EoS from terrestrial experiments: static and dynamic polarizations of nuclear density

XIAMEN-CUSTIPEN WORKSHOP ON
THE EOS OF DENSE NEUTRON-RICH MATTER
IN THE ERA OF GRAVITATIONAL WAVE ASTRONOMY
January 3-7, 2019, Xiamen, China

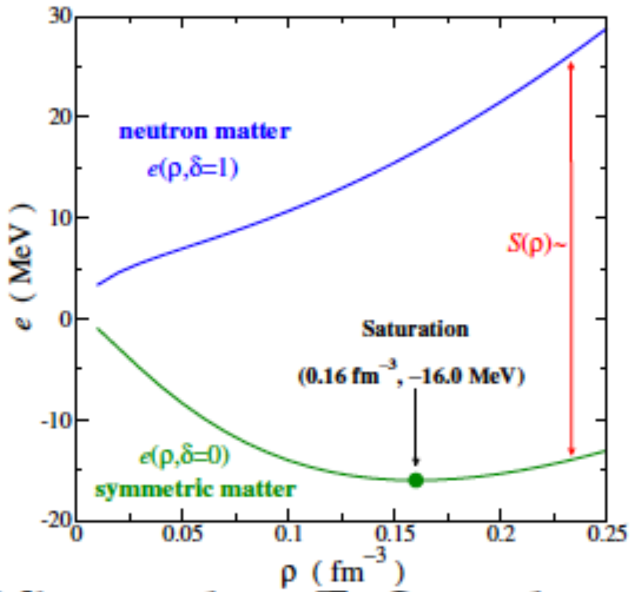
Hiroyuki Sagawa (RIKEN Nishina Center /University of Aizu)

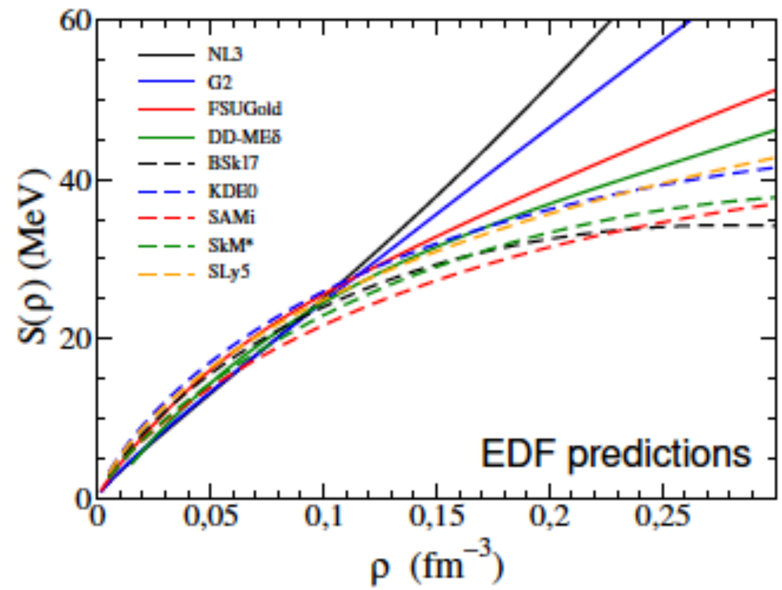
1. Incompressibility and ISGMR
2. IAS and CSB and CIB interactions
3. Proton scattering and nuclear density polarization

to Advance Knowledge for Humanity
THE UNIVERSITY OF AIZU
1993

RIKEN NiSHiNA CENTER

The Nuclear Equation of State: Infinite System





* The nuclear EoS can be written in good approximation as:

E/A= $e(\rho,\beta) \approx e(\rho,\beta=0) + S(\rho)\beta^2$ where $\beta \equiv \dfrac{\rho_n - \rho_p}{\rho_n + \rho_p}$

