Recent work on universal relations for neutron stars

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A brief history of (selected) universal relations

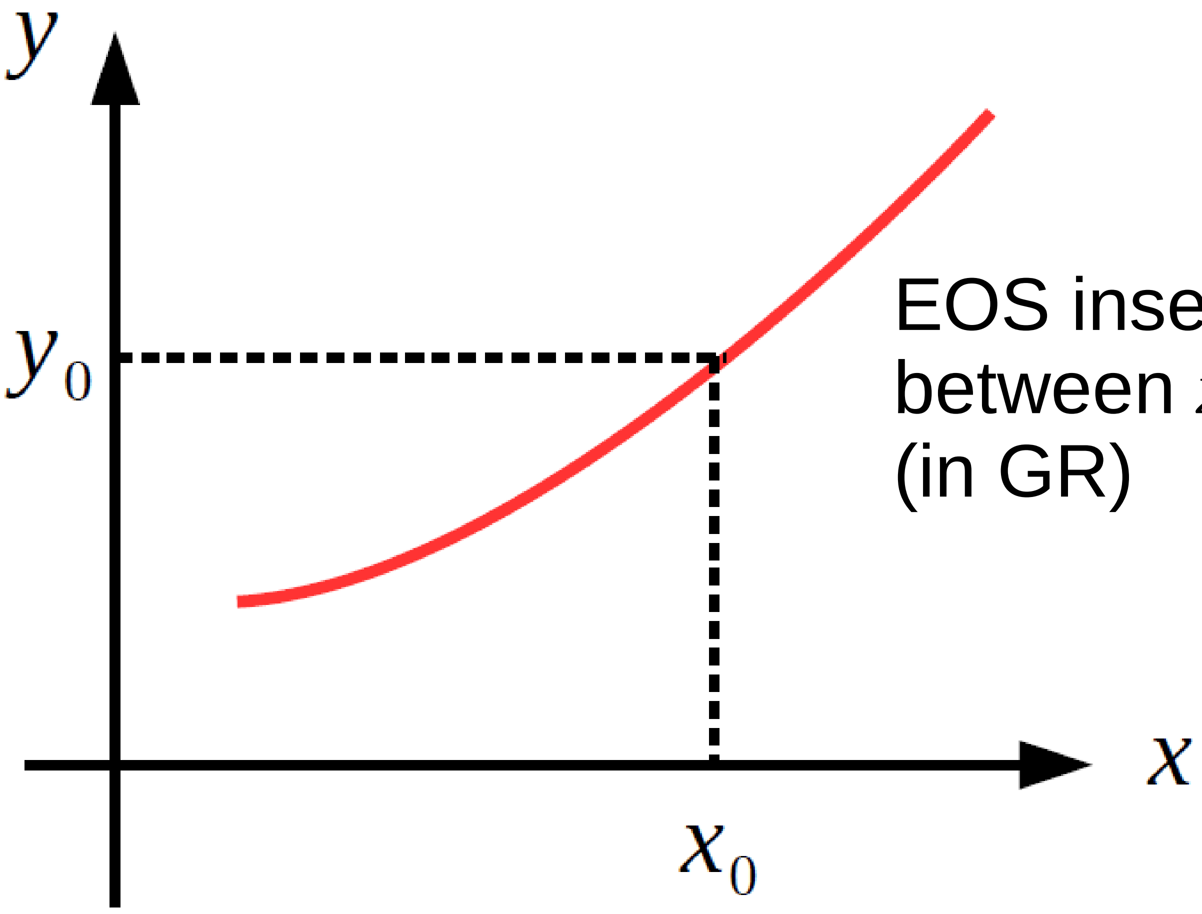
- It is well known that many observables of neutron stars depend sensitively on EOS (Good for constraining EOS)

- There also exist various approximately EOS-insensitive relations connecting different quantities of neutron stars. They are called universal relations
(Definitions? EOS-insensitive to ~O(1%) level?)

- Potential applications?

* If one of the quantities can be measured, the other one can be inferred from the relation

* If both quantities can be measured together, then we can test for GR…..or may be some exotic microphysics?

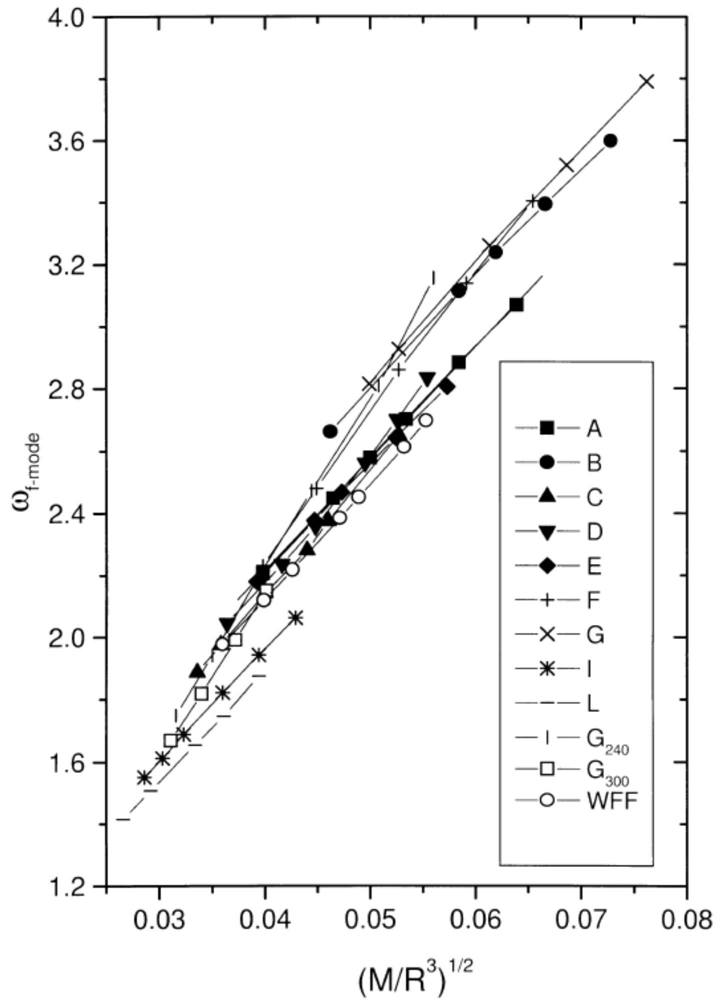


EOS insensitive relation between x and y (in GR)

- f-mode universal relations

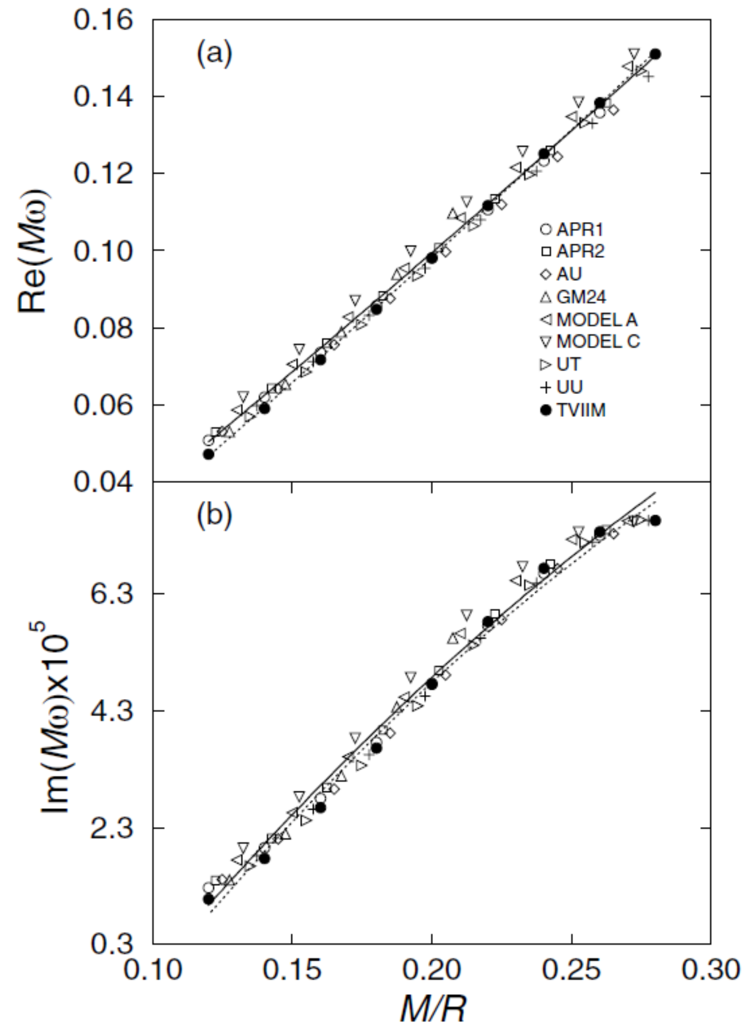
Andersson & Kokkotas,
MNRAS, 299, 1059 (1998)

$$f\,(\mathrm{kHz}) \approx 0.78 + 1.635 \left(\frac{M_{1.4}}{R_{10}^{3}} \right)^{1/2}$$



