Correlated Structure of Nuclear Symmetry Energy from Covariant Nucleon Self-Energy

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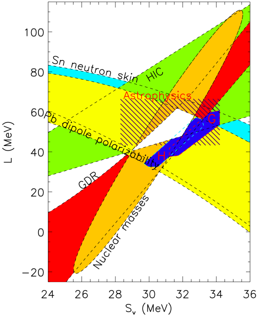
Symmetry Energy in Nuclear Matter

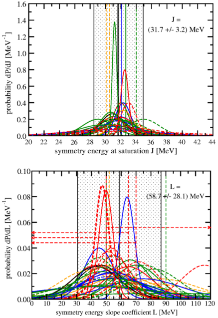
- Equation of state for isospin asymmetric nuclear matter

$$E_b(\rho_b,\delta) = E_0(\rho_b) + E_S(\rho_b)\delta^2 + S_4(\rho_b)\delta^4 + \mathcal{O}(4), \quad L = 3\rho_0 \left.\frac{\partial E_S(\rho_b)}{\partial \rho_b}\right|_{\rho_b=\rho_0},$$

- Important to understand
  - nuclear structure: fission properties, density distribution, collective excitation, ...
  - nuclear astrophysics: NS's radius, crust-core transition density, cooling rate, ...
  - heavy ion reactions: isospin diffusion, DR(n/p), ...



☼ *Lattimer:ApJ2013*



☼ *Oertel:RMP2017*

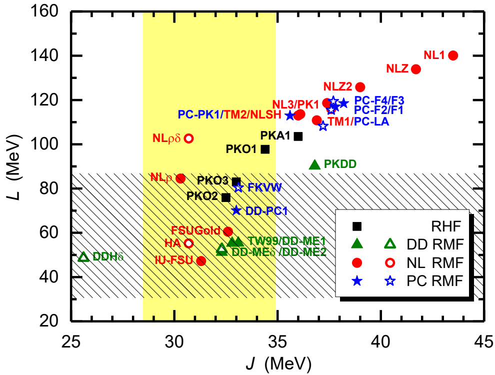
To improve the nuclear many-body models, information of symmetry energy is essential

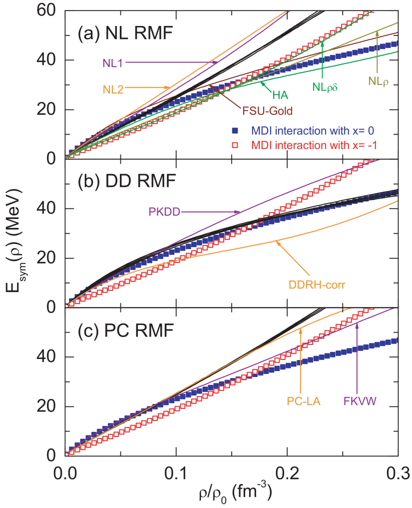
New interest in studying ingredients of effective nuclear force: momentum-dependence, exchange term, tensor force, short range correlation, ...

Covariant density functional (CDF) theory:

☼ *Walecka (1974), Serot (1986), Reihard (1989), Ring (1996), Bender (2003), Meng (2006), Liang (2015)*

- spin-orbit coupling
- pseudo-spin symmetry
- consistent treatment of time-odd fields
- connection to QCD





☼ *L. W. Chen, C. M. Ko and B. A. Li, Phys. Rev. C* ***76****, 054316 (2007).*

Symmetry Energy Studied in RHF Theory

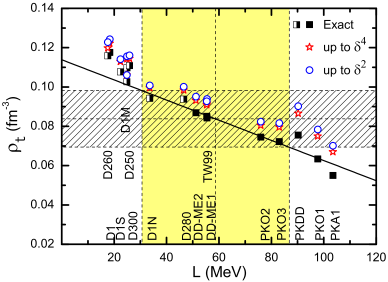
Relativistic Hartree-Fock (RHF) theory:

☼ *Bouyssy (1987), Bernardos (1993), Shi (1995), Marcos (2004), Long (2006-2010)*

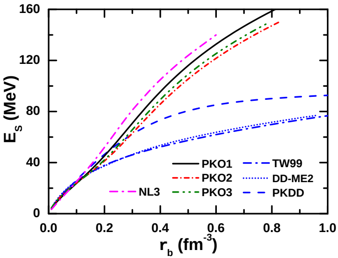
- nonlocal Fock terms
- π-PV and ρ-T couplings
- tensor force involved naturally

Improved isospin related structure descriptions

☼ *W.H.Long:PLB2006, H.Z.Liang:PRL2008, Q.Zhao:JPG2015*



☼ *Z.W. Liu, Z. Qian, R.Y. Xing, J.R. Niu, and BYS, PRC* ***97****, 025801 (2018).*