* SNM can be expanded around ρ_0 and define some useful

$$S(\rho) = J + L\left(\frac{\rho-\rho_0}{3\rho_0}\right) + \frac{1}{2}K_{sym}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2$$

$$\text{where } J{=}S(\rho_0),\; L{=}3\rho_0 \left.\frac{\partial S}{\partial \rho}\right|_{\rho_0},\; K_{sym} = 9\rho_0^2 \left.\frac{\partial^2 S}{\partial \rho^2}\right|_{\rho_0}$$

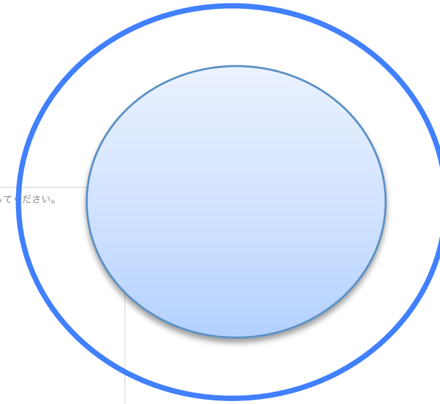
イメージを表示できません。メモリ不足のためにイメージを開くことができないか、イメージが破損している可能性があります。コンピューターを再起動して再度ファイルを開いてください。それでも赤い x が表示される場合は、イメージを削除して挿入してください。

Nuclear Matter EOS → Supernova Explosion

イメージを表示できません。メモリ不足のためにイメージを開くことができないか、イメージが破損している可能性があります。コンピューターを再起動して再度ファイルを開いてください。それでも赤い x が表示される場合は、イメージを削除して挿入してください。

Isoscalar Giant Monopole Resonances

Isoscalar Compressional Dipole Resonances



Incompressibility K

$$E_{ISGMR} = \sqrt{\frac{\hbar^2 K_A}{m < r^2 >_m}},$$

$$K_A = K_\infty + K_{surf} A^{-1/3} + K_\tau \delta^2 + K_{Coul} \frac{Z^2}{A^{4/3}},$$

Self consistent HF+RPA calculations

Self consistent RMF+RPA calculations

(α,α') experimtent

The nuclear incompressibility from ISGMR

We can give credit to the idea that the link should be provided microscopically through the Energy Functional E[ρ].

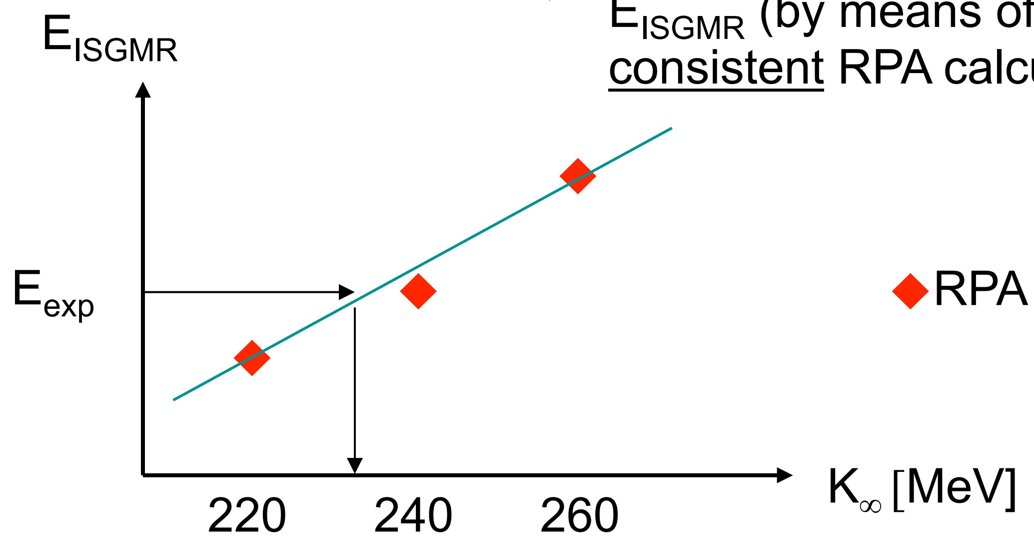
IT PROVIDES AT THE SAME TIME

- K_∞ in nuclear matter (analytic)
- E_{ISGMR} (by means of <u>self-consistent</u> RPA calculations)