Tsui & Leung,
MNRAS, 357, 1029 (2005)

$$M\,\omega = a\left(\frac{M}{R}\right)^{2} + b\left(\frac{M}{R}\right) + c$$

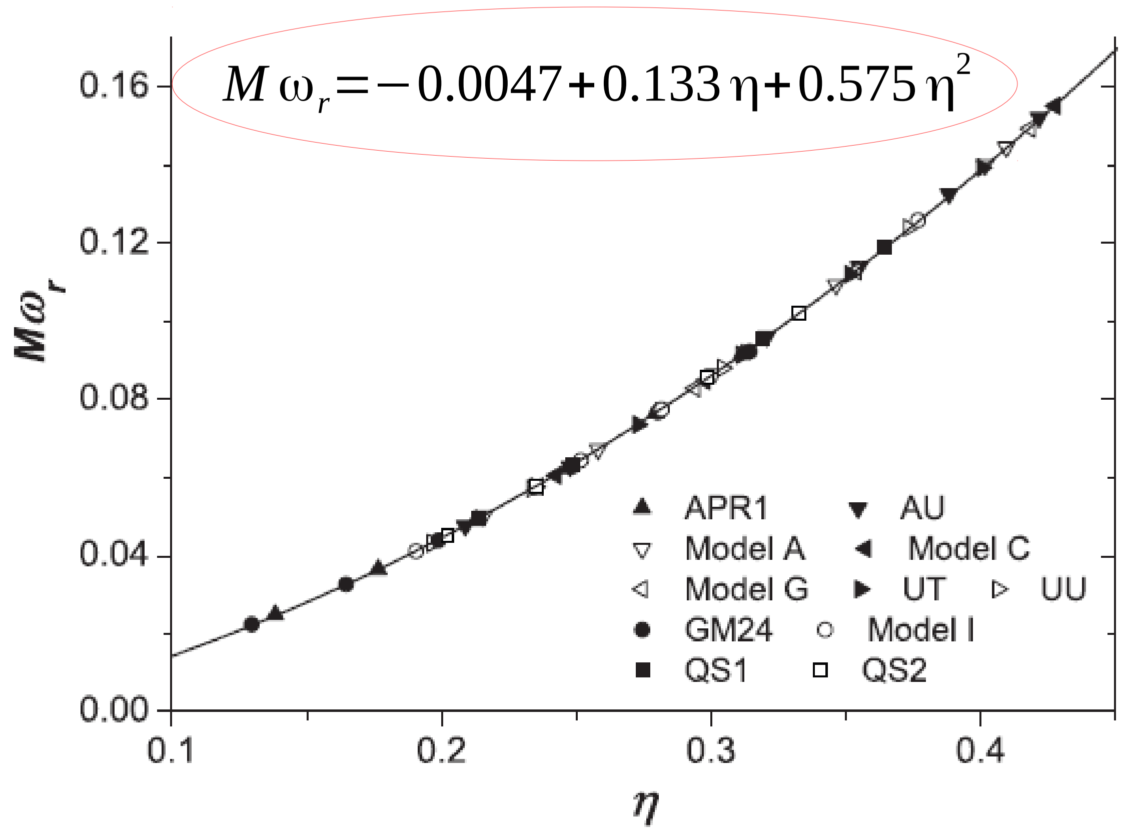


- Improved f-mode universal relations

* Motivated by

(1) Empirical relations between NS's moment of inertia (I) and compactness (M/R)
[Bejger & Haensel (2002); Lattimer & Schutz (2005)]

(2) I carries richer information about the mass distribution



Effective compactness

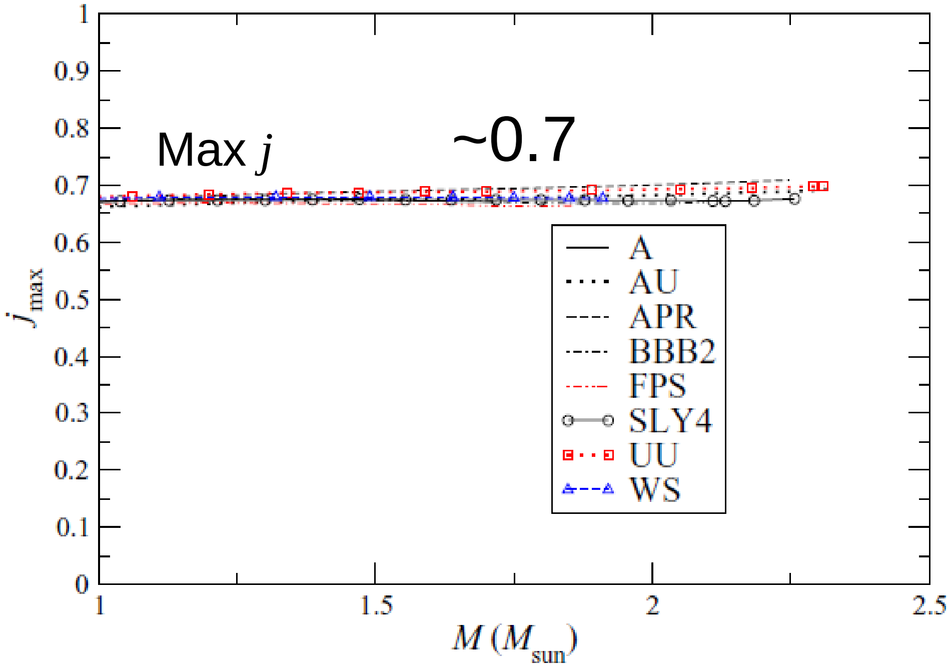
$$\eta \equiv \sqrt{\frac{M^3}{I}}$$

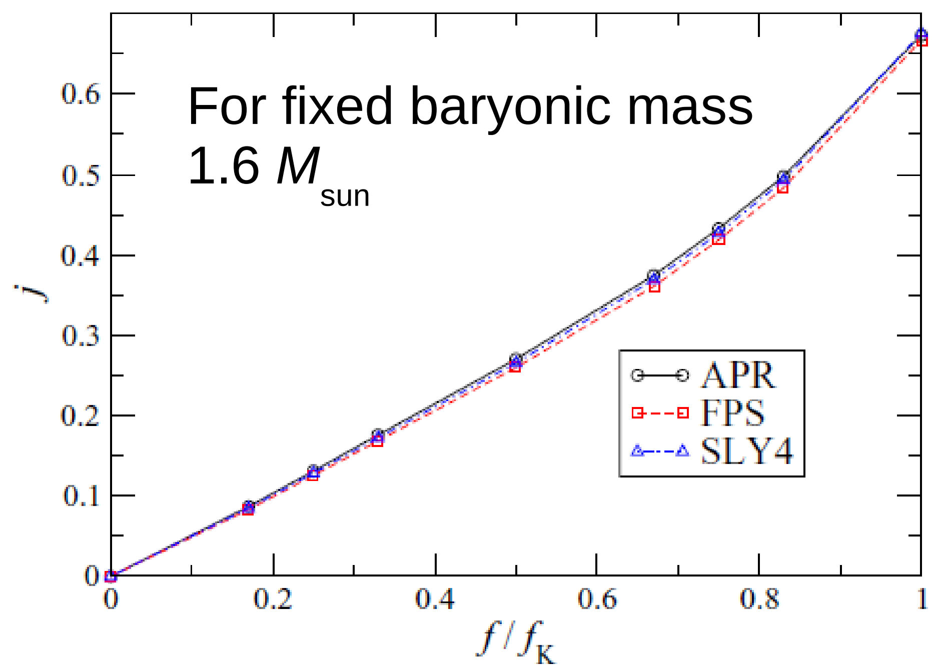
Lau, Leung, LML (2010)

* M, R, and I can be inferred from f-mode observations

- "Universal" relation for the spin parameter of rotating neutron stars?

Dimensionless spin parameter: $j \equiv \frac{J}{M^2}$ (G=c=1)





Lo & LML (2011)

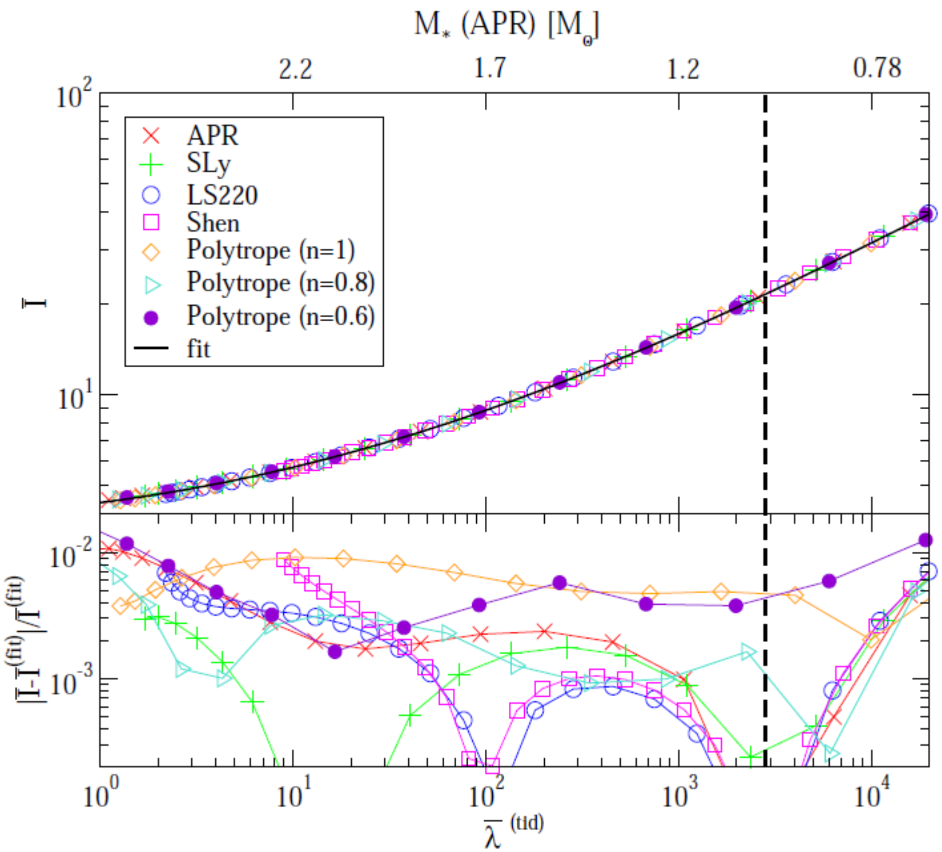
* Quark stars (MIT bag) and a small number of recent nuclear matter EOS can have higher j_{max}