→ All mesons account for isospin properties via Fock terms

→ Significant contributions to the symmetry energy from isoscalar meson exchange diagrams

☼ *BYS, W.H. Long, J. Meng, and U. Lombardo, PRC* ***78****, 065805 (2008).*

☼ *W.H. Long, BYS, K. Hagino, and H. Sagawa, PRC* ***85****, 025806 (2012).*

☼ *BYS, Q. Zhao, and W.H. Long, EPJConf* ***117****, 07011 (2016).*

Relativistic formalism to quantify tensors in Fock diagrams of π-PV, σ-S, ω-V, ρ-T couplings:

☼ *L. J. Jiang, S. Yang, BYS, W. H. Long, et al., PRC* ***91****, 034326 (2015).*

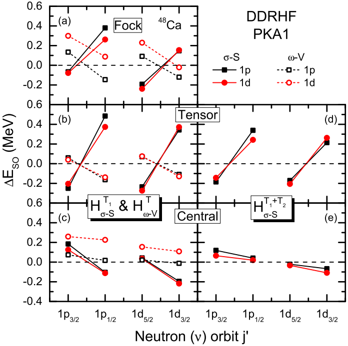
☼ *Y. Y. Zong and BYS, Chin. Phys. C* ***42****, 024101 (2018).*

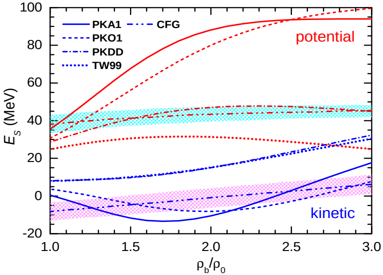
Second-Order Irreducible Tensor S_{12} for π-PV:

$$S_{12} = 3\left(\gamma_0 \boldsymbol{\Sigma}_1 \cdot \boldsymbol{q}\right)\left(\gamma_0 \boldsymbol{\Sigma}_2 \cdot \boldsymbol{q}\right) - \left(\gamma_0 \boldsymbol{\Sigma}_1\right) \cdot \left(\gamma_0 \boldsymbol{\Sigma}_2\right) \boldsymbol{q}^2$$

→ The tensors are involved naturally by the Fock diagrams and quantified by the relativistic formalism without introducing any additional free parameters.

Two evidence: Spin dependence, Tensor Sum Rule





Kinetic and potential symmetry energy:

☼ *Q. Zhao, BYS, W. H. Long, J. Phys. G* ***42****, 095101 (2015).*

		TW99	PKDD	PKO1	PKA1	BHF
J	$T=0$	51.0	50.8	38.8	42.4	44.2
	$T=1$	-26.2	-22.1	-8.1	-5.7	-9.0
	kin	8.0	8.1	3.7	0.5	-1.0

BHF: ☼ *I. Vidaña+:PRC2011* CFG: ☼ *Or Hen+:PRC2015*

Impact of Tensor Force on Nuclear Symmetry Energy

Tensor force effects on kinetic symmetry energy:

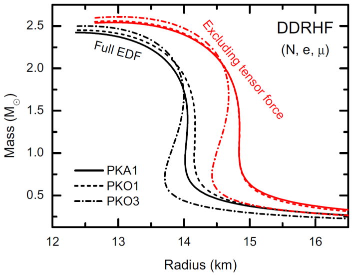
		TW99	PKDD	PKO1	PKA1
E_S	kin	8.0	8.1	3.7	0.5
	kin-T_1			-7.3	-9.7
	kin-T_2			-0.4	-0.6

☼ *Or Hen et al., PRC* ***91****, 025803 (2015).* -10±7.5 MeV

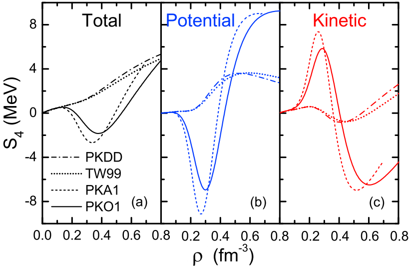
☼ *B. J. Cai, B. A. Li, PRC* ***92****, 011601 (2015).* −14.28±11.60 MeV

☼ *B. J. Cai, B. A. Li, PRC* ***93****, 014619 (2016).* -16.94±13.66 MeV

Effects of Fock terms to E_S: Soften due to tensor part



☼ *L. J. Jiang et al., PRC* ***91****, 025802 (2015).*



Nuclear fourth-order symmetry energy:

S_4 suppressed in RHF, but $S_{4,\text{kin}}$ enhanced at ρ_0

☼ *Z.W. Liu et al., PRC* ***97****, 025801 (2018).*

		TW99	PKDD	PKO1	PKA1
S_4	kin	0.5	0.5	1.2	2.1
	kin-T			0.2	0.3

☼ *B. J. Cai, B. A. Li, PRC* ***92****, 011601 (2015).* 7.18±2.52 MeV

Properties of E_S at Saturation Density: J and L

Methods to analyse the structure of symmetry energy:

- In terms of energy density functional: kinetic and potential, spin-isospin ✓
- In terms of components of nuclear force: central and tensor part ✓
- In terms of single-particle energy: nucleon self-energy

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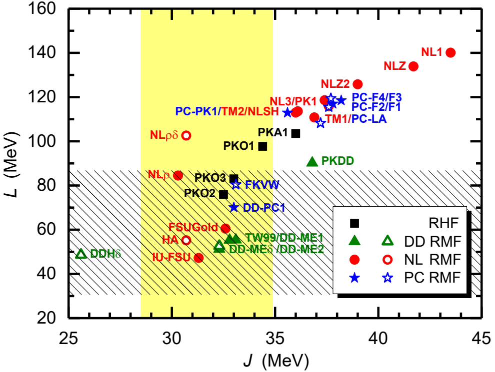
Properties of Symmetry Energy at Saturation Density in CDF theory:



From a covariant nucleon self-energy, try to understand:

- Origin of model dependence of L
- Correlations between L and J

☼ *Z. W. Liu, Q. Zhao and BYS, arXiv:1809.03837.*

Hugenholtz-Van Hove (HVH) Theorem

- Hugenholtz-Van Hove (HVH) theorem ☼ *N. M. Hugenholtz, L. Van Hove, Physica* **24** *(1958) 363.*
  Relations between binding energy per nucleon and single-nucleon Fermi energy
- Symmetry Energy from HVH theorem ☼ *B. J. Cai, L. W. Chen, Phys. Lett. B* **711** *(2012) 104.*

$$E_b + \rho_b \frac{\partial E_b}{\partial \rho_b} = \varepsilon_F \xrightarrow[\text{to asymmetry}]{\text{expansion}} E_S(\rho_b) = \frac{1}{4}\frac{d}{d\delta}\left[\sum_\tau \tau \varepsilon_F^\tau(\rho_b, \delta, k_F^\tau)\right]\Big|_{\delta=0}$$

$$\frac{d\varepsilon_F^\tau}{d\delta}\Big|_{\delta=0} = \frac{\tau k_F}{3}\left(\frac{\partial \varepsilon}{\partial k}\right)\Big|_{k=k_F} + \frac{\partial \varepsilon_F^\tau}{\partial \delta}\Big|_{\delta=0} \Longrightarrow E_S(\rho_b) = \underbrace{E_S^{\text{kin}}(\rho_b) + E_S^{\text{mon}}(\rho_b)}_{\partial k} + \underbrace{E_S^{\text{1st}}(\rho_b)}_{\partial \delta}$$

- Kinetic part, k-dependence of self-energy, δ-dependence of self-energy

$$E_S^{\text{kin}}(\rho_b) = \frac{k_F k_F^*}{6E_F^*}, \qquad E_S^{1st}(\rho_b) = \frac{1}{2}\frac{g_\rho^2}{m_\rho^2}\rho_b + E_S^{1st,E}$$

$$E_S^{\text{mon}}(\rho_b) = \frac{k_F k_F^*}{6E_F^*}\frac{\partial \Sigma_V}{\partial k}\Big|_{k=k_F} + \frac{k_F M_F^*}{6E_F^*}\frac{\partial \Sigma_S}{\partial k}\Big|_{k=k_F} + \frac{k_F}{6}\frac{\partial \Sigma_0}{\partial k}\Big|_{k=k_F},$$

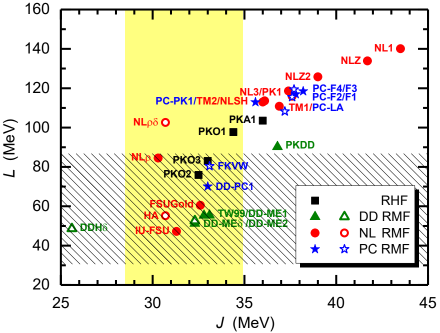
- Symmetry energy at saturation density

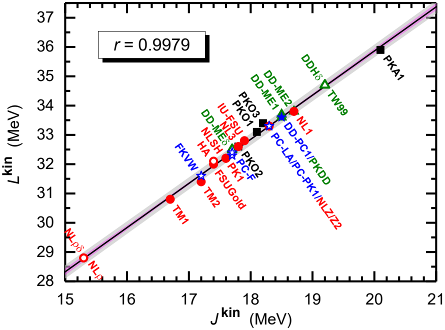
$$J = J^{\text{kin}} + J^{\text{mom}} + J^{\text{1st}}$$

$$L = L^{\text{kin}} + L^{\text{mom}} + L^{\text{1st}} + L^{\text{cross}} + L^{\text{2nd}}$$

