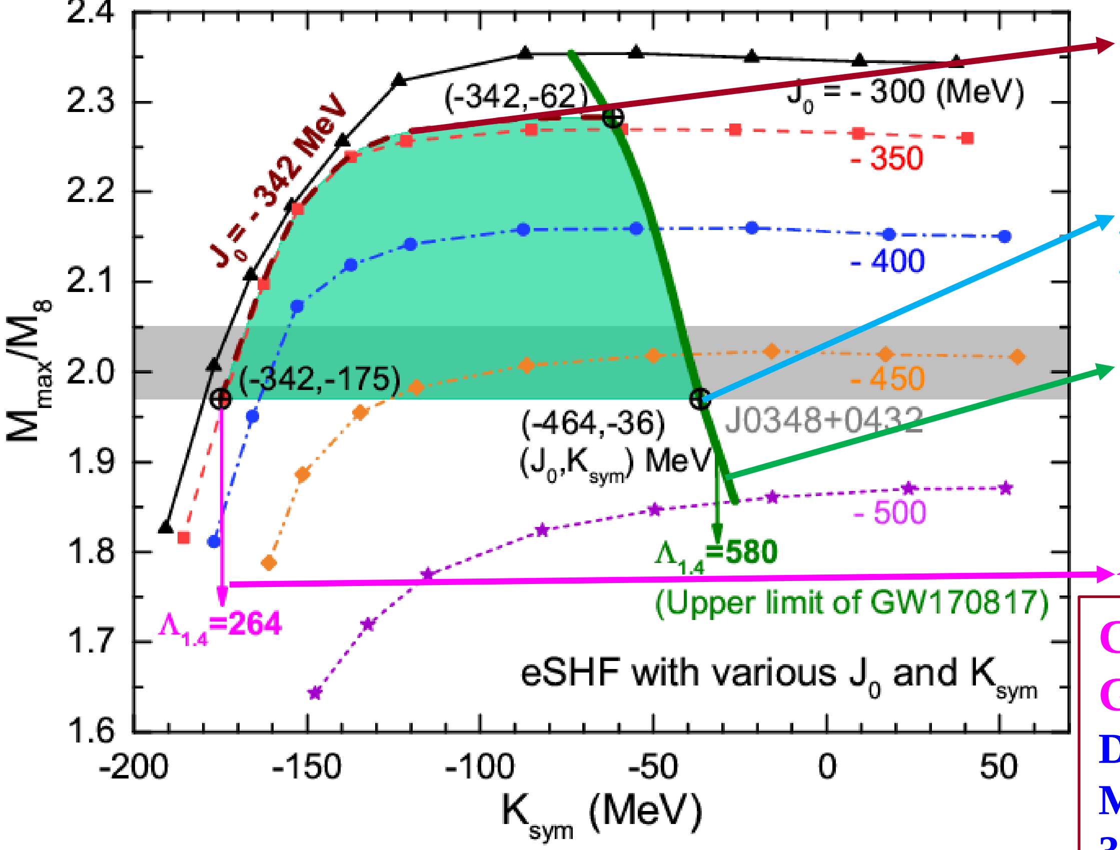
$E_{sym}(\rho_c)$= 26.65 MeV
$L(\rho_c)$= 47.3 MeV

Pressure of SNM is very sensitive to J0 but essentially independent of Ksym

-550 MeV ~ < J_0 < -342 MeV: Flow Data in HIC's

J0 and Ksym: Flow data, NStar Mass, Λ

Y. Zhou/LWC/Z. Zhang, to be submitted



J0 = -342 MeV

J0 = -464 MeV
Ksym < - 36 MeV

$\Lambda_{1.4}$ < 580

GW170817 (LIGO/Virgo)

$\Lambda_{1.4}$ > 264

Consistent with EM counterpart of GW170817, see, e.g.,
D. Radice et al., ApJL852, L29(2018);
M.W. Coughlin et al., MNRAS 480, 3871(2018)