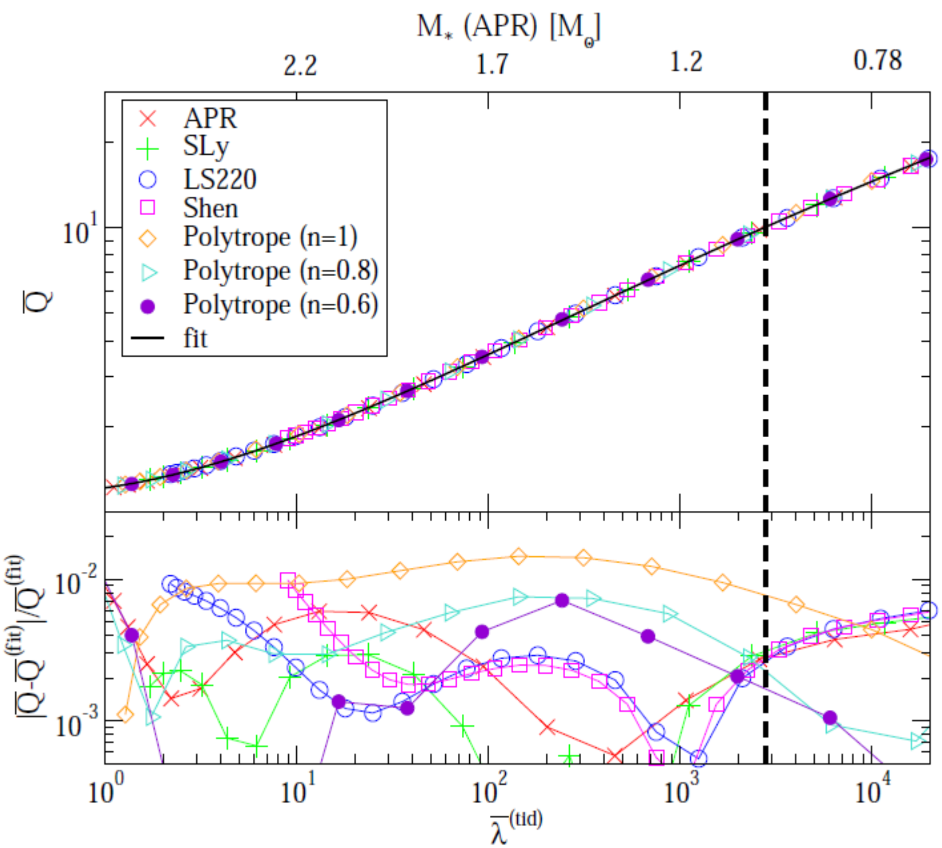
- I-Love-Q universal relations

$$\bar{I} \equiv \frac{I}{M^3} \quad , \quad \bar{Q} \equiv -\frac{Q}{M^3 j^2} \quad , \quad \bar{\lambda} \equiv \frac{\lambda}{M^5}$$

- Q = rotation-induced quadrupole moment
- λ = tidal deformability
- j = spin parameter

Yagi & Yunes (2013)





Our recent work I:
Compact stars with crystalline quark matter and broken I-Love relation

(with Shu-Yan Lau and Pui-Tang Leung)

Compact stars with crystalline quark matter

* Crystalline color superconducting phase is suggested to be a possible phase of dense, but not asymptotically dense quark matter

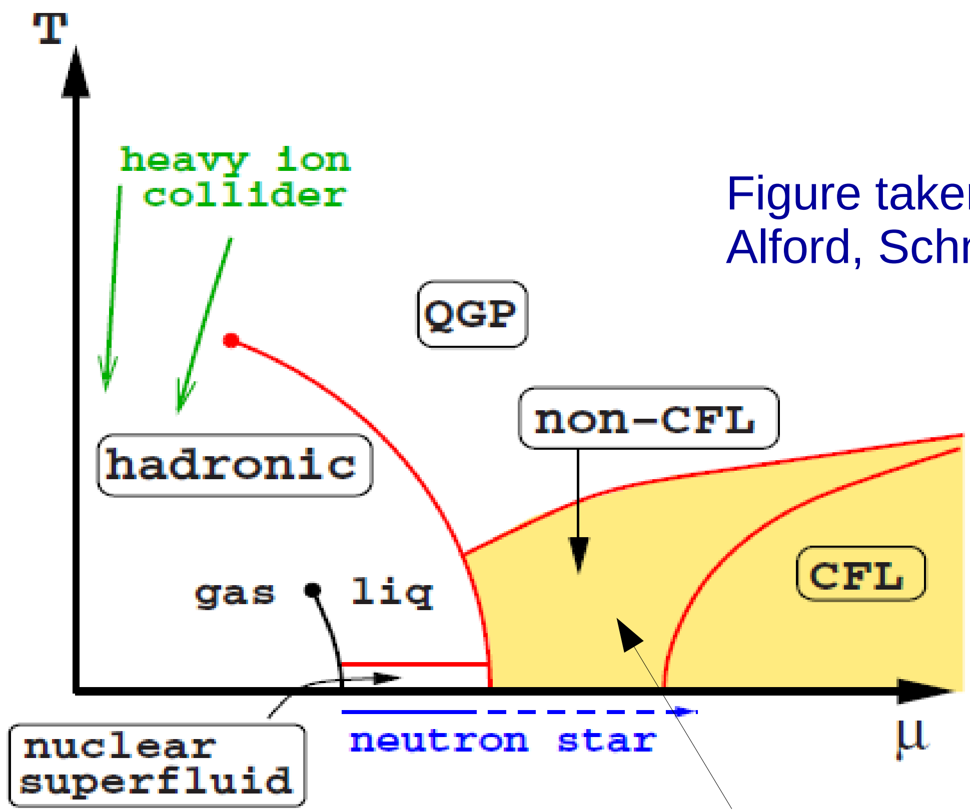


Figure taken from
Alford, Schmitt, Rajagopal, Schäfer (2008)

Crystalline phase?

* Crystalline color superconducting (CCS) quark matter is extremely rigid

* The shear modulus of CCS quark matter can be up to 1000 times larger than neutron star crust

$$\mu = 2.47\ \mathrm{MeV/fm^3} \left(\frac{\Delta}{10\ \mathrm{MeV}} \right)^2 \left(\frac{\mu_q}{400\ \mathrm{MeV}} \right)^2$$

Mannarelli, Rajagopal, Sharma
PRD, 76, 074026 (2007)

Δ = Gap parameter (~5-25 MeV)
μ_q = quark chemical potential

Tidal deformation of compact stars with CCS quark matter?

* We have formulated and studied the tidal deformability of compact stars with CCS quark matter by considering the effect of elasticity:

$$\delta G_{\alpha\beta} = 8\pi \delta T_{\alpha\beta} \quad ; \quad \delta(\nabla_\alpha T^{\alpha\beta}) = 0$$

$$T_{\alpha\beta} = T^{\text{bulk}}_{\alpha\beta} + T^{\text{shear}}_{\alpha\beta}$$

Shear modulus
(Elasticity enters at the perturbation level)

$$\delta T^{\text{shear}}_{\alpha\beta} = -2\mu \delta \Sigma_{\alpha\beta}$$

Lau, Leung, LML (2017, 2018)

Remarks:

* Tidal deformability of neutron stars with solid crust has been studied before [Penner, Andersson, Samuelsson, Hawke, Jones (2011)]

* Our formulation is somewhat different in the sense that our resulting equations are closer to the corresponding Newtonian equations

* Comparing to pure fluid stars, the resulting equations and boundary conditions for solid quark stars (and hybrid stars with a solid core) are more complicated

We consider bare solid quark star and hybrid star models:

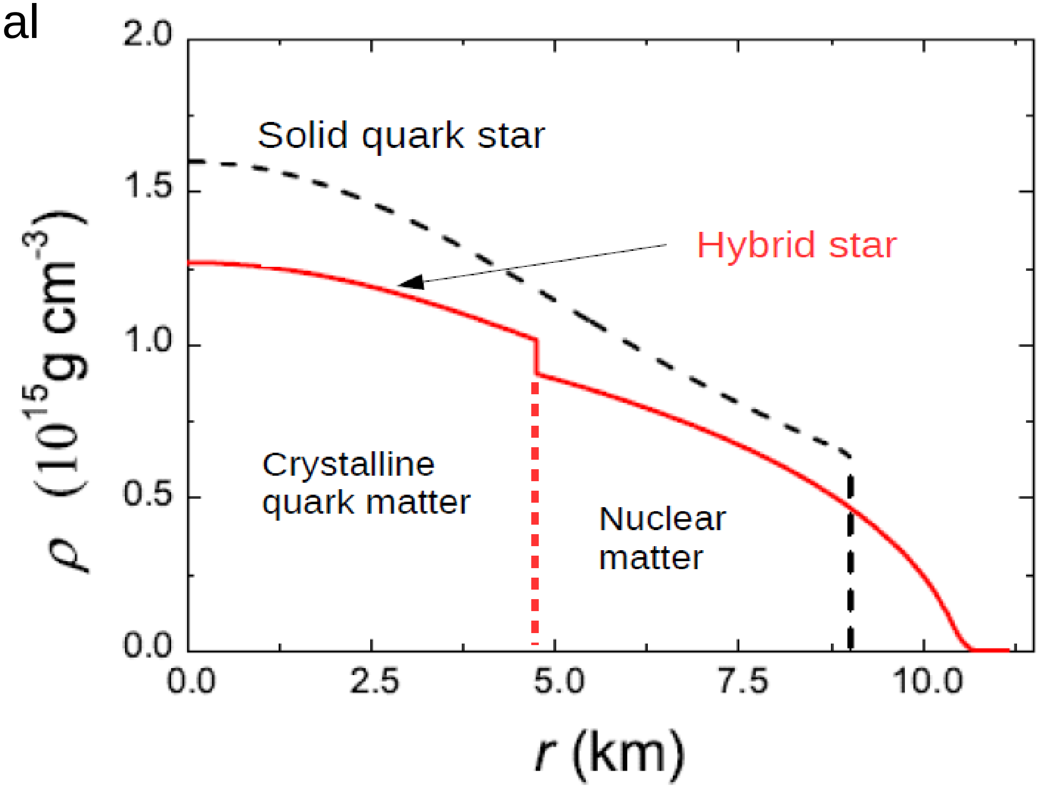
Quark matter is described by the phenomenological model

$$\Omega_{QM} = -\frac{3}{4\pi^2} a_4 \mu_q^4 + \frac{3}{4\pi^2} a_2 \mu_q^2 + B_{eff}$$

Alford, Braby, Paris, Reddy (2005)

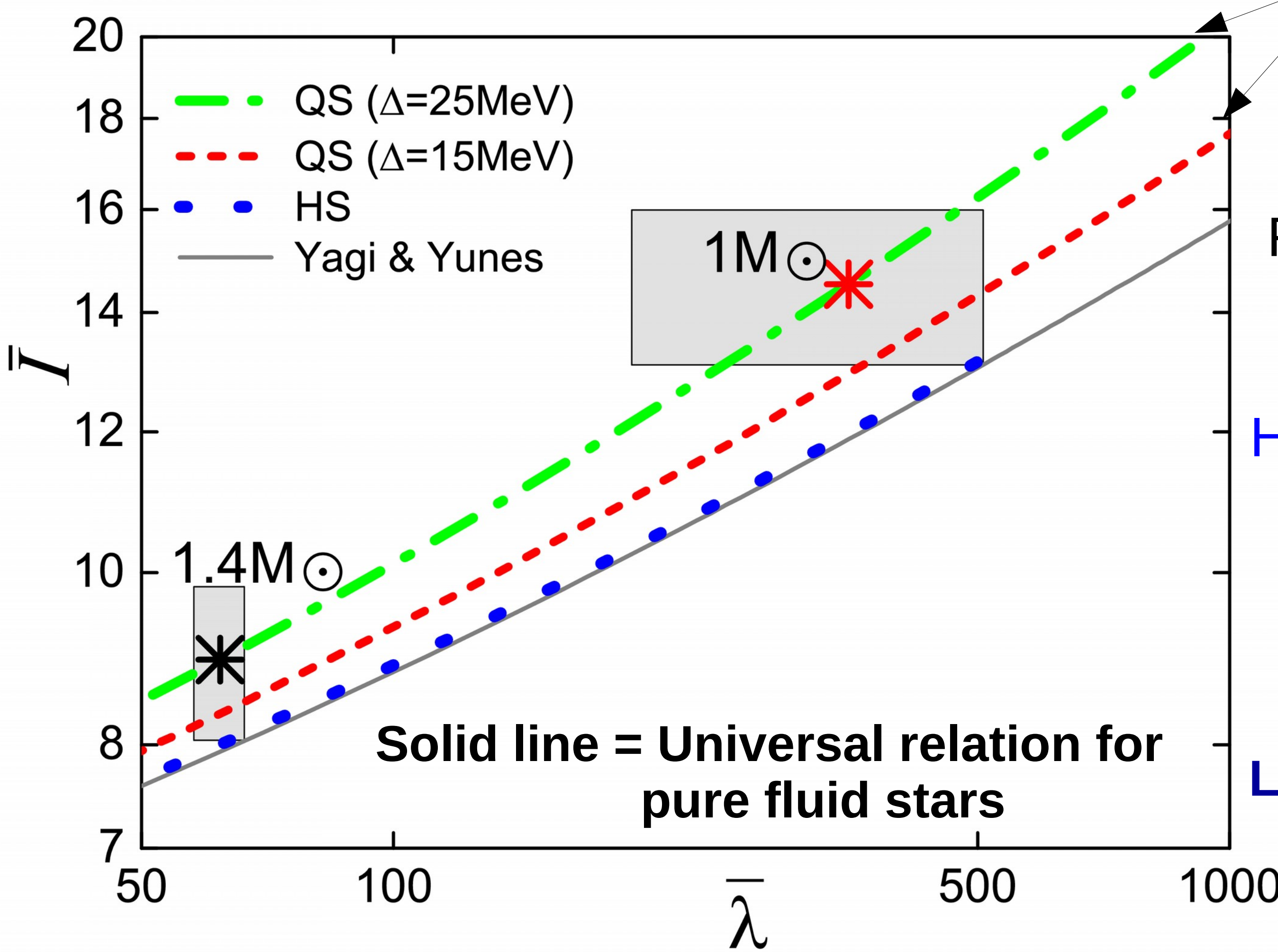
Quark chemical potential

For hybrid stars:
* Nuclear matter part is described by APR EOS
* Phase transition is implemented using Maxwell construction



- Broken I-Love relation

QS = Bare solid quark stars



Recall: shear modulus increases with Δ

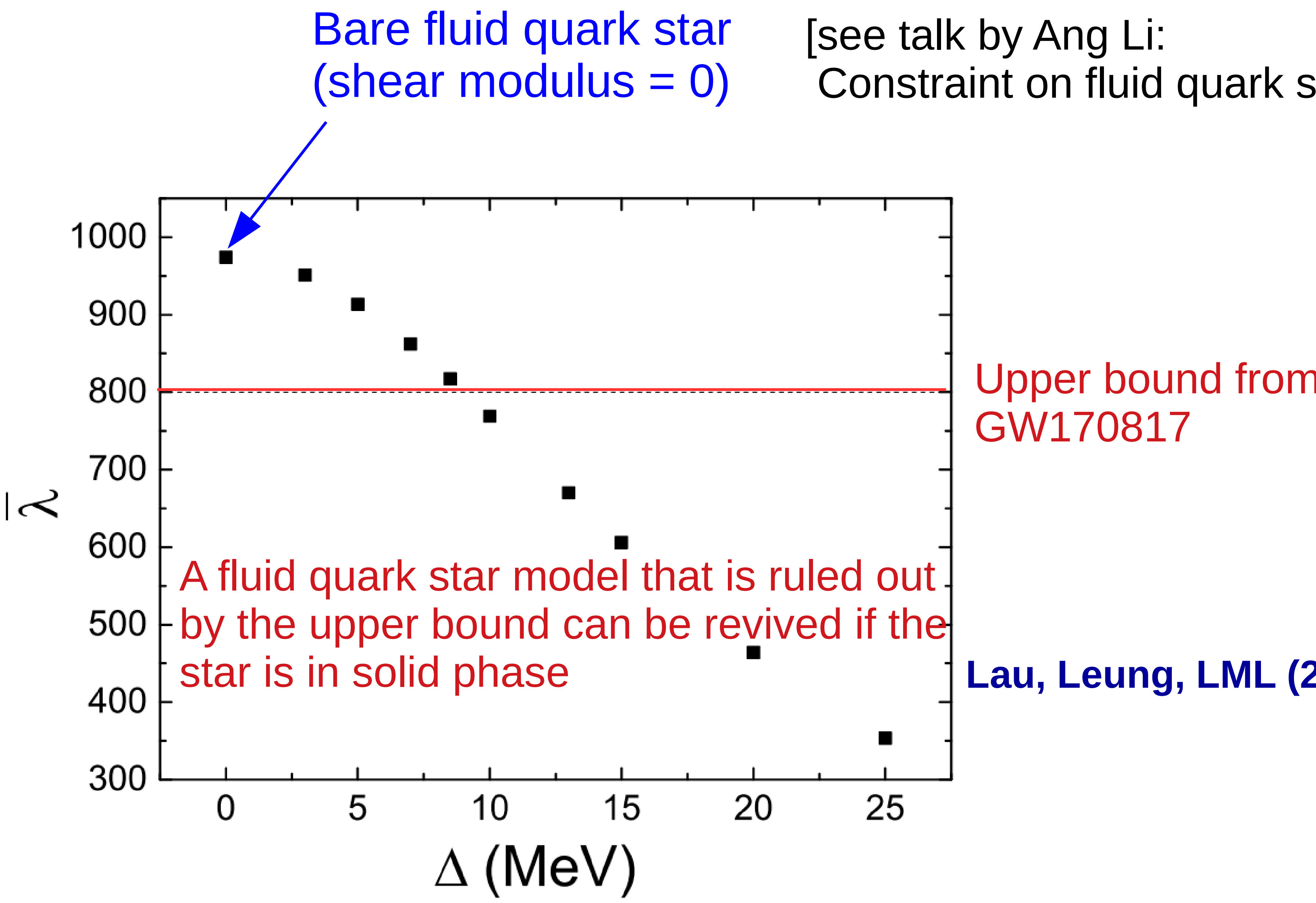
HS = Hybrid stars with a solid core

Lau, Leung, LML (2017)

