

# Stiff Symmetry Energy from Thick Isovector Aura in Charge-Exchange Reactions

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in the Era of Gravitational Wave Astronomy

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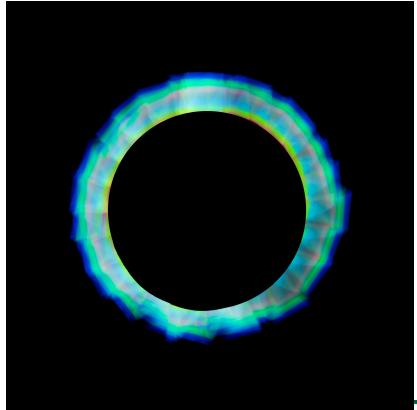


# Aura

## Historically Kirlian Photography



Living Being



Nucleus

# Charge Symmetry & Charge Invariance

Charge symmetry: invariance of nuclear interactions under  $n \leftrightarrow p$  interchange

An isoscalar quantity  $F$  does not change under  $n \leftrightarrow p$  interchange. E.g. nuclear energy. Expansion in asymmetry  $\eta = (N - Z)/A$ , for smooth  $F$ , yields even terms only:

$$F(\eta) = F_0 + F_2 \eta^2 + F_4 \eta^4 + \dots$$

An isovector quantity  $G$  changes sign. Example:  
 $\rho_{np}(r) = \rho_n(r) - \rho_p(r)$ . Expansion with odd terms only:

$$G(\eta) = G_1 \eta + G_3 \eta^3 + \dots$$

Note:  $G/\eta = G_1 + G_3 \eta^2 + \dots$

In nuclear practice, analyticity requires shell-effect averaging!

Charge invariance: invariance of nuclear interactions under rotations in  $n$ - $p$  space. Isospin  $\vec{T} = \sum_{i=1}^A \vec{\tau}_i$  SU(2)



# Charge Symmetry & Charge Invariance

Charge symmetry:

$n \leftrightarrow p$  invariance

Charge invariance:

symmetry under

rotations in

$n$ - $p$  space

Isospin doublets

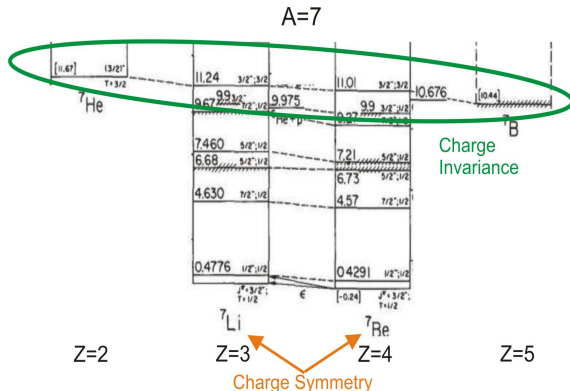
$$p : (\tau, \tau_z) = \left(\frac{1}{2}, \frac{1}{2}\right)$$

$$n : (\tau, \tau_z) = \left(\frac{1}{2}, -\frac{1}{2}\right)$$

Net isospin

$$\vec{T} = \sum_{i=1}^A \vec{\tau}_i$$

Isobars: Nuclei with the same  $A$



$$T = \frac{3}{2}, \dots$$

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Nuclear states:  $(T, T_z), \quad T \geq |T_z| = \frac{1}{2}|N - Z|$