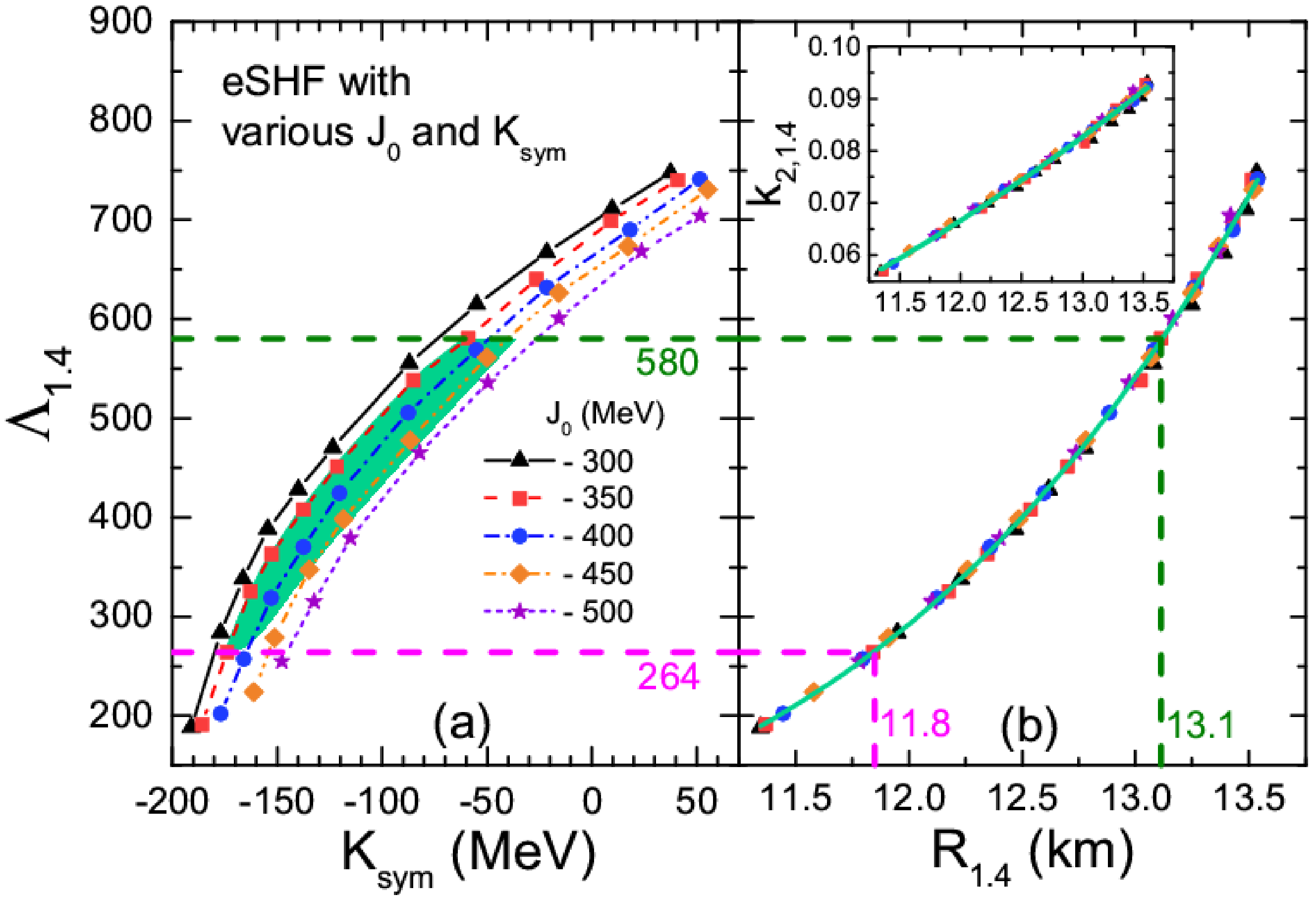
K_{sym} affects strongly M_{max}
for K_{sym} < - 100 MeV

Flow Data in HIC +Mmax:
K_{sym} > -175 MeV, $\Lambda_{1.4}$ >264

Flow Data in HIC+Mmax+ $\Lambda_{1.4}$:
-464 MeV < J_0 < -342 MeV:
-175 MeV < K_{sym} < -36 MeV:

R1.4: Flow data, NStar Mass, Λ

Y. Zhou/LWC/Z. Zhang, to be submitted



$R_{1.4}$: 11.8-13.1 km !