[see also talk by Sofia Han:
Broken I-Love relation for fluid stars with sequential QCD phase transitions]

- Implications from GW170817

[see talk by Ang Li: Constraint on fluid quark stars]

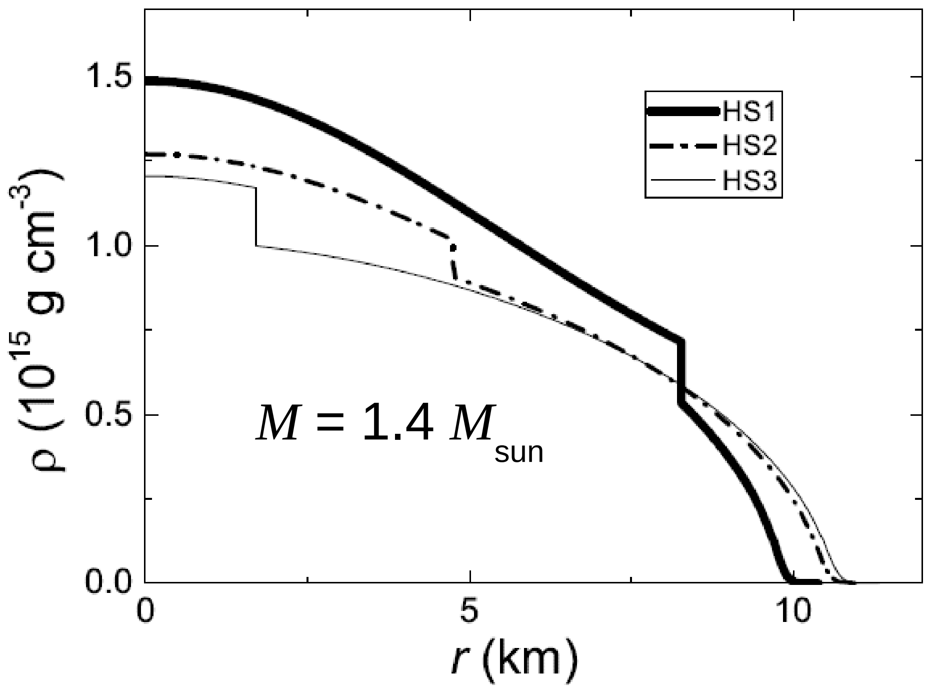


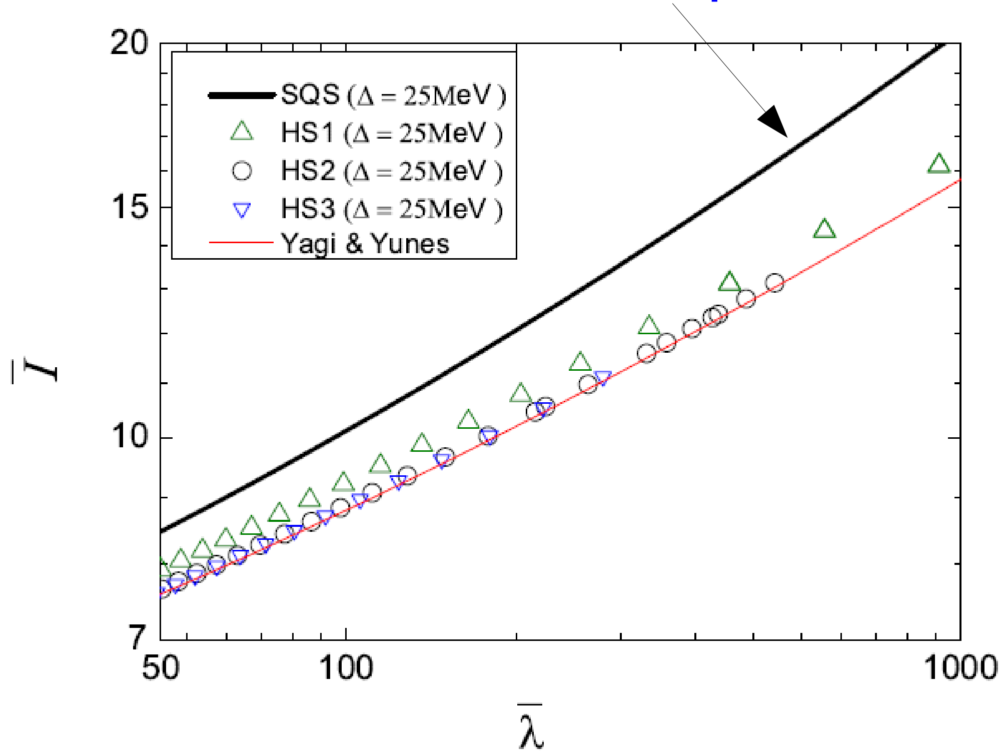
Lau, Leung, LML (2018)

- Hybrid stars

* Solid core has little effect (~1%) on the tidal deformability if the core radius $R_{\text{core}} < 0.7\ R$

Bare solid quark stars





* No deviation from the I-Love relation does not necessarily rule out CCS quark matter

* If a hybrid star model is ruled out by the GW170817 upper bound, then the conclusion will still hold even if the core is in a solid phase (unless $R_{\text{core}} \sim R$)

Our recent work II:
New universal relations for rapidly rotating neutron stars

(with Shun-Sun Luk)

Innermost stable circular orbits (ISCO) around rotating neutron stars

- ISCO is an important prediction of GR concerning the strong field spacetime around a compact object

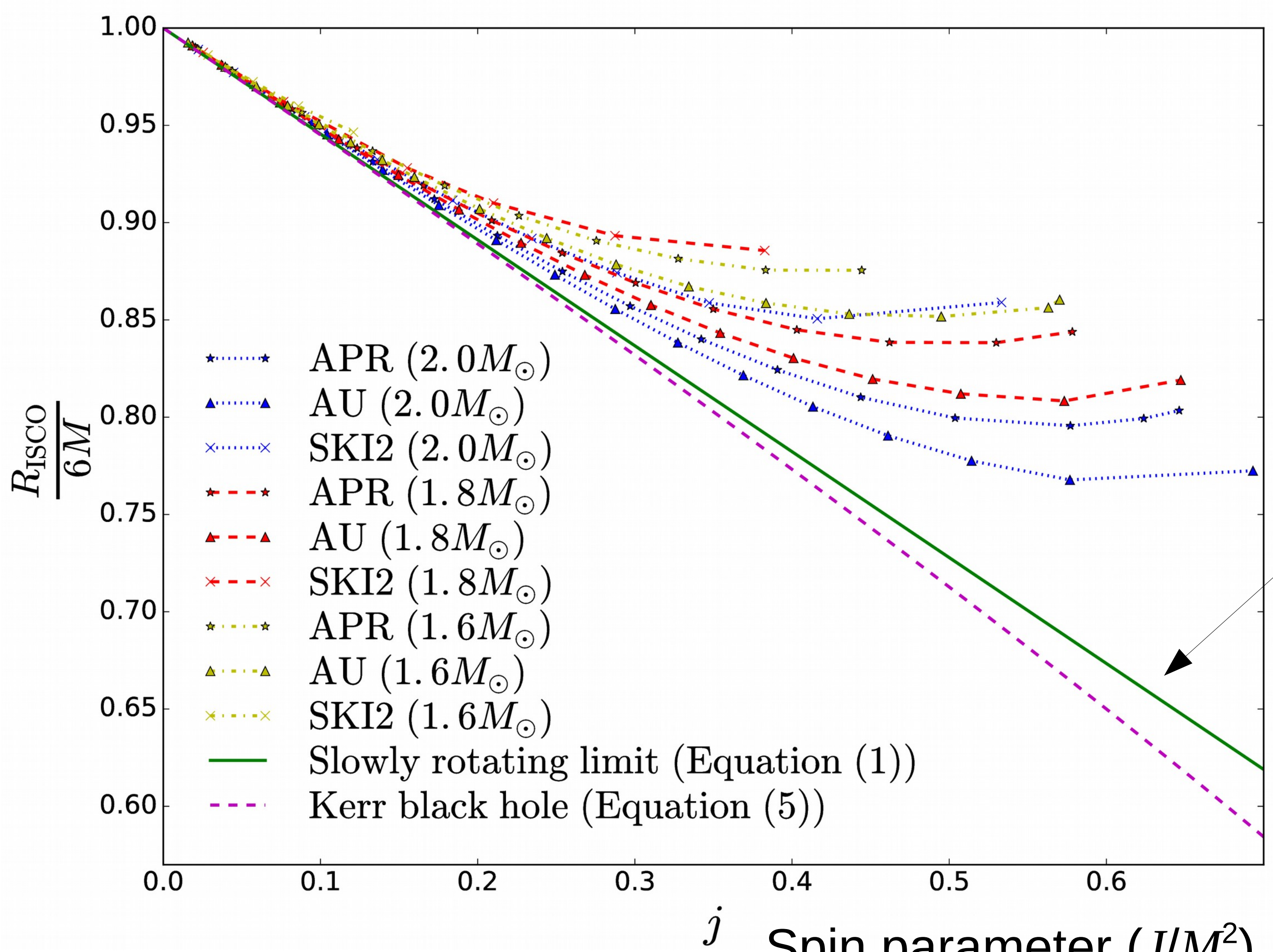
- ISCO may be closely related to kHz quasi-periodic oscillations observed from low-mass X-ray binaries

- ISCO around a Kerr black hole can be determined analytically [Bardeen, Press, Teukolsky (1972)]

- ISCO around rapidly rotating neutron stars need to be obtained numerically (Open source codes: LORENE/rotstar, RNS ...)