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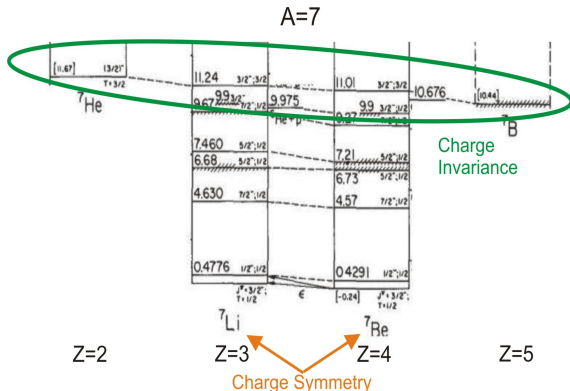
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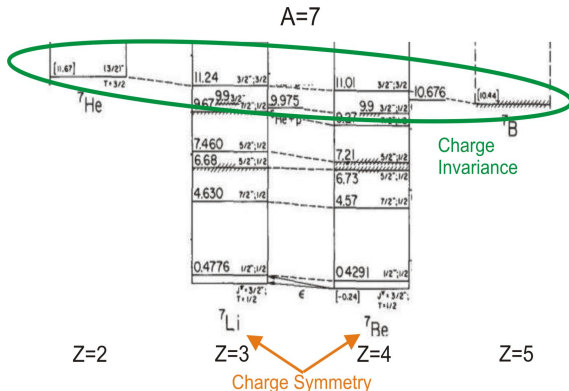
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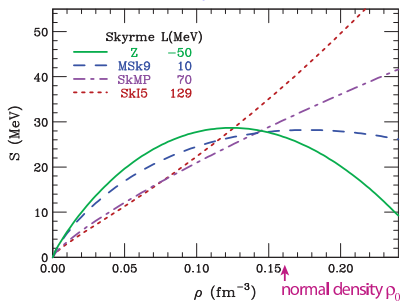
# Examples: Nuclear Energy, Densities

$$\frac{E}{A}(\rho_n, \rho_p) = \frac{E_0}{A}(\rho) + \mathcal{S}(\rho) \left( \frac{\rho_n - \rho_p}{\rho} \right)^2 + \mathcal{O}(\dots^4)$$

symmetric matter

(a)symmetry energy

$$\rho = \rho_n + \rho_p$$



Net  $\rho = \rho_n + \rho_p$  isoscalar

Difference  $\rho_n - \rho_p$  isovector

$\rho_a = \frac{A}{N-Z} (\rho_n - \rho_p)$  isoscalar

$$\rho_{n,p}(r) = \frac{1}{2} \left[ \rho(r) \pm \frac{N-Z}{A} \rho_a(r) \right]$$

Energy min in Thomas-Fermi:

$$\rho_a(r) \propto \frac{\rho(r)}{S(\rho(r))}$$

low  $S \Leftrightarrow$  high  $\rho_a$

$$S(\rho) = S(\rho_0) + \frac{L}{3} \frac{\rho - \rho_0}{\rho_0} + \dots$$

Unknown:  $S(\rho_0)$  ?  $L$  ?



# Stiffness of EOS & Mass & Radius of $n$ -Star

$$\frac{E}{A} = \frac{E_0}{A}(\rho) + S(\rho) \left( \frac{\rho_n - \rho_p}{\rho} \right)^2$$

$$S \simeq a_a^V + \frac{L}{3} \frac{\rho - \rho_0}{\rho_0}$$

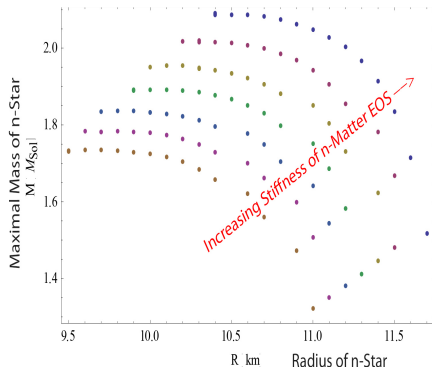
In neutron matter:

$$\rho_p \approx 0 \text{ \& } \rho_n \approx \rho.$$

$$\text{Then, } \frac{E}{A}(\rho) \approx \frac{E_0}{A}(\rho) + S(\rho)$$

Pressure:

$$P = \rho^2 \frac{d}{d\rho} \frac{E}{A} \simeq \rho^2 \frac{dS}{d\rho} \simeq \frac{L}{3\rho_0} \rho^2$$



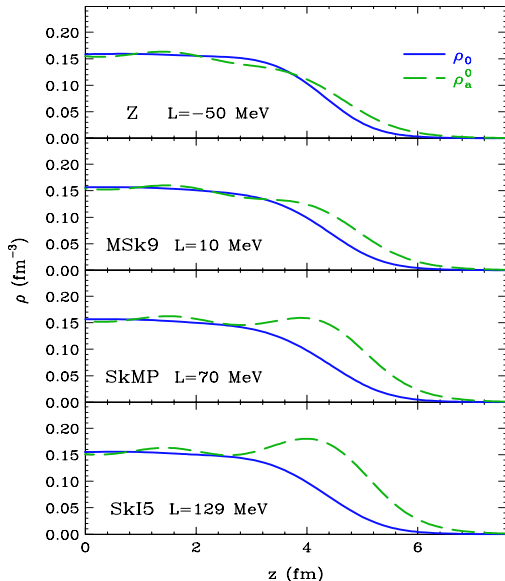
Schematic Calculation by Stephen Portillo (Harvard U)