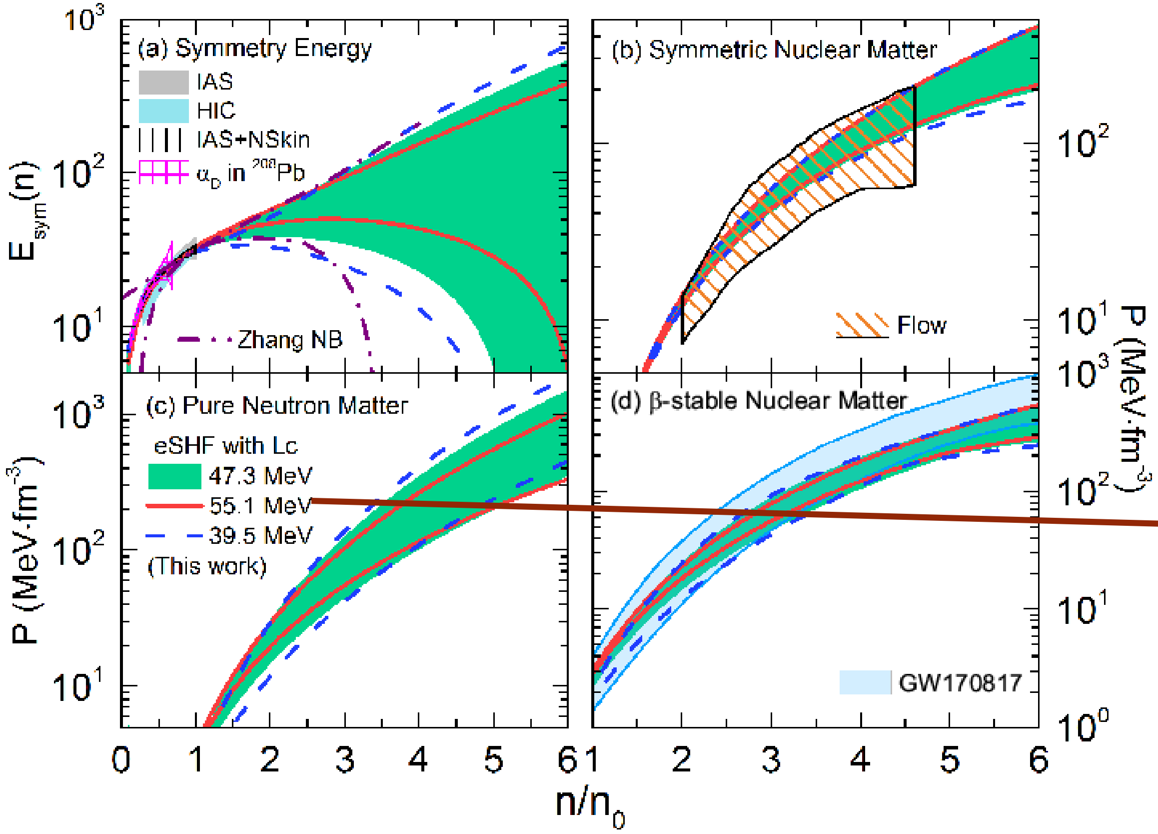
$$\Lambda_{1.4} = (1.24 \pm 0.12) \times 10^{-6} \cdot R_{1.4}^{(7.76 \pm 0.04)}$$

$$k_{2,1.4} = (7.62 \pm 0.53) \times 10^{-5} \cdot R_{1.4}^{(2.73 \pm 0.03)}$$

$$\Lambda = \frac{2}{3} k_2 (R/M)^5$$

EOS: Flow data, NStar Mass, Λ

Y. Zhou/LWC/Z. Zhang, to be submitted



$L(\rho_c)$=47.3+/-7.8 MeV using α_D of ^{208}Pb (Z. Zhang, LWC, PRC90, 064317(2014))

$L(\rho_c)$ indeed affects the extraction of Esym at high densitities but does not change much the Nstar matter EOS!

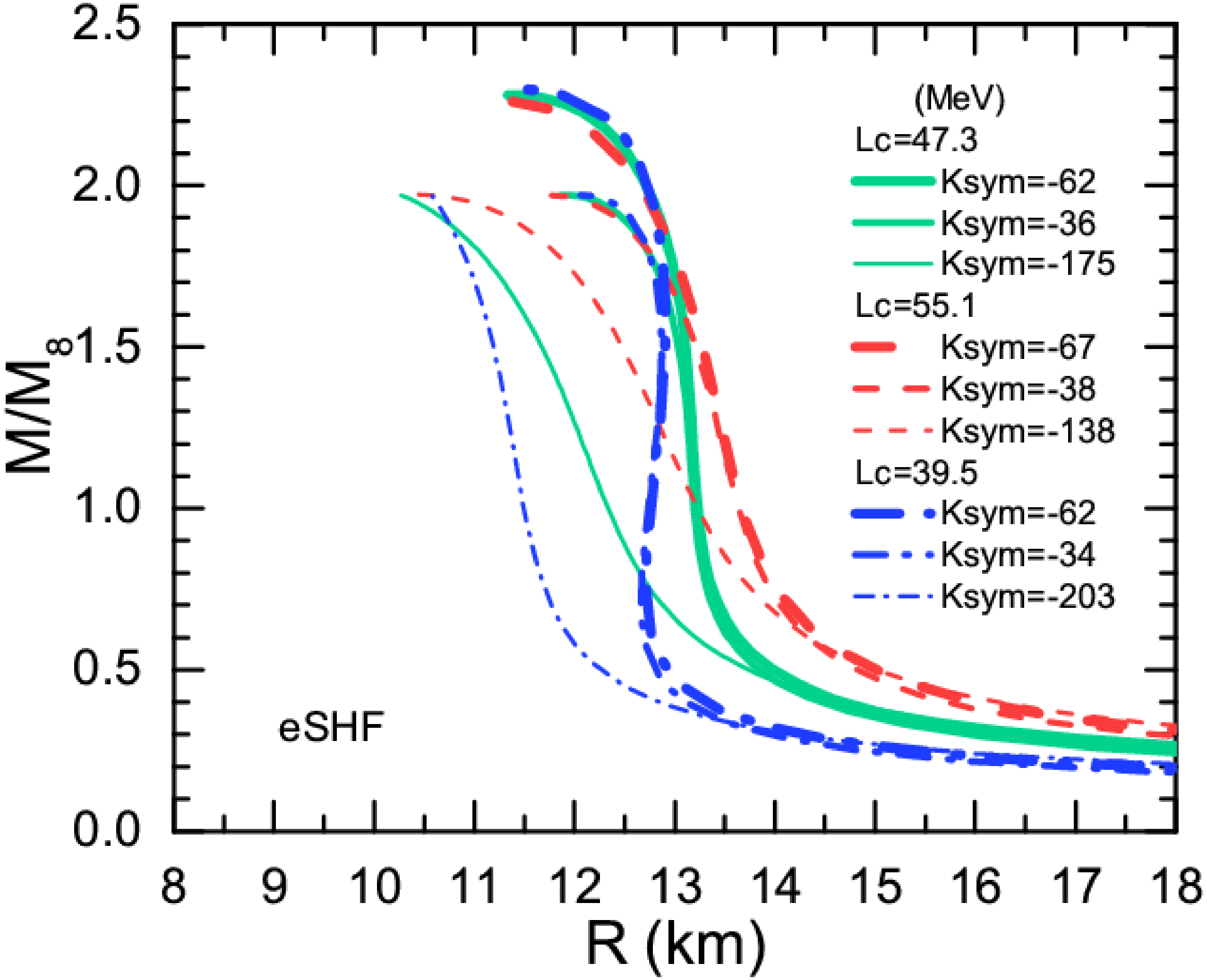
$L(\rho_c)$=47.3 MeV:
J0:[-464,-342] MeV,
K_{sym}:[-175,-36] MeV
$E_{sym}(2\rho_0)$:[39.4, 54.5] MeV