Correlated Structure of Nuclear Symmetry Energy

$$J = \boxed{J^{\rm kin}} + J^{\rm mom} + J^{\rm 1st}, \qquad L = \boxed{L^{\rm kin}} + L^{\rm mom} + L^{\rm 1st} + L^{\rm cross} + L^{\rm 2nd}$$



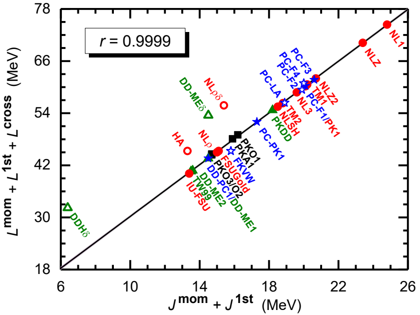


$$\begin{aligned}
\boldsymbol{k}^{*,\tau} &= \boldsymbol{k} + \hat{\boldsymbol{k}}\Sigma_V^\tau(\rho,\delta,|\boldsymbol{k}|),\\
M_D^{*,\tau} &= M + \Sigma_S^\tau(\rho,\delta,|\boldsymbol{k}|),\\
\varepsilon^{*,\tau} &= \varepsilon^\tau - \Sigma_0^\tau(\rho,\delta,|\boldsymbol{k}|),
\end{aligned}$$

$$L^{\rm kin} = \frac{3}{2}J^{\rm kin} + \frac{k_F}{6}\left[\frac{M_F^{*2}}{\varepsilon_F^{*2}}\frac{k_F}{\varepsilon_F^*} - \frac{1}{2}\frac{k_F^*}{\varepsilon_F^*}\right]$$

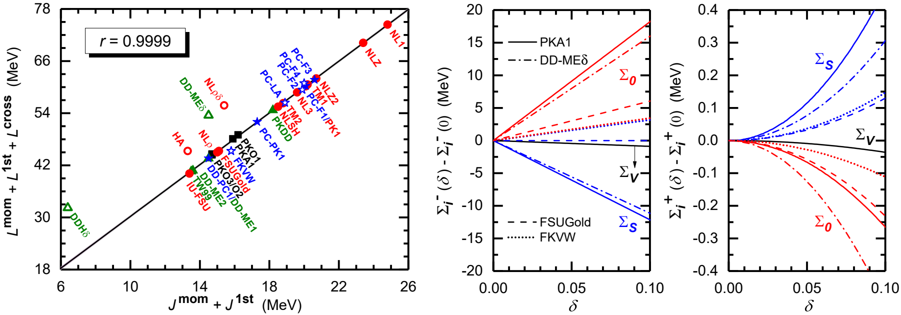
Correlated Structure of Nuclear Symmetry Energy

$$J = J^{\text{kin}} + \boxed{J^{\text{mom}} + J^{\text{1st}}}, \qquad L = L^{\text{kin}} + \boxed{L^{\text{mom}} + L^{\text{1st}} + L^{\text{cross}}} + L^{\text{2nd}}$$



Correlated Structure of Nuclear Symmetry Energy

$$J = J^{\rm kin} + \boxed{J^{\rm mom} + J^{\rm 1st}}, \qquad L = L^{\rm kin} + \boxed{L^{\rm mom} + L^{\rm 1st} + L^{\rm cross}} + L^{\rm 2nd}$$



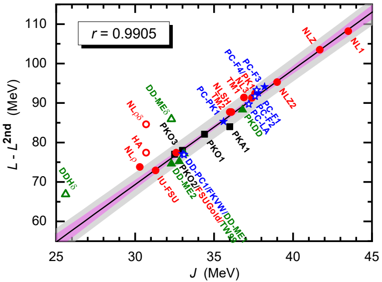
$$L^{\rm 1st} = 3J^{\rm 1st} + \frac{3}{2}\left[\frac{k_F^*}{\varepsilon_F^{*3/2}}\Sigma_S^{{\rm sym},1} - \frac{M_F^*}{\varepsilon_F^{*3/2}}\Sigma_V^{{\rm sym},1}\right]^2_{|\boldsymbol{k}|=k_F} - \frac{k_F M_F^*}{\varepsilon_F^{*2}}\left[\frac{k_F^*}{\varepsilon_F^*}\Sigma_S^{{\rm sym},1} - \frac{M_F^*}{\varepsilon_F^*}\Sigma_V^{{\rm sym},1}\right]_{|\boldsymbol{k}|=k_F}$$

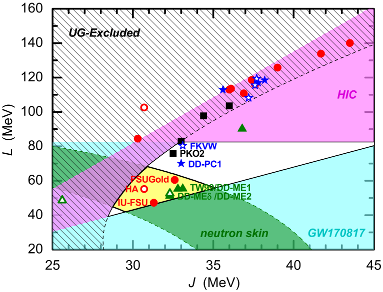
$$\Sigma_{\mathcal{O}}^{{\rm sym},1} = \frac{\partial}{\partial\delta}\frac{\Sigma_{\mathcal{O}}^-}{2}\bigg|_{\delta=0}, \qquad \Sigma_{\mathcal{O}}^- = \Sigma_{\mathcal{O}}^n - \Sigma_{\mathcal{O}}^p$$