Stiffer symmetry energy correlates with  
larger max mass of neutron star & larger radii





# Relation between $\rho$ , $\rho_a$ & $S(\rho)$

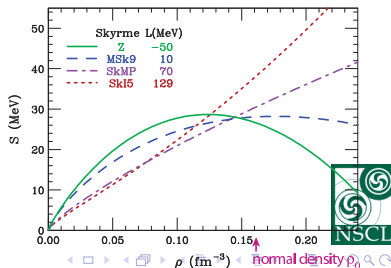


Results f/different Skyrme  
ints in half- $\infty$  matter

PD&Lee NPA818(09)36

Isoscalar ( $\rho=\rho_n+\rho_p$ ; blue) &  
isovector ( $\rho_a\propto\rho_n-\rho_p$ ; green)  
densities displaced  
relative to each other.

As  $S(\rho)$  changes,  $\rho_a(r) \propto \frac{\rho(r)}{S(\rho(r))}$ ,  
so does displacement.



# Probing Independently 2 Densities

Jefferson Lab

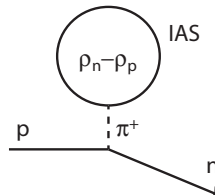
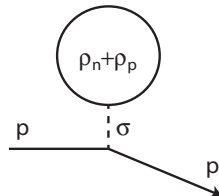
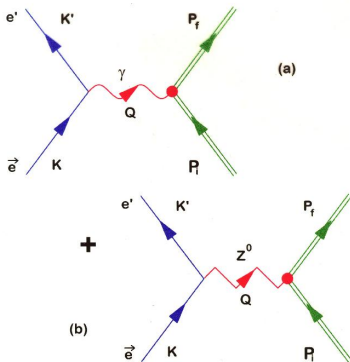
Direct:  $\sim p$

Interference:  $\sim n$

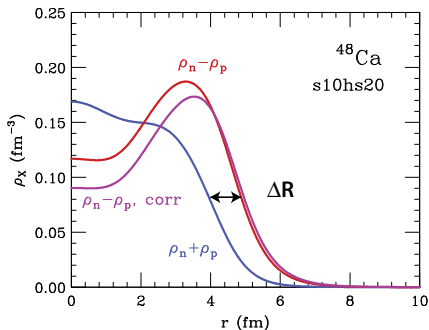
PD, Singh, Lee NPA958(17)147  
[after Dao Tien Kho]

elastic:  $\sim p + n$

charge exchange:  $\sim n - p$



# Expectations on Isovector Aura?



Much Larger Than Neutron Skin!

Surface radius  $R \simeq \sqrt{\frac{5}{3}} \langle r^2 \rangle^{1/2}$

rms neutron skin

$$\langle r^2 \rangle_{\rho_n}^{1/2} - \langle r^2 \rangle_{\rho_p}^{1/2}$$

$$\simeq 2 \frac{N-Z}{A} \left[ \langle r^2 \rangle_{\rho_n - \rho_p}^{1/2} - \langle r^2 \rangle_{\rho_n + \rho_p}^{1/2} \right]$$

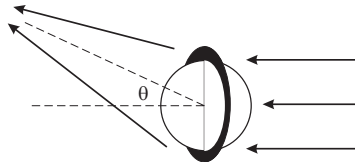
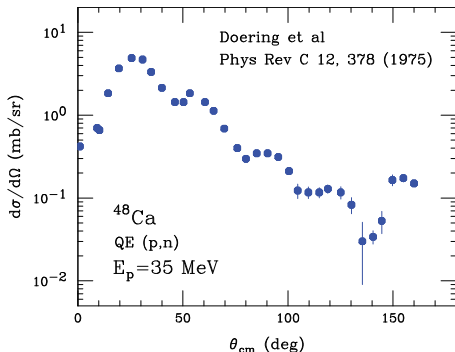
rms isovector aura

Estimated  $\Delta R \sim 3 \left( \langle r^2 \rangle_{\rho_n}^{1/2} - \langle r^2 \rangle_{\rho_p}^{1/2} \right)$  for  $^{48}\text{Ca}/^{208}\text{Pb}$ !