$L(\rho_c)$=39.5 MeV:
J0:[-475,-342] MeV,
K_{sym}:[-203,-34] MeV
$E_{sym}(2\rho_0)$:[33.0, 51.3] MeV

$L(\rho_c)$=55.1 MeV:
J0:[-455,-342] MeV,
K_{sym}:[-138,-38] MeV
$E_{sym}(2\rho_0)$:[46.9, 57.6] MeV

M-R: Flow data, NStar Mass, Λ

Y. Zhou/LWC/Z. Zhang, to be submitted



$L(\rho_c)$=47.3 MeV:
$\Lambda_{1.4}$ > 264
R1.4:[11.8,13.1] km

$L(\rho_c)$=39.5 MeV:
$\Lambda_{1.4}$ > 193
R1.4:[11.1,12.9] km

$L(\rho_c)$=55.1 MeV:
$\Lambda_{1.4}$ > 379
R1.4:[12.6,13.3] km

- ☐ **$\Lambda_{1.4}$ > 193**
- ☐ **R1.4 ≈ 12.2 +/- 1.1 km**

EOS: Flow data, NStar Mass, Λ

- ☐ $J_0 \approx -408.5$ +/- 66.5 MeV and $K_{sym} \approx -118.5$ +/- 84.5 MeV:
  Data of finite nuclei + Flow Data in HIC + Observed NStar Largest Mass + Tidal Deformability of Neutron Star (from recent GW170817 signal) analyzed simultaneously within the same EDF – extended SHF

- ☐ $E_{sym}(2\rho_0) \approx 45.3$ +/- 12.3 MeV: R1.4 ≈ 12.2 +/- 1.1 km

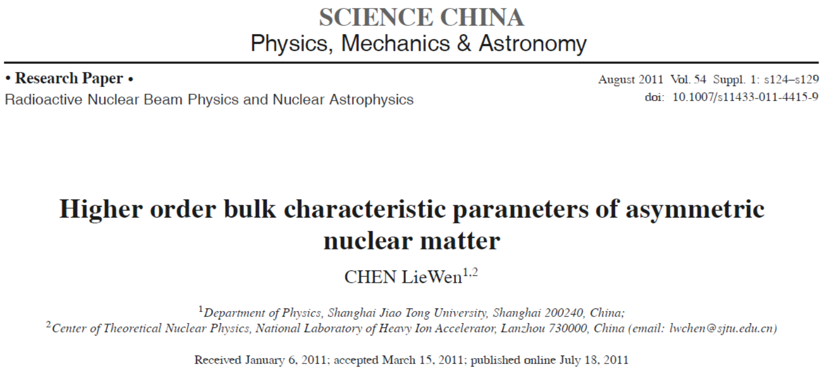
In this workshop:

- ➢ **B.A. Li/N.B. Zhang:** $E_{sym}(2\rho_0) \approx 47.2$ +/- 9.9 MeV
- ➢ **J. Margueron:** $J_0 \approx -300$ +/- 400 MeV and $K_{sym} \approx -100$ +/- 100 MeV
- ➢ **H. Sagawa:** $K_{sym} \approx -100$ +/- 40 MeV
- ➢ **W. Trautmann:** $E_{sym}(2\rho_0) \approx 55$ +/- 5 MeV