☼ *Z. W. Liu, Q. Zhao and BYS, arXiv:1809.03837.*

Two underlying linear correlations construct the fundamental correlation between L and J in CDF framework.

$$J = \boxed{J^{\text{kin}} + J^{\text{mom}} + J^{\text{1st}}}, \qquad L = \boxed{L^{\text{kin}} + L^{\text{mom}} + L^{\text{1st}} + L^{\text{cross}}} + L^{\text{2nd}}$$





$$\boxed{L = 2.93J - 18.60 + L^{\text{1st},\delta} + L^{\text{2nd}}\ \text{MeV}}$$

$$L = (3+\gamma)J - (1+\gamma)S_0 - \frac{1}{6}\gamma\rho_0(a_0 - \eta_1 a_1 + \eta_1 a_2)$$ ☼ *Holt:PLB2018*

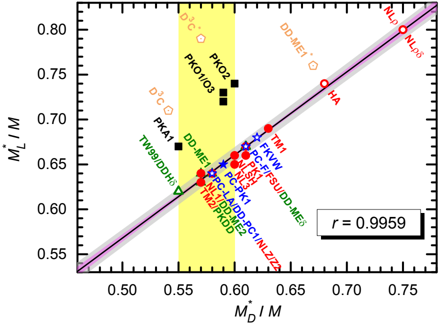
$$L^{\text{2nd}} = 3\left[\frac{M_F^*}{\varepsilon_F^*}\Sigma_S^{\text{sym},2} + \Sigma_0^{\text{sym},2} + \frac{k_F^*}{\varepsilon_F^*}\Sigma_V^{\text{sym},2}\right]_{|\boldsymbol{k}|=k_F}$$

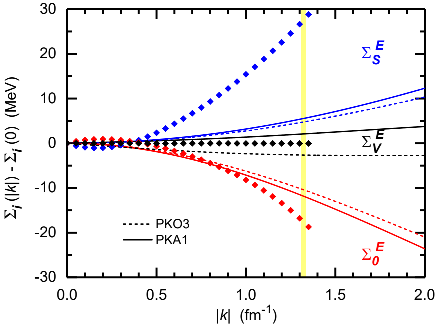
$$\Sigma_{\mathcal{O}}^{\text{sym},2} = \frac{1}{2}\frac{\partial^2}{\partial\delta^2}\frac{\Sigma_{\mathcal{O}}^{+}}{2}\bigg|_{\delta=0}, \qquad \Sigma_{\mathcal{O}}^{+} = \Sigma_{\mathcal{O}}^{n} + \Sigma_{\mathcal{O}}^{p}$$

Main factors to break the correlation:

- $\Sigma_{\mathcal{O}}^{\text{sym},1}$: isovector scalar coupling
- $\Sigma_{\mathcal{O}}^{\text{sym},2}$: strong model dependence

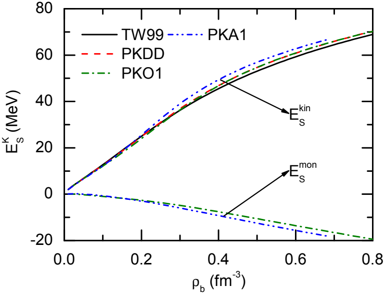
Landau Mass in CDF Theory





☼ *Z. W. Liu, Q. Zhao and BYS, arXiv:1809.03837.*

☼ *T. Katayama, K. Saito, PRC 88, 035805 (2013).*



k-dependenct contribution and Landau mass:

$$E_S^K = E_S^{\text{kin}} + E_S^{\text{mon}} = \frac{k_F^2}{6M_L^*}$$

The negative E_S^{mon} due to the k-dependence of self-energies in RHF, lead to larger M_L^*

Neutrino Emissivity in DUrca Process

- Non-relativistic expression: ☼ *J.M. Lattimer et al., PRL* **66** *(2017) 2701.*

$$Q^{(D)} = \frac{457\pi}{10080} G_F^2 \cos^2\theta_c (F_V^2 + 3G_A^2) \frac{m_n^* m_p^* m_l^*}{\hbar^{10} c^3} (k_B T)^6 \Theta_{npe}$$

Adopting a non-relativistic approximation of the matrix element.