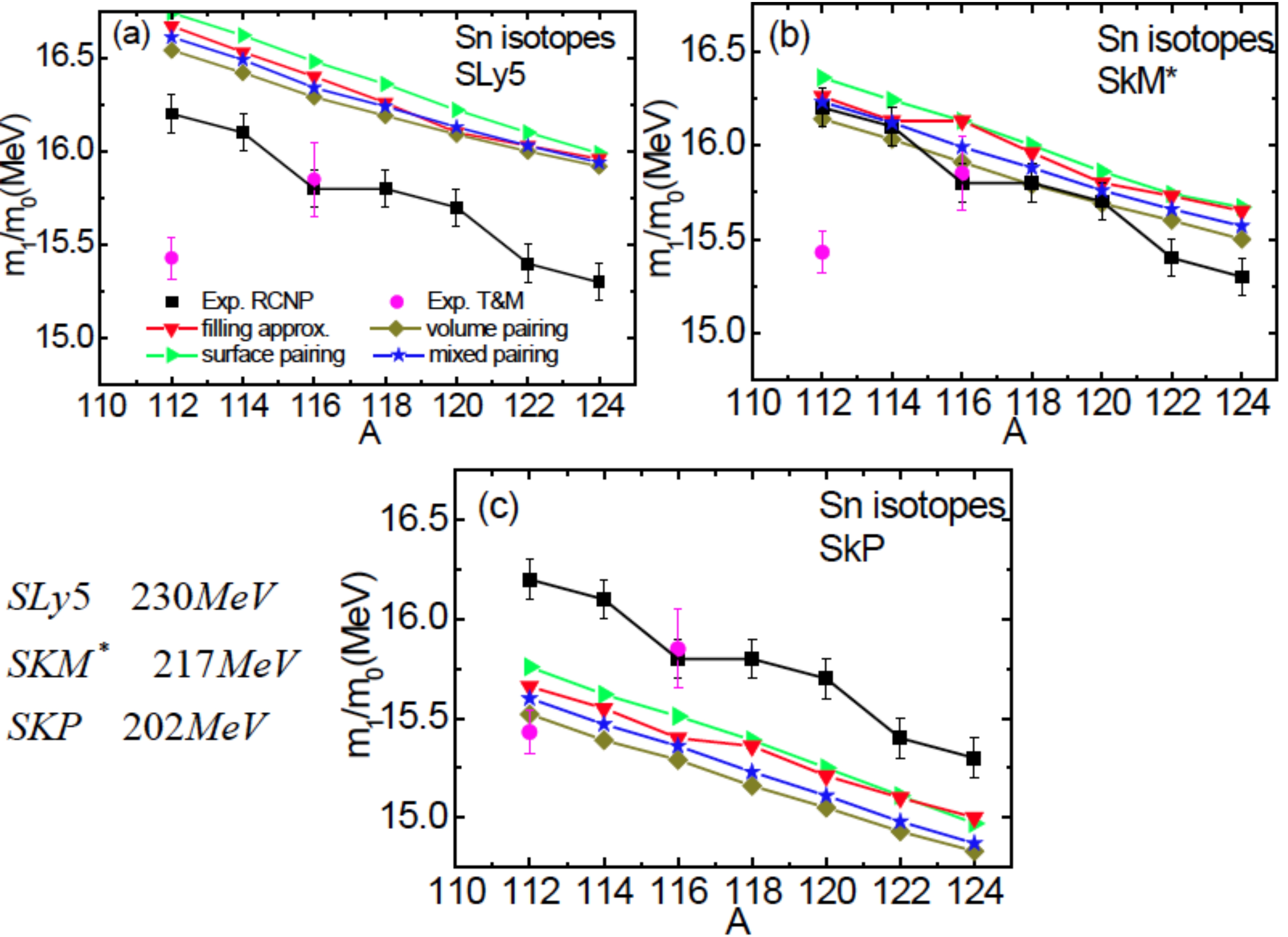
Skyrme

Gogny

RMF



Extracted value of K_∞


(a)
Sn isotopes
SLy5
(b)
Sn isotopes
SkM*
(c)
Sn isotopes
SkP
Exp. RCNP
Exp. T&M
filling approx.
volume pairing
surface pairing
mixed pairing
m1/m0(MeV)
A
SLy5 230MeV
SKM* 217MeV
SKP 202MeV