- EOS dependence of ISCO around rapidly rotating neutron stars

Circumferential ISCO radius



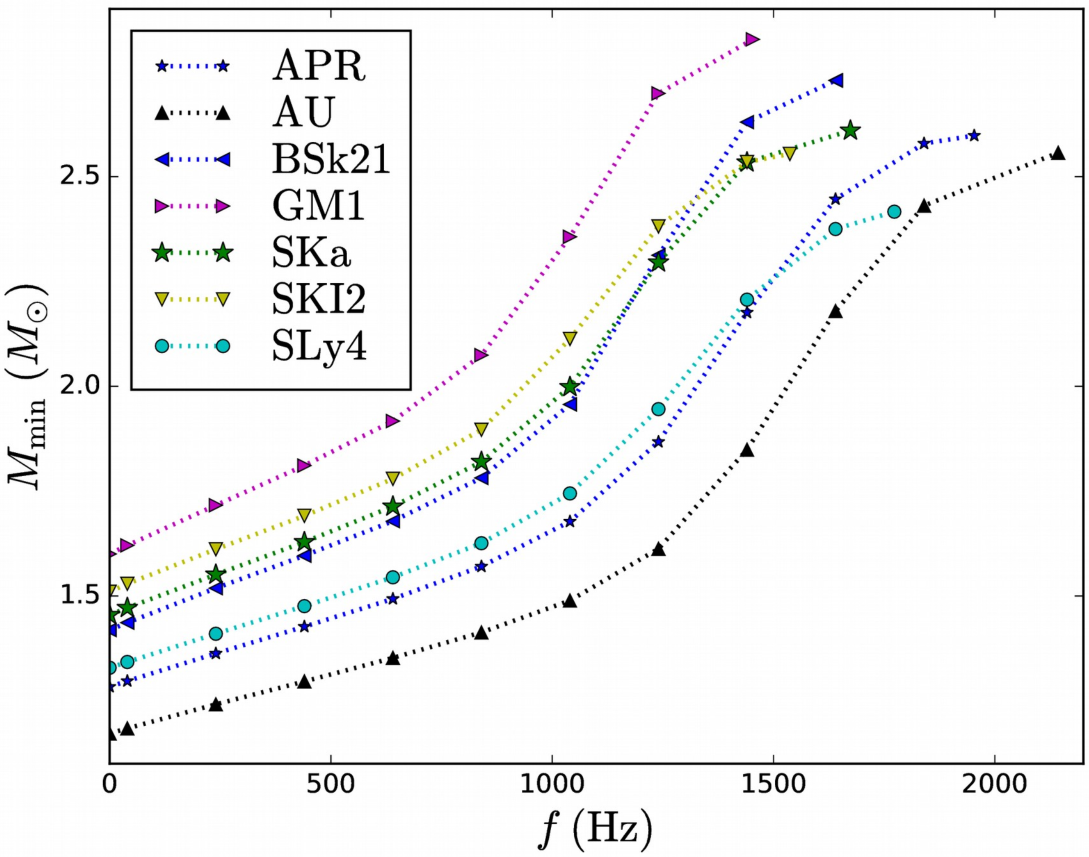
Spin parameter (J/M^2)

Slowly rotating limit:

$$R_{\rm ISCO} = 6M\left[1 - j\left(\frac{2}{3}\right)^{3/2}\right]$$

- Minimum mass for the appearance of ISCO

* For given spin frequency f and EOS, $R_{\mathrm{ISCO}} > R$ only if $M > M_{\mathrm{min}}$



* ISCO depends sensitively on EOS and stellar parameters

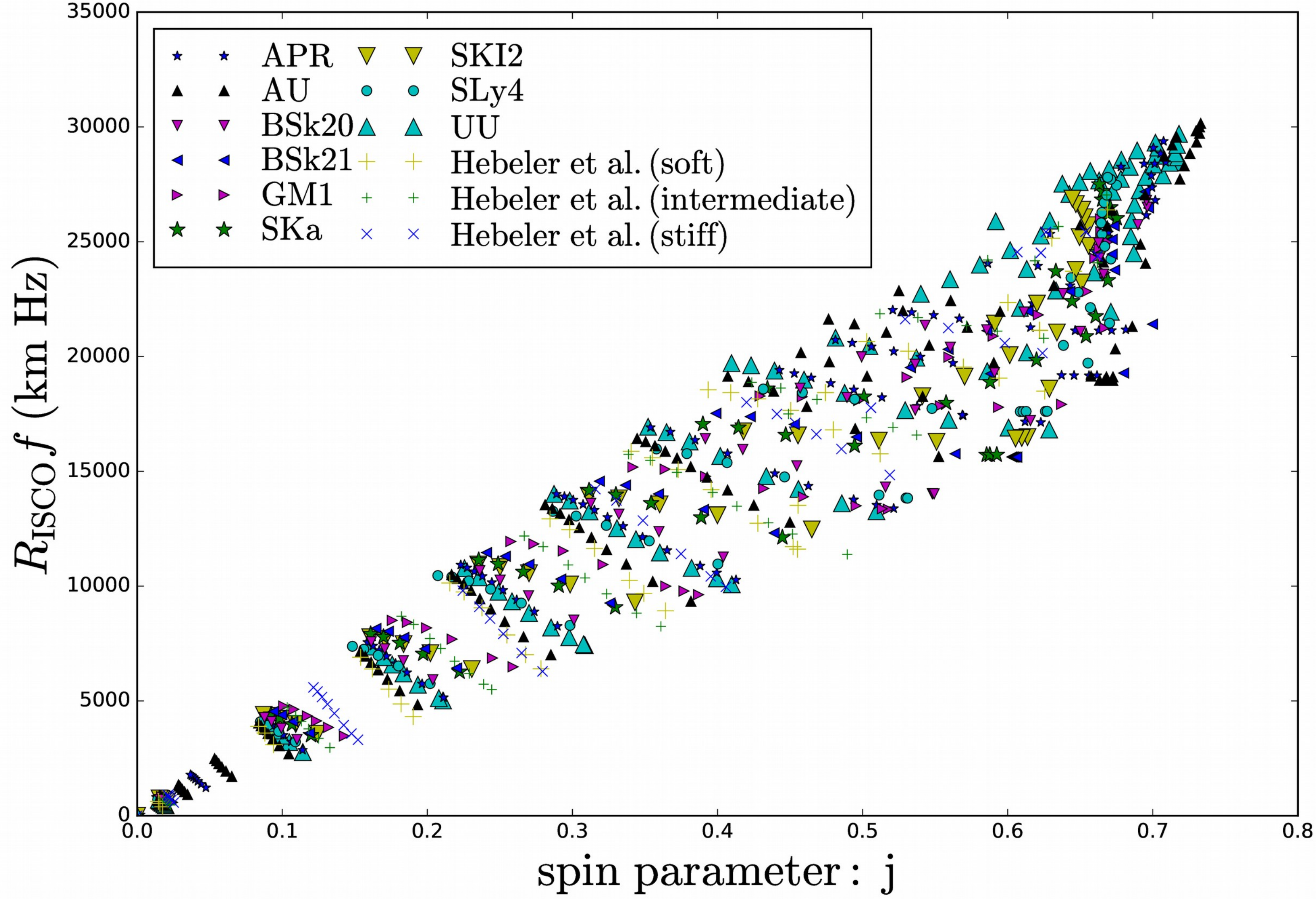
Question: **Are there universal relations connecting ISCO?**

Rule of thumb:

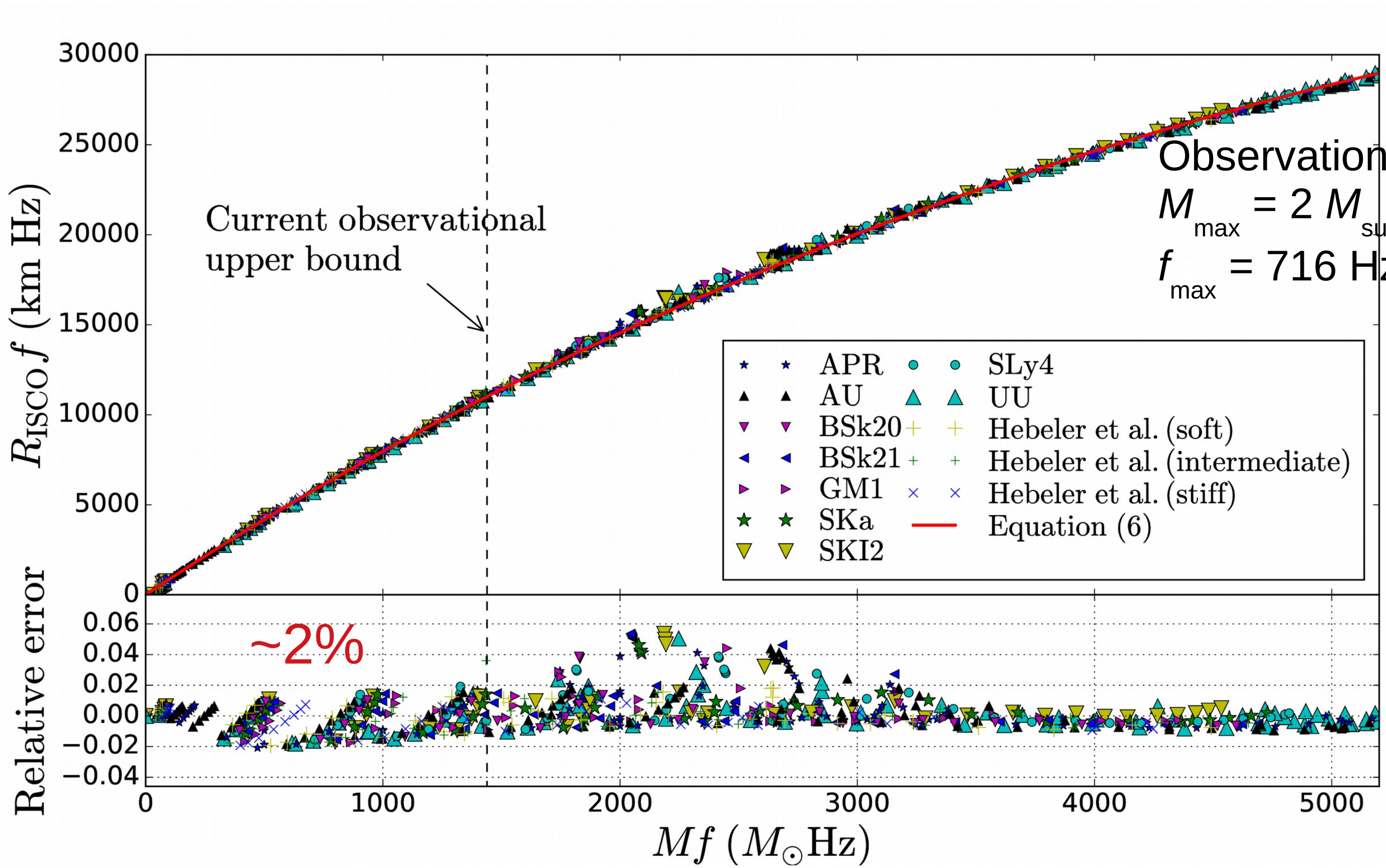
Try scaled dimensionless physical quantities relevant to the problem (M, R, f, j, $R_{\rm ISCO}$, $f_{\rm ISCO}$ …..)

- Failed attempt

Note: $R_{ISCO}f$ is dimensionless in $G=c=1$ units



- Universal relation for R_{ISCO} (ISCO circumferential radius)



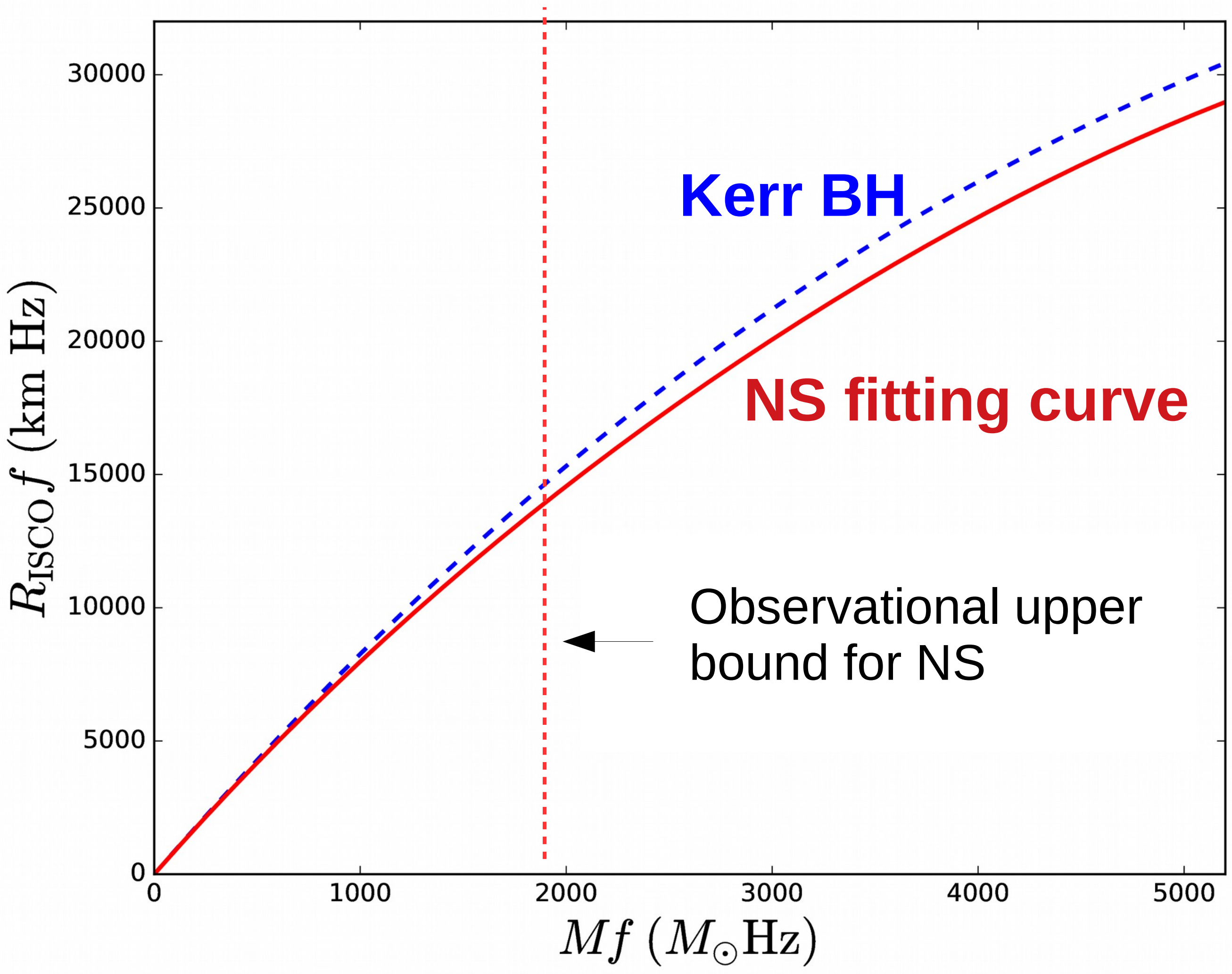
Observational upper bound set by
$M_{max} = 2\ M_{sun}$ (PSR J1614-2230)
f_{max} = 716 Hz (PSR J1748-2446ad)

Luk & LML (2018)

Fitting curve (red line): $$y = a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4, \quad (6)$$

where $y = R_{\text{ISCO}} f$ and $x = Mf$. The fitting parameters are $a_1 = 8.809$, $a_2 = -9.166 \times 10^{-4}$, $a_3 = 8.787 \times 10^{-8}$, and $a_4 = -6.019 \times 10^{-12}$.