- Relativistic expression: ☼ *L.B. Leinson and A. Pérez, PLB* **518** *(2001) 15.*

$$Q^{(D)} = \frac{457\pi}{10080} \frac{G_F^2 \cos^2\theta_c}{\hbar^{10} c^3} (k_B T)^6 [F_V G_A ((\varepsilon_{F_n} + \varepsilon_{F_p}) p_{Fl}^2 - (\varepsilon_{F_n} - \varepsilon_{F_p})(p_{F_n}^2 - p_{F_p}^2)) + 2G_A^2 \mu_l M_n^* M_p^*$$
$$+ (F_V^2 + G_A^2)(\mu_l (2\varepsilon_{F_n}\varepsilon_{F_p} - M_n^* M_p^*) + \varepsilon_{F_n} p_{Fl}^2) - \frac{1}{2}(\varepsilon_{F_n} + \varepsilon_{F_p})(p_{F_n}^2 - p_{F_p}^2 + p_{Fl}^2)] \Theta_{npe}$$

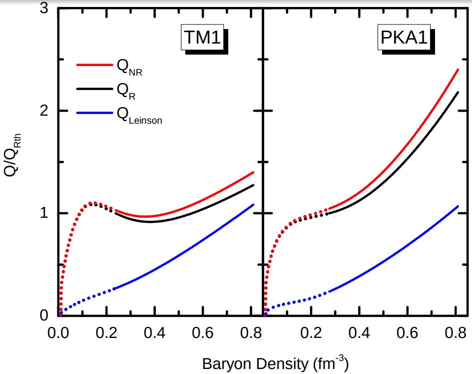
Relativistic matrix element, but approximation in phase space integration

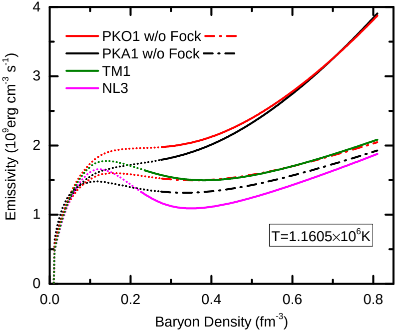
- New expression for neutrino emissivity in DUrca processes:

$$Q^{(D)} = Q_1^{(D)} + Q_2^{(D)}$$
$$Q_1^{(D)} = \frac{457\pi}{10080} \frac{G_F^2 \cos^2\theta_c}{\hbar^{10} c^3} (k_B T)^6 [2(F_V^2 + G_A^2) + \hat{M}_n \hat{M}_p (G_A^2 - F_V^2)] m_n^* m_p^* m_l^* \Theta_{npe}$$
$$Q_2^{(D)} = -\frac{45[2\pi^2\zeta(5) + 63\zeta(7)]}{16\pi^5} G_F^2 \cos^2\theta_c \frac{m_n^* m_p^*}{\hbar^{10} c^5} \hat{P}_n \hat{P}_p \left[\frac{|\mathbf{p}_{nF}|}{|\mathbf{p}_{pF}|}(F_V + G_A)^2 + \frac{|\mathbf{p}_{pF}|}{|\mathbf{p}_{nF}|}(F_V - G_A)^2\right] (k_B T)^7 \Theta_{npe}$$

Relativistic matrix element, more attention to angular integration in phase space

Neutrino Emissivity in DUrca Process





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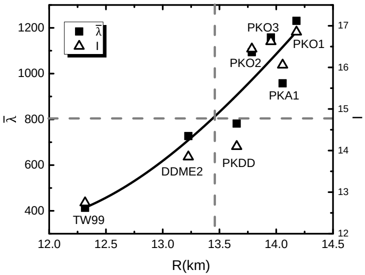
$$Q_2^{(D)} = -\frac{45[2\pi^2\zeta(5)+63\zeta(7)]}{16\pi^5}G_F^2\cos^2\theta_c\frac{m_n^*m_p^*}{\hbar^{10}c^5}\hat{P}_n\hat{P}_p\left[\frac{|\mathbf{p}_{nF}|}{|\mathbf{p}_{pF}|}(F_V+G_A)^2+\frac{|\mathbf{p}_{pF}|}{|\mathbf{p}_{nF}|}(F_V-G_A)^2\right](k_BT)^7\Theta_{npe}$$

Relativistic matrix element, more attention to angular integration in phase space

Summary and Outlook

- The symmetry energy in CDF theory, illustrating the effects of exchange terms, also the tensor force effects in kinetic part, impact on Landau mass and neutrino emissivity
- The correlated structure between L and J is revealed in terms of the nucleon self-energy
- Possible way to improve the CDFs in the market, from a viewpoint of nucleon self-energy, constrained by either ab initios or experiments



Still on the way to establish an unified CDF both for finite nuclei and for astrophysics

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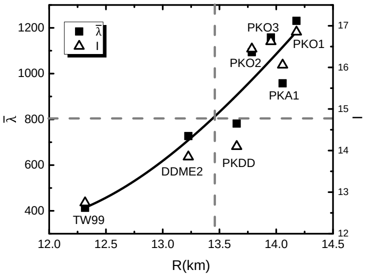


Still on the way to establish an unified CDF both for finite nuclei and for astrophysics

Thank you for your attention!