……

High-Order Bulk Characteristic Parameters

Estimates from systematic analysis based on Skyrme-Hartree-Fock energy density functionals



SCIENCE CHINA
Physics, Mechanics & Astronomy

• **Research Paper** •
Radioactive Nuclear Beam Physics and Nuclear Astrophysics

August 2011 Vol. 54 Suppl. 1: s124–s129
doi: 10.1007/s11433-011-4415-9

Higher order bulk characteristic parameters of asymmetric nuclear matter

CHEN LieWen[1,2]

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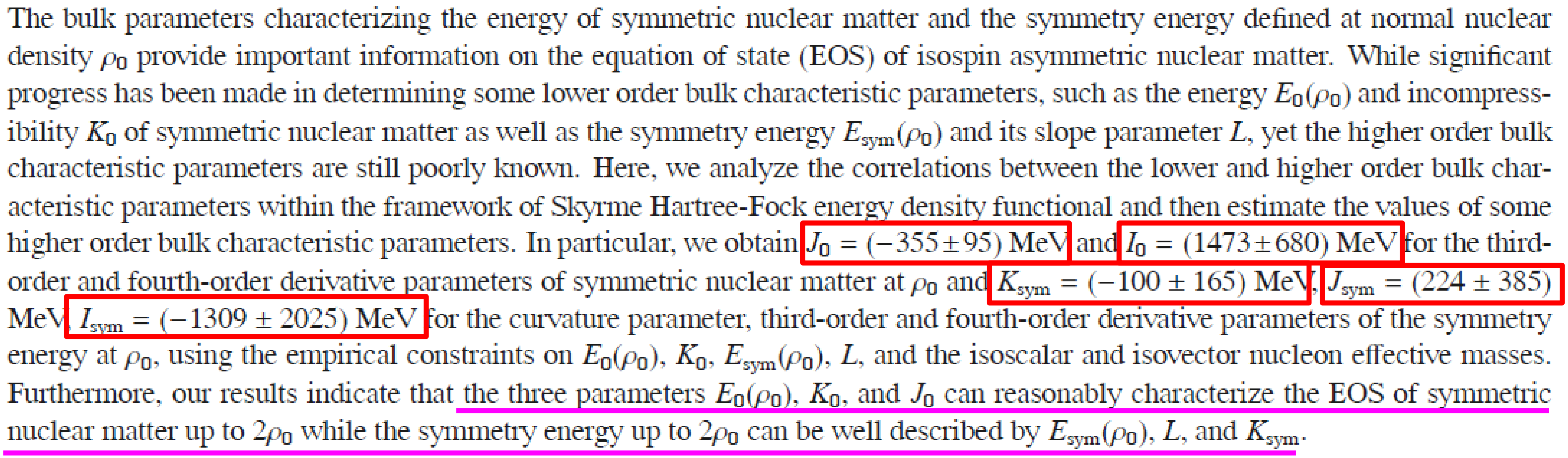
[2] *Center of Theoretical Nuclear Physics, National Laboratory of Heavy Ion Accelerator, Lanzhou 730000, China (email: lwchen@sjtu.edu.cn)*

Received January 6, 2011; accepted March 15, 2011; published online July 18, 2011