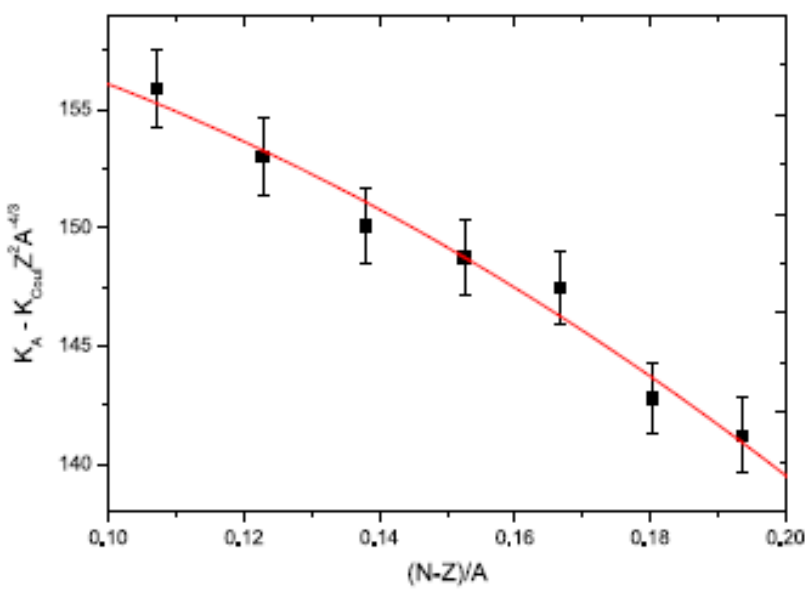
Extracting K_τ from data

$$K_A = K_\infty + K_{surf} A^{-1/3} + K_\tau \delta^2 + K_{Coul} \frac{Z^2}{A^{4/3}}$$

Using this formula globally is dangerous and should not be done (cf. M. Pearson, S. Shlomo and D. Youngblood) but one can use it locally.

K_{Coul} can be calculated and ETF calculations point to $K_{surf} \approx -K_\infty$.

$$K_A - K_{Coul} \frac{Z^2}{A^{4/3}} = K_\infty (1 - A^{-1/3}) + K_\tau \delta^2$$



TERS week ending 19 OCTOBER 2007

K_τ = -500 ± 50 MeV

in the Even-A $^{112-124}$Sn Isotopes

ncompressibility

Correlation between Isospin GMR and nuclear matter properties

1.Nuclear incompressibility K is determined empirically with the ISGMR in ^{208}Pb to be

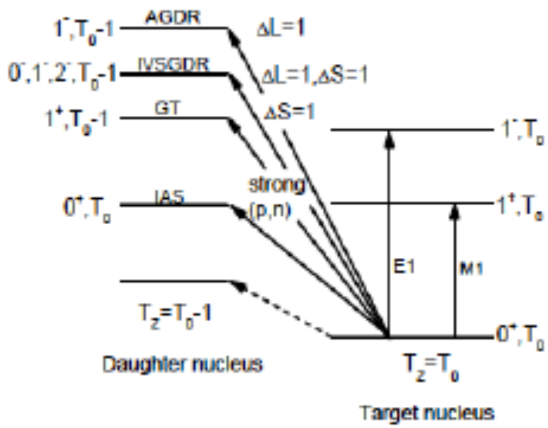
K~230MeV(Skyrme,Gogny), K~250MeV(RMF). K=(240 +/-10 +/- 10)MeV