Selected CDF Effective Lagrangians

Table: Bulk properties of symmetric nuclear matter at saturation point

	Fock	σ-NL	ω-NL	DD	π-PV	ρ-T	ρ_0 (fm^{-3})	E_B/A (MeV)	K (MeV)	J (MeV)	L (MeV)	Reference
PKA1	✓	×	×	✓	✓	✓	0.160	-15.83	230.0	36.0	104	*Long:2007*
PKO1	✓	×	×	✓	✓	×	0.152	-16.00	250.2	34.4	98	*Long:2006*
PKO2	✓	×	×	✓	×	×	0.151	-16.03	249.6	32.5	76	*Long:2008*
PKO3	✓	×	×	✓	✓	×	0.153	-16.04	262.5	33.0	83	*Long:2008*
NL1	×	✓	×	×	×	×	0.152	-16.43	211.2	43.5	140	*Reinhard:1986*
NL3	×	✓	×	×	×	×	0.148	-16.25	271.7	37.4	118	*Lalazissis:1997*
NL-SH	×	✓	×	×	×	×	0.146	-16.33	354.9	36.1	114	*Sharma:1993*
TM1	×	✓	✓	×	×	×	0.145	-16.26	281.2	36.9	111	*Sugahara:1994*
PK1	×	✓	✓	×	×	×	0.148	-16.27	282.7	37.6	116	*Long:2004*
TW99	×	×	×	✓	×	×	0.153	-16.25	240.3	32.8	55	*Typel:1999*
DD-ME1	×	×	×	✓	×	×	0.152	-16.20	244.7	33.1	56	*Nikšić:2002*
DD-ME2	×	×	×	✓	×	×	0.152	-16.11	250.3	32.3	51	*Lalazissis:2005*
PKDD	×	×	×	✓	×	×	0.150	-16.27	262.2	36.8	90	*Long:2004*

Relatively large values of K and J systematically in RMF with nonlinear self-coupling of mesons (NLRMF)

☼ *B. Y. Sun et al., PRC* **78***(2008)065805; W. H. Long et al., PRC* **85***(2012)025806; L. J. Jiang et al., PRC* **91***(2015)025802.*

Relativistic Formalism of Tensors

Relativistic formalism to quantify tensors in Fock diagrams of π-PV, σ-S, ω-V, ρ-T couplings:

☼ *L. J. Jiang, S. Yang, B. Y. Sun, W. H. Long, and H. Q. Gu, Phys. Rev. C* **91**, *034326 (2015).*

$$\mathcal{H}^T_{\pi\text{-PV}} = -\frac{1}{2}\left[\frac{f_\pi}{m_\pi}\bar\psi\gamma_0\Sigma_\mu\vec\tau\psi\right]_1\cdot\left[\frac{f_\pi}{m_\pi}\bar\psi\gamma_0\Sigma_\nu\vec\tau\psi\right]_2 D^{T,\,\mu\nu}_{\pi\text{-PV}}(1,2), \tag{2}$$

$$\mathcal{H}^T_{\sigma\text{-S}} = -\frac{1}{4}\left[\frac{g_\sigma}{m_\sigma}\bar\psi\gamma_0\Sigma_\mu\psi\right]_1\left[\frac{g_\sigma}{m_\sigma}\bar\psi\gamma_0\Sigma_\nu\psi\right]_2 D^{T,\,\mu\nu}_{\sigma\text{-S}}(1,2), \tag{3}$$

$$\mathcal{H}^T_{\omega\text{-V}} = +\frac{1}{4}\left[\frac{g_\omega}{m_\omega}\bar\psi\gamma_\lambda\gamma_0\Sigma_\mu\psi\right]_1\left[\frac{g_\omega}{m_\omega}\bar\psi\gamma_\delta\gamma_0\Sigma_\nu\psi\right]_2 D^{T,\,\mu\nu\lambda\delta}_{\omega\text{-V}}(1,2), \tag{4}$$

$$\mathcal{H}^T_{\rho\text{-T}} = +\frac{1}{2}\left[\frac{f_\rho}{2M}\bar\psi\sigma_{\lambda\mu}\vec\tau\psi\right]_1\cdot\left[\frac{f_\rho}{2M}\bar\psi\sigma_{\delta\nu}\vec\tau\psi\right]_2 D^{T,\,\mu\nu\lambda\delta}_{\rho\text{-T}}(1,2), \tag{5}$$

where $\Sigma^\mu = \left(\gamma^5, \boldsymbol{\Sigma}\right)$, and D^T (ϕ for σ and π, ϕ' for ω and ρ) read as,