LWC, arXiv:1101.2384

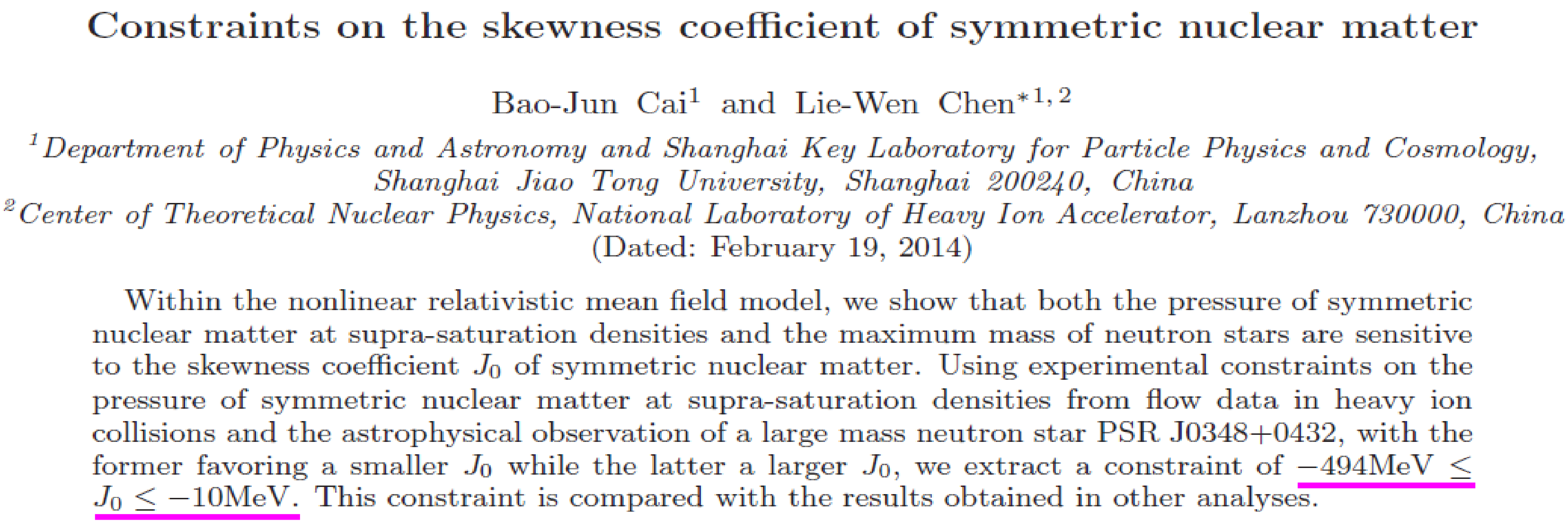


The bulk parameters characterizing the energy of symmetric nuclear matter and the symmetry energy defined at normal nuclear density ρ_0 provide important information on the equation of state (EOS) of isospin asymmetric nuclear matter. While significant progress has been made in determining some lower order bulk characteristic parameters, such as the energy $E_0(\rho_0)$ and incompressibility K_0 of symmetric nuclear matter as well as the symmetry energy $E_{\rm sym}(\rho_0)$ and its slope parameter L, yet the higher order bulk characteristic parameters are still poorly known. Here, we analyze the correlations between the lower and higher order bulk characteristic parameters within the framework of Skyrme Hartree-Fock energy density functional and then estimate the values of some higher order bulk characteristic parameters. In particular, we obtain $J_0 = (-355 \pm 95)$ MeV and $I_0 = (1473 \pm 680)$ MeV for the third-order and fourth-order derivative parameters of symmetric nuclear matter at ρ_0 and $K_{\rm sym} = (-100 \pm 165)$ MeV, $J_{\rm sym} = (224 \pm 385)$ MeV, $I_{\rm sym} = (-1309 \pm 2025)$ MeV for the curvature parameter, third-order and fourth-order derivative parameters of the symmetry energy at ρ_0, using the empirical constraints on $E_0(\rho_0)$, K_0, $E_{\rm sym}(\rho_0)$, L, and the isoscalar and isovector nucleon effective masses. Furthermore, our results indicate that the three parameters $E_0(\rho_0)$, K_0, and J_0 can reasonably characterize the EOS of symmetric nuclear matter up to $2\rho_0$ while the symmetry energy up to $2\rho_0$ can be well described by $E_{\rm sym}(\rho_0)$, L, and $K_{\rm sym}$.

High-Order Bulk Characteristic Parameters

J0 from NL-RMF models

Cai/LWC, arXiv:1402.4242
Nucl. Sci. Tech., 2017



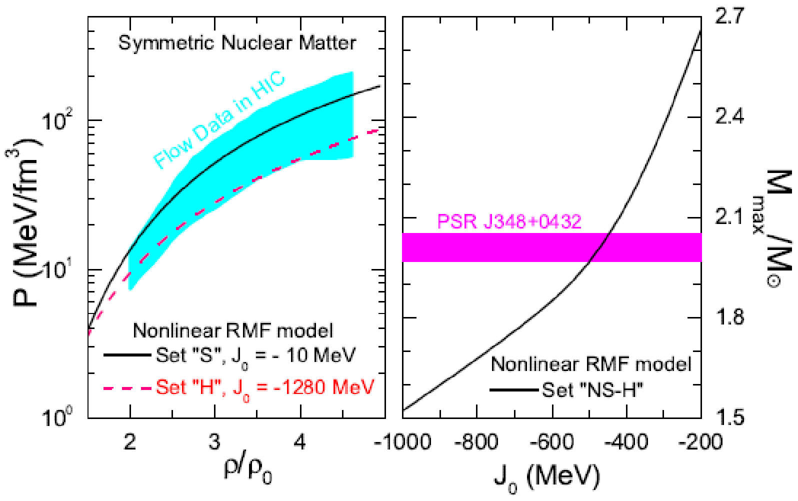
Constraints on the skewness coefficient of symmetric nuclear matter