Even before consideration of Coulomb effects that further enhances difference!



# Direct Reaction Primer



DWBA:

$$\frac{d\sigma}{d\Omega} \propto \left| \int dr \psi_f^* U_1 \psi_i \right|^2$$

Lane Potential

$$U = U_0 + \frac{4\pi T}{A} U_1$$

$$U_0 \propto \rho \quad U_1 \propto \rho_n - \rho_p$$

It is common to assume the same geometry for  $U_0$  &  $U_1$ , implicitly  $\rho$  &  $\rho_a$ , e.g. Koning&Delaroche NPA713(03)231



# Cross-Section Sensitivity to Isovector Surface

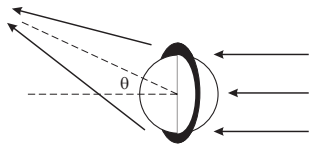
Koning-Delaroche

NPA713(03)231

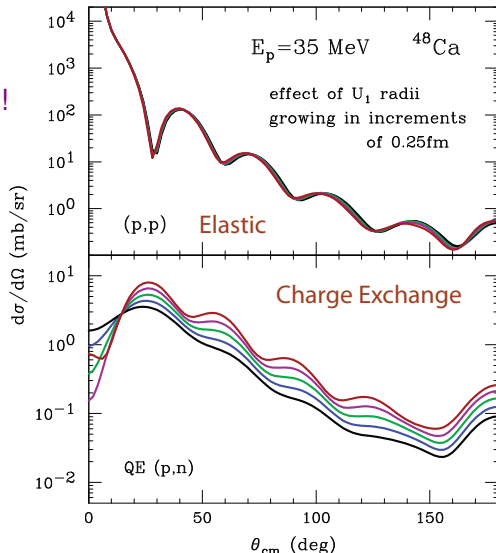
same radii  $R$  for  $U_0$  &  $U_1$ !

$$U_1(r) \propto \frac{U_{01}}{1 + \exp \frac{r-R}{a}}$$

$$R \rightarrow R + \Delta R_1$$

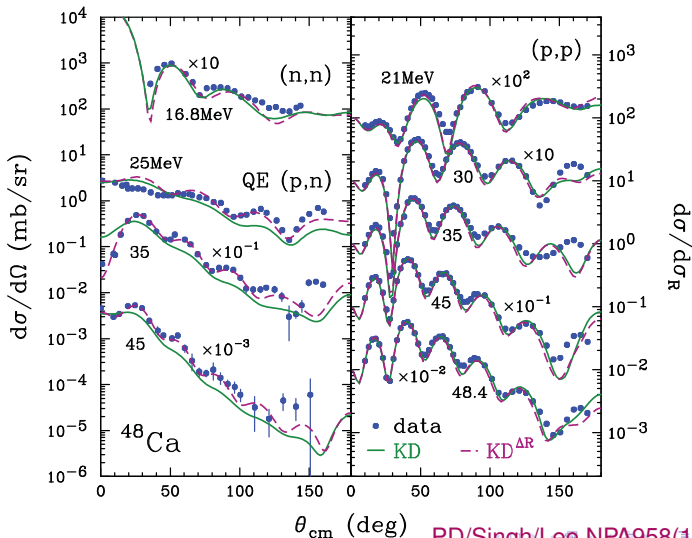


charge-exchange cs  
oscillations grow



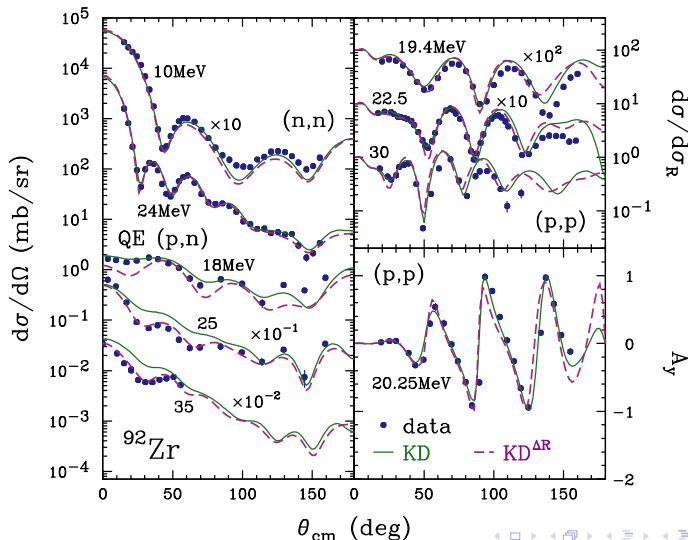
# Simultaneous Fits to Elastic & Charge-Change: $^{48}\text{Ca}$