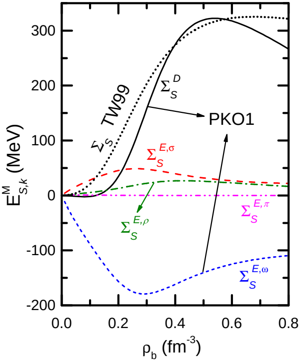
$$D^{T,\,\mu\nu}_{\phi}(1,2) = \left[\partial^\mu(1)\partial^\nu(2) - \frac{1}{3}g^{\mu\nu}m^2_\phi\right]D_\phi(1,2) + \frac{1}{3}g^{\mu\nu}\delta(x_1 - x_2),$$

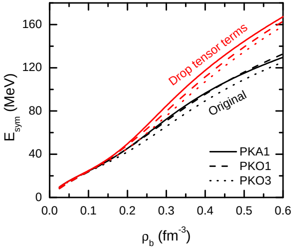
$$D^{T,\,\mu\nu\lambda\delta}_{\phi'}(1,2) = \left[\partial^\mu(1)\partial^\nu(2)g^{\lambda\delta} - \frac{1}{3}G^{\mu\nu\lambda\delta}m^2_{\phi'}\right]D_{\phi'}(1,2) + \frac{1}{3}G^{\mu\nu\lambda\delta}\delta(x_1 - x_2).$$

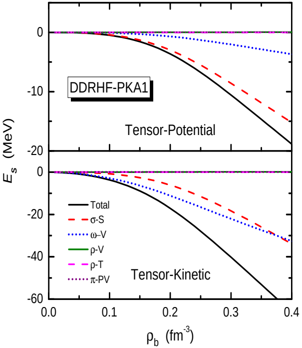
$$G^{\mu\nu\lambda\delta} \equiv \left(g^{\mu\nu}g^{\lambda\delta} - \frac{1}{3}g^{\mu\lambda}g^{\nu\delta}\right)$$

$$E_S(\rho_b) = E_{S,k} + E^D_{S,T=0} + E^E_{S,T=0} + E^D_{S,T=1} + E^E_{S,T=1}$$

☼ *Qian Zhao, Bao Yuan Sun, Wen Hui Long, J. Phys. G* **42**, *095101 (2015).*







		TW99	PKDD	PKO1	PKA1	BHF
	kin	5.9	5.0	-34.5	-69.6	14.9
L	$T=0$	62.2	78.2	67.5	71.3	69.1
	$T=1$	-12.8	7.0	64.8	103.2	-17.5

Large model dependence in kinetic and T=1 potential parts: Significant $E_S^{E,\sigma+\omega}$

Stiff E_S in RHF: Too strong T=1 ☹

☼ *I. Vidaña et al., PRC* **84**, *062801(R) (2011).*

Effective Mass in RHF Theory

- Definition of the Effective Mass:

$$\frac{M^*_\tau}{M} = 1 - \frac{dU_\tau(k,\epsilon(k))}{d\epsilon_\tau} = k\frac{dk}{d\epsilon_\tau} = \left[1 + \frac{M}{k}\frac{dU_\tau}{dk}\right]^{-1}$$

- Non-relativistic Mass: Shrödinger equivalent potential U^τ_e

$$E - mass : \frac{\bar{M}}{M} = \left[1 - \frac{\partial U^\tau}{\partial \epsilon^\tau}\right], \quad K - mass : \frac{\tilde{M}}{M} = \left[1 + \frac{M}{k}\frac{\partial U^\tau}{\partial k}\right]^{-1}$$

$$\frac{M^*_{NR}}{M} = \frac{\bar{M}}{M}\cdot\frac{\tilde{M}}{M} = \left[1 - \frac{\Sigma_0}{M}\right]\left[1 + \left(\frac{M^*}{k}\frac{\partial\Sigma_S}{\partial k} + \frac{E^*}{k}\frac{\partial\Sigma_0}{\partial k} + \frac{k^*}{k}\frac{\partial\Sigma_V}{\partial k} + \frac{\Sigma_V}{k}\right)\right]^{-1}$$

- Landau Mass: $\epsilon + M = E^* + \Sigma_0$

$$M^*_L = k\frac{dk}{d\epsilon^\tau} = k\left[\frac{k^*}{E^*} + \frac{M^*}{E^*}\frac{\partial\Sigma_{S,E}}{\partial k} + \frac{k^*}{E^*}\frac{\partial\Sigma_{V,E}}{\partial k} + \frac{\partial\Sigma_{0,E}}{\partial k}\right]^{-1}$$

- Relativistic Mass:

☼ *W. H. Long et al., Phys. Lett. B 640, 150 (2006).*

$$\frac{M^*_R}{M} = 1 - \frac{d}{d\epsilon}\left[U^\tau_e - \frac{\epsilon^2}{2M}\right], \quad M^*_R = M^*_{NR} + \epsilon = M^*_g \rightarrow \text{group mass}$$