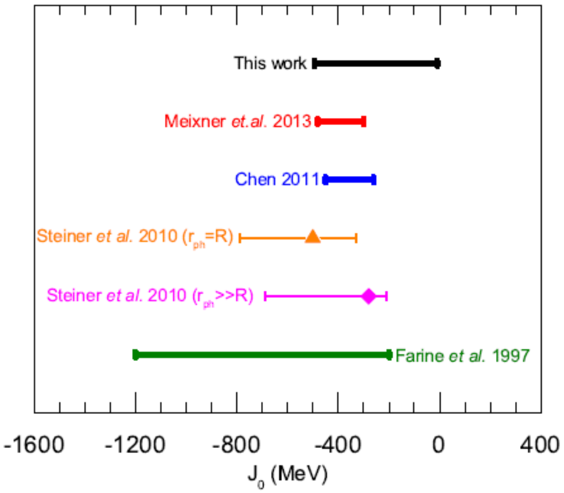
Bao-Jun Cai[1] and Lie-Wen Chen*[1, 2]

[1] *Department of Physics and Astronomy and Shanghai Key Laboratory for Particle Physics and Cosmology, Shanghai Jiao Tong University, Shanghai 200240, China*
[2] *Center of Theoretical Nuclear Physics, National Laboratory of Heavy Ion Accelerator, Lanzhou 730000, China*

(Dated: February 19, 2014)

Within the nonlinear relativistic mean field model, we show that both the pressure of symmetric nuclear matter at supra-saturation densities and the maximum mass of neutron stars are sensitive to the skewness coefficient J_0 of symmetric nuclear matter. Using experimental constraints on the pressure of symmetric nuclear matter at supra-saturation densities from flow data in heavy ion collisions and the astrophysical observation of a large mass neutron star PSR J0348+0432, with the former favoring a smaller J_0 while the latter a larger J_0, we extract a constraint of $-494\mathrm{MeV} \leq J_0 \leq -10\mathrm{MeV}$. This constraint is compared with the results obtained in other analyses.





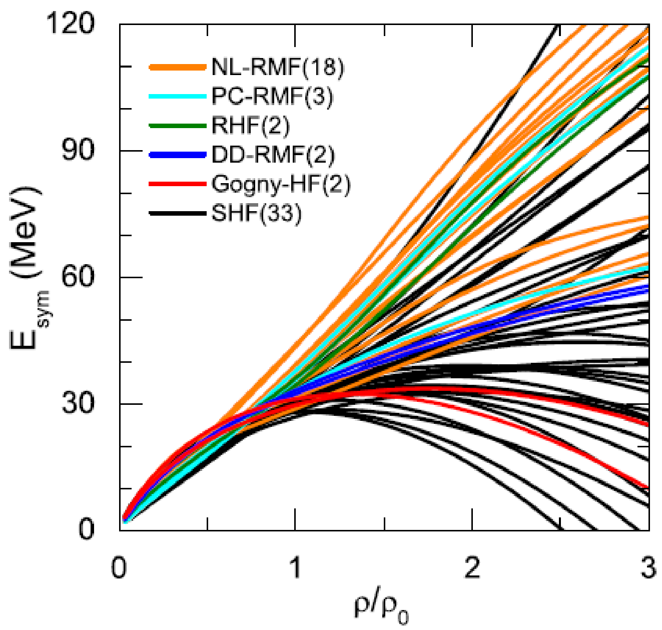
High-Order Bulk Characteristic Parameters