Different radii for densities/potentials:  $R_a = R + \Delta R$



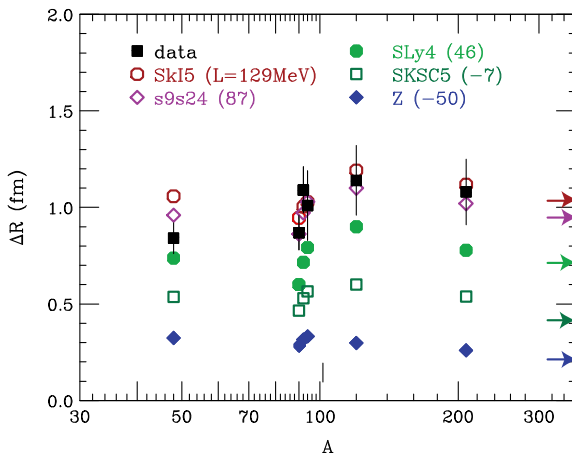
# Simultaneous Fits to Elastic & Charge-Change: $^{92}\text{Zr}$

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# Thickness of Isovector Aura

6 targets analyzed, differential cross section + analyzing power



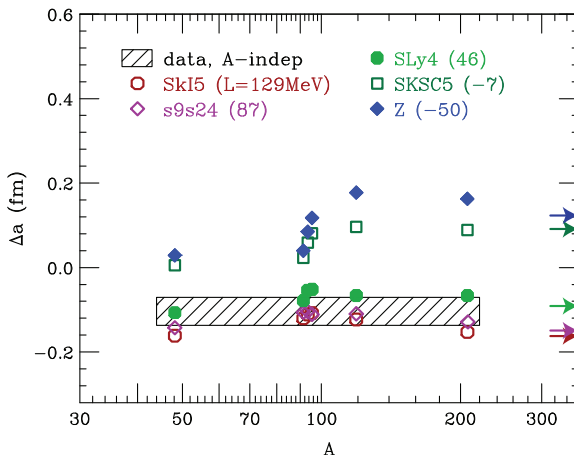
Colored: Skyrme predictions. Arrows: half-infinite matter

Thick  $\sim 0.9$  fm isovector aura!  $\sim$ Independent of  $A$ .





# Difference in Surface Diffuseness



Colored: Skyrme predictions. Arrows: half-infinite matter

Sharper isovector surface than isoscalar!



# Bayesian Inference

Probability density in parameter space  $p(x)$  updated as experimental data on observables  $E$ , value  $\bar{E}$  with error  $\sigma_E$ , get incorporated

Probability  $p$  is updated iteratively, starting with prior  $p_{\text{prior}}$   
 $p(a|b)$  - conditional probability

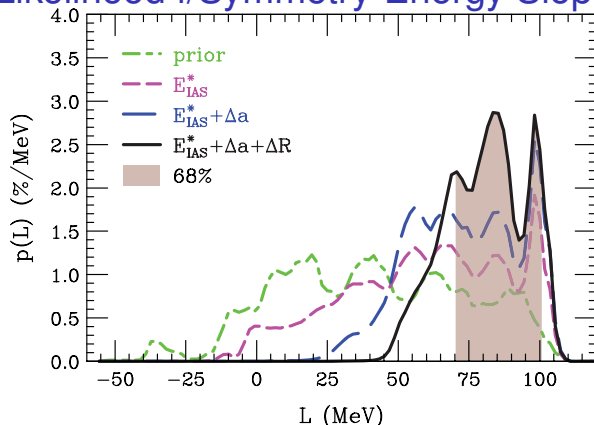
$$p(x|\bar{E}) \propto p_{\text{prior}}(x) \int dE e^{-\frac{(E-\bar{E})^2}{2\sigma_E^2}} p(E|x)$$

For large number of incorporated data,  $p$  becomes independent of  $p_{\text{prior}}$