- Connection to Kerr black hole

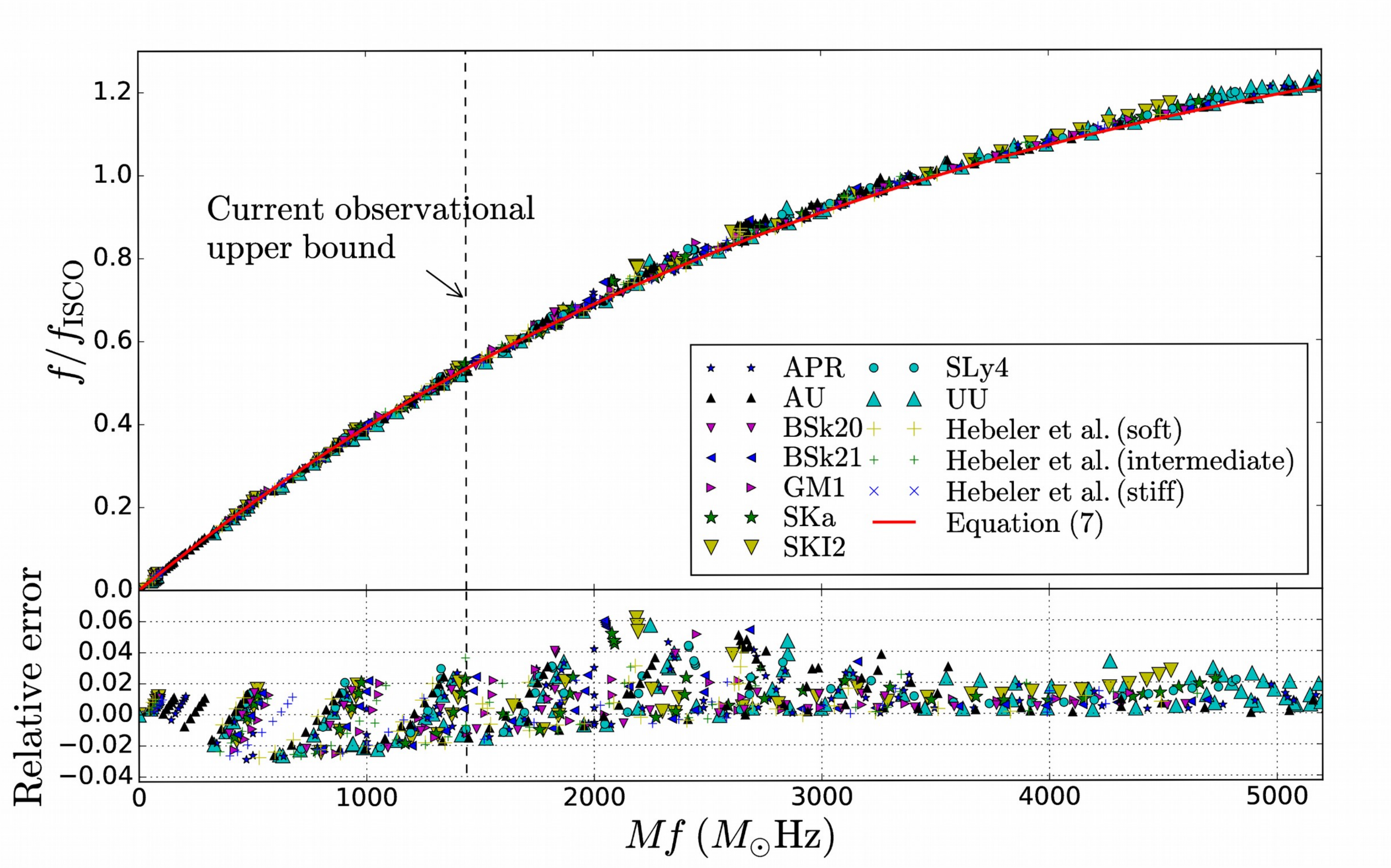


fractional difference within 6%

For black hole, we use the horizon frequency $f_H=\Omega_H/2\pi$

$$2M\Omega_{\rm H} = \frac{j}{1+\sqrt{1-j^2}}$$

- Universal relation for $f_{\rm ISCO}$ (ISCO orbital frequency)



Fitting curve (red line):

$$y = b_1 x + b_2 x^2 + b_3 x^3 + b_4 x^4, \quad (7)$$

where $y = f/f_{\rm ISCO}$ and $x = Mf$. The fitting parameters are given by $b_1 = 4.497 \times 10^{-4}$, $b_2 = -6.130 \times 10^{-8}$, $b_3 = 4.527 \times 10^{-12}$, and $b_4 = -1.446 \times 10^{-16}$.

- Application of the ISCO universal relations

* The ISCO universal relations connect f_{ISCO} and R_{ISCO} to the mass and spin frequency of the star (M, f)

* The relations can be applied to neutron stars in low-mass X-ray binaries (LMXBs)

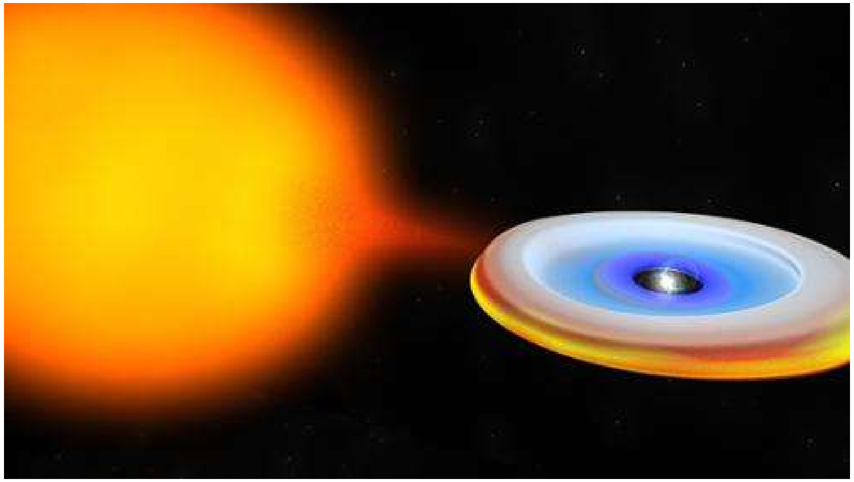


Image credit: Bill Saxton, NRAO/AUI/NSF

If ISCO exists: $$R \leq R_{\mathrm{ISCO}} \leq R_{\text{inner radius of disk}}$$

* kHz quasi-periodic oscillations (kHz QPOs) are well observed in these systems

* QPO models are generally associated with the orbital motion and/or oscillations near the inner edge of the accreting disk (no general consensus yet….)

* We consider the system 4U 0614+09

- Highest QPO frequency measured ≈ 1220 Hz Boutelier, Barret, Miller (2009)

- Spin frequency of NS = 415 Hz Strohmayer, Markwardt, Kuulkers (2008)
  (inferred from X-ray burst oscillations)

* Assume the existence of ISCO in the system and ISCO frequency f_{ISCO} = 1220 Hz, the ISCO universal relations give

$$M = 2.0\, M_{\mathrm{sun}}$$
$$R_{\mathrm{ISCO}} = 16\,\mathrm{km} \quad \Rightarrow \quad R < 16\ \mathrm{km}$$

* We have bypassed the assumption of slow rotation and the spin parameter in obtaining the NS mass

Boutelier, Barret, Miller (2009)

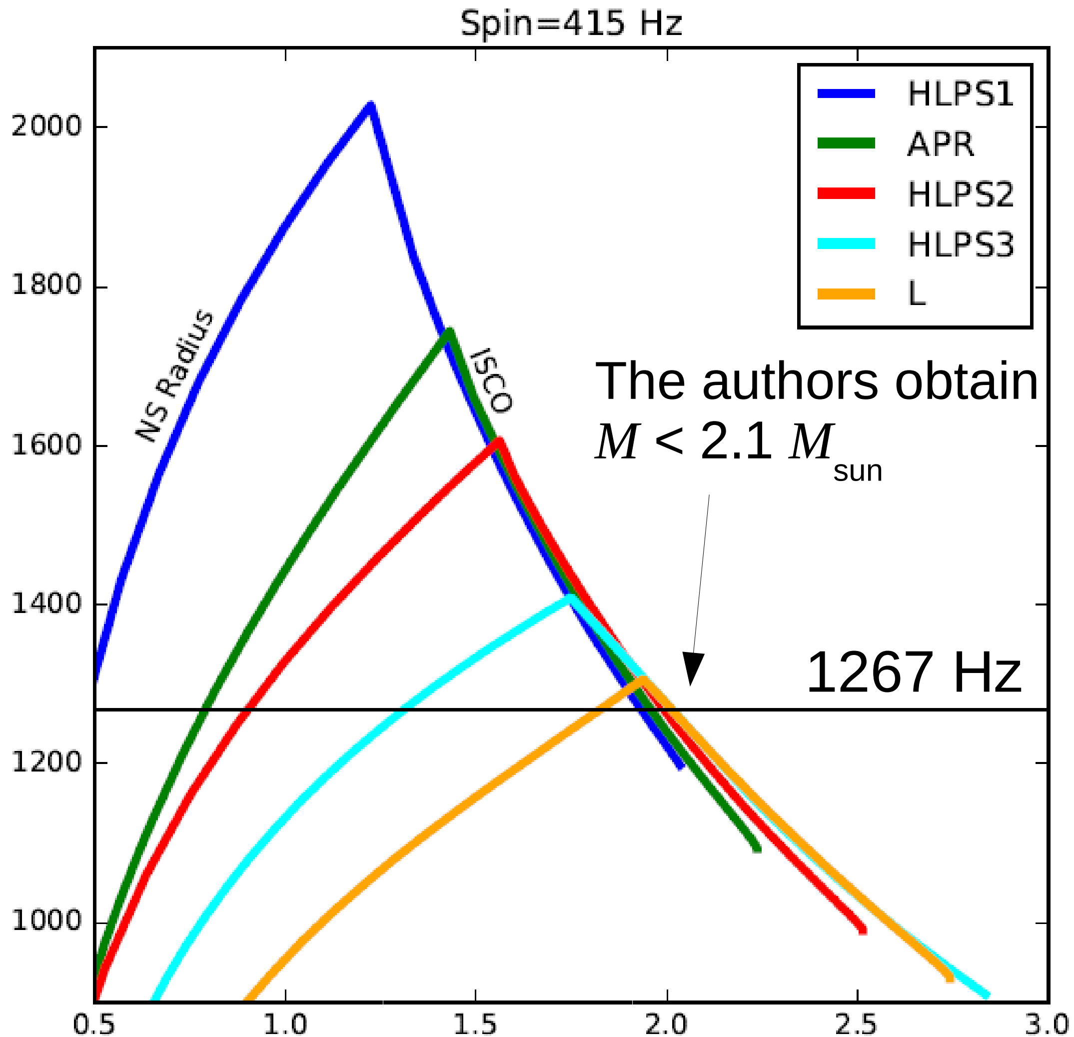
Note: If the QPO frequency corresponds to orbit outside the ISCO, then the value of M is an upper bound

* Recent update of 4U 0614+09

van Doesburgh, van der Klis, Morsink (2018)

Highest QPO frequency:
1220 Hz → 1267 Hz

Maximum orbital frequency (Hz)



Mass (M_{sun})

Summary

* Solid quark stars composed of CCS quark matter can break the universal I-Love relation for fluid stars

* Fluid quark star model ruled out by GW upper bound on the tidal deformability can be revived if the entire star is in a solid phase (depending on the shear modulus)

* Hybrid star model ruled out by the GW170817 upper bound would still be ruled out even if the core is solid (….unless $R_{core} \sim R$)

* Universal relations for the ISCO radius and frequency around rapidly rotating neutron stars are proposed

* Assuming the highest QPO frequency of the system 4U 0614+09 to be the ISCO frequency, we determine the NS mass to be 2.0 M_{sun}

Thank you!