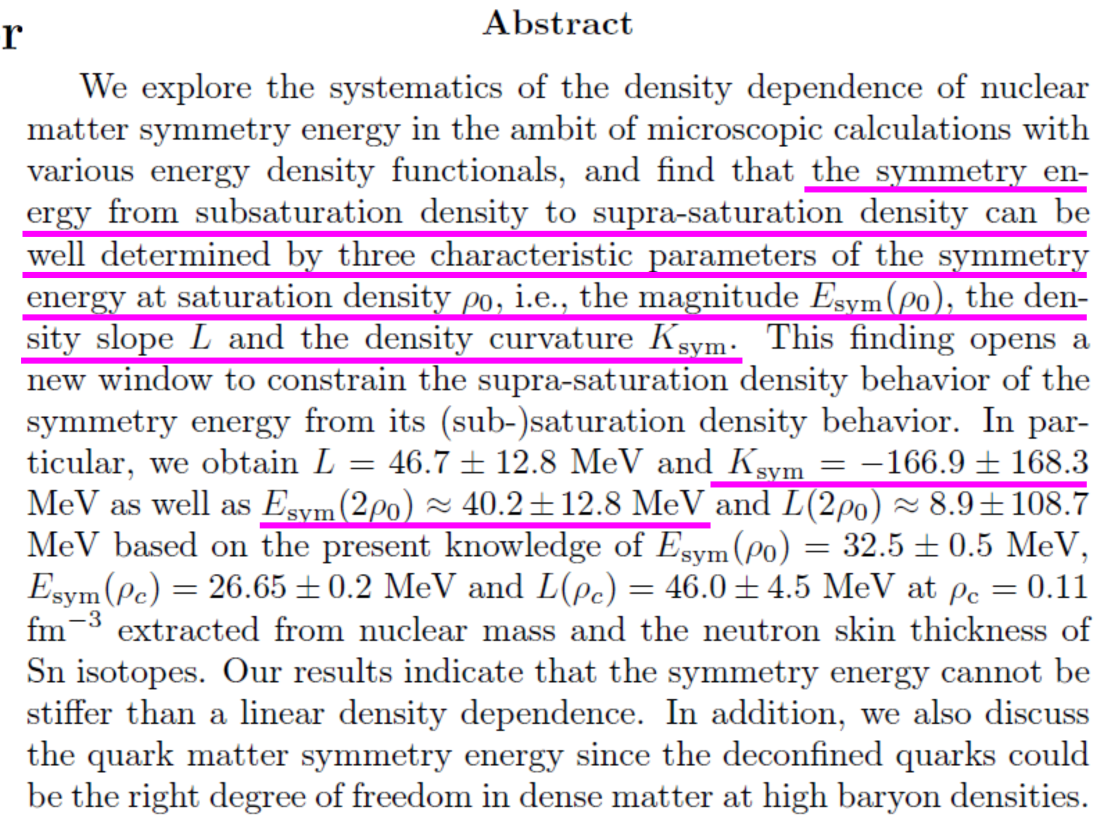
Ksym from Esym systematics

EPJ Web of Conferences **88**, 00017 (2015)
DOI: 10.1051/epjconf/20158800017

Symmetry energy systematics and its high density behavior

LIE-WEN CHEN*





Abstract

We explore the systematics of the density dependence of nuclear matter symmetry energy in the ambit of microscopic calculations with various energy density functionals, and find that the symmetry energy from subsaturation density to supra-saturation density can be well determined by three characteristic parameters of the symmetry energy at saturation density ρ_0, i.e., the magnitude $E_{\rm sym}(\rho_0)$, the density slope L and the density curvature $K_{\rm sym}$. This finding opens a new window to constrain the supra-saturation density behavior of the symmetry energy from its (sub-)saturation density behavior. In particular, we obtain $L = 46.7 \pm 12.8$ MeV and $K_{\rm sym} = -166.9 \pm 168.3$ MeV as well as $E_{\rm sym}(2\rho_0) \approx 40.2 \pm 12.8$ MeV and $L(2\rho_0) \approx 8.9 \pm 108.7$ MeV based on the present knowledge of $E_{\rm sym}(\rho_0) = 32.5 \pm 0.5$ MeV, $E_{\rm sym}(\rho_c) = 26.65 \pm 0.2$ MeV and $L(\rho_c) = 46.0 \pm 4.5$ MeV at $\rho_c = 0.11$ fm^{-3} extracted from nuclear mass and the neutron skin thickness of Sn isotopes. Our results indicate that the symmetry energy cannot be stiffer than a linear density dependence. In addition, we also discuss the quark matter symmetry energy since the deconfined quarks could be the right degree of freedom in dense matter at high baryon densities.

Outline

- **Nuclear matter EOS and the symmetry energy (Esym)**
- **Dense nuclear matter from Nuclear experiments +
Observed neutron star largest mass +
Tidal deformability from GW170817**
- **Summary and outlook**

Summary and outlook

- A lower limit of $\Lambda_{1.4}$ >193 for the tidal deformability of 1.4 solar mass neutron star can be obtained by using the flow data in HIC and the observed largest mass of neutron stars
- The skewness coefficient J0 of symmetric nuclear matter can be constrained to be J0 = -408.5 +/- 66.5 MeV by using the flow data in HIC, the observed largest mass of neutron stars, and the tidal deformability from GW170817
- The density curvature parameter Ksym of the symmetry energy can be constrained to be Ksym = -118.5 +/- 84.5 MeV, $E_{sym}(2\rho_0) \approx$ 45.3 +/- 12.3 MeV, R1.4 ≈ 12.2 +/- 1.1 km, by using the flow data in HIC, the observed largest mass of neutron stars, and the tidal deformability from GW170817, ruling out too stiff and too soft high density behaviors of symmetry energy.
- More data from experiments and observations are epected to improve the constraints on the EOS. Heavier Nstar will lead more stringent constraints

Acknowledgements

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Jun Xu, Rui Wang (SINAP, CAS, Shanghai)
Gao-Chan Yong (IMP, Lanzhou)
Zhen Zhang (Sen-Yat-Sen U, Zhuhai)
Xin Wang, Ying Zhou, Jie Pu, Si-Pei Wang, Xu-Run Huang (SJTU, Shanghai)

Funding:

National Natural Science Foundation of China
Major State Basic Research Development Program (973 Program) in China
Shanghai Rising-Stars Program
Shanghai “Shu-Guang” Project
Shanghai “Eastern Scholar”
Science and Technology Commission of Shanghai Municipality

谢 谢！
Thanks！