In here,  $p_{\text{prior}}$  and  $p(E|x)$  are constructed from all Skyrme ints in literature, and their linear interpolations.  $p_{\text{prior}}$  is made uniform in plane of symmetry-energy parameters ( $L, S_0$ )



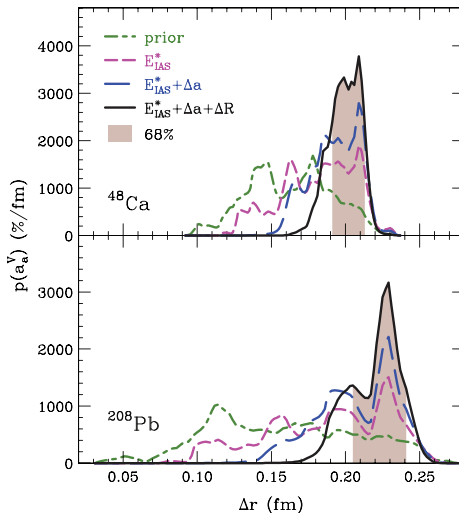
# Likelihood f/Symmetry-Energy Slope



$E_{IAS}^*$  - from excitations to isobaric analog states  
in PD&Lee NPA922(14)1

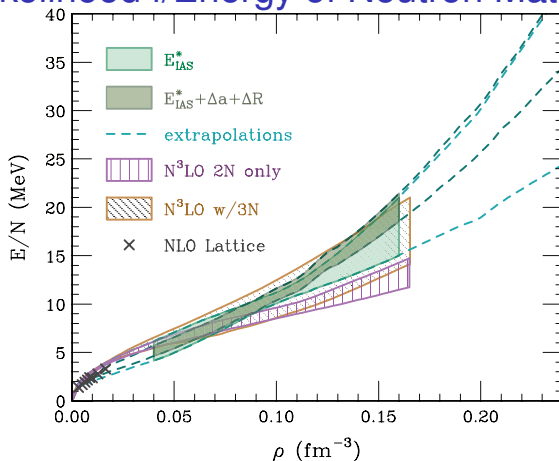
Oscillations in prior of no significance  
- represent availability of Skyrme parametrizations

# Likelihood $f$ /Neutron-Skin Values



Sizeable  $n$ -Skins

# Likelihood $f$ /Energy of Neutron Matter

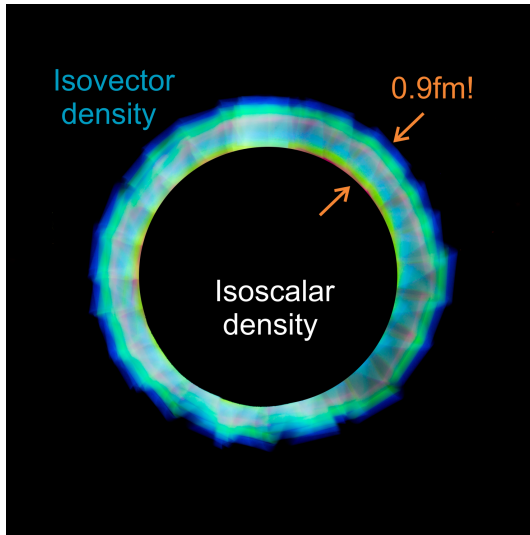


$E_{\text{IAS}}^*$  - from excitations to isobaric analog states  
in PD&Lee NPA922(14)1

Some oscillations due to prior



# Isovector Aura



## Conclusions

- Symmetry-energy polarizes nuclear densities, pushing isovector density out to region of low isoscalar density
- For large  $A$ , displacement of isovector relative to isoscalar surface is expected to be roughly independent of nucleus and depend on slope of symmetry energy
- Surface displacement can be studied in comparative analysis of data on elastic scattering and quasielastic charge-exchange reactions
- Such an analysis produces thick isovector aura  
 $\Delta R \sim 0.9 \text{ fm!}$
- Symmetry & neutron energies are stiff!  
 $L = (70 - 100) \text{ MeV}$ ,  $S(\rho_0) = (33.5 - 36.5) \text{ MeV}$  at 68% level
- Now: Novel error analysis

PD, Lee & Singh NPA818(09)36, 922(14)1, 958(17)147 + in progress  
DOE DE-SC0019209 + CUSTIPEN



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