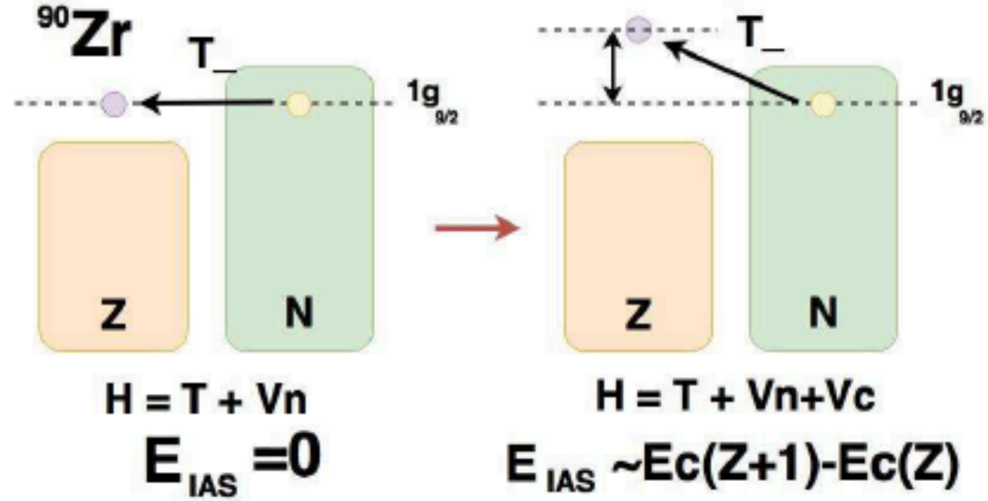
2. Combining ISGMR data of Sn and Cd isotopes(RCNP) K=(225 +/-10)MeV

3. $K_\tau = -(500 \pm 50)\text{MeV}$ is extracted from isotope dependence of ISGMR.

$$K_\tau = Ksym + 3L - \frac{27 L \rho_{nm}^2}{K} \frac{d^3 H_{nm}}{d\rho^3}\bigg|_{\rho=\rho_{nm}}$$

The isobaric analog state energy: ΔE_d





- **Definition:** $(N, Z+1) \to (N+1, Z)$: T_0 g.s. isospin of $(N+1, Z)$, its IAS in $(N, Z+1)$ will be the lowest state where $T = T_0$.
- **Analog state** can be defined: $|A\rangle = \dfrac{T_-|0\rangle}{\langle 0|T_+T_-|0\rangle}$
- **Displacement energy**

$$E_{IAS} \approx \Delta E_d \equiv E_A - E_0 = \langle A|\mathcal{H}|A\rangle - \langle 0|\mathcal{H}|0\rangle = \frac{\langle 0|[T_+[\mathcal{H}, T_-]]|0\rangle}{\langle 0|T_+T_-|0\rangle}$$

E^{exp}_{IAS} easy to measure and depends only on isospin symmetry symmetry breaking terms: Coulomb and to less extent (few %) strong interaction

Coulomb direct displacement energy

$$\left\langle \left[T_+ , [H, T_-] \right] \right\rangle \Rightarrow$$

$$\Delta E_d \approx \Delta E_d^{C,\mathrm{direct}} = \frac{1}{N-Z} \int \left[\rho_n(\vec{r}) - \rho_p(\vec{r}) \right] U_C^{\mathrm{direct}}(\vec{r}) d\vec{r}$$

where $U_C^{\mathrm{direct}}(\vec{r}) = \int \frac{e^2}{|\vec{r}_1 - \vec{r}|} \rho_{ch}(\vec{r}_1) d\vec{r}_1$

Assuming a uniform neutron and proton distributions of radius R_n and R_p respectively, and $\rho_{ch} \approx \rho_p$ one can find

$$\Delta E_d \approx \Delta E_d^{C,\mathrm{direct}} \approx \frac{6}{5} \frac{Ze^2}{R_p} \left(1 - \frac{1}{2} \frac{N}{N-Z} \frac{R_n - R_p}{R_p} \right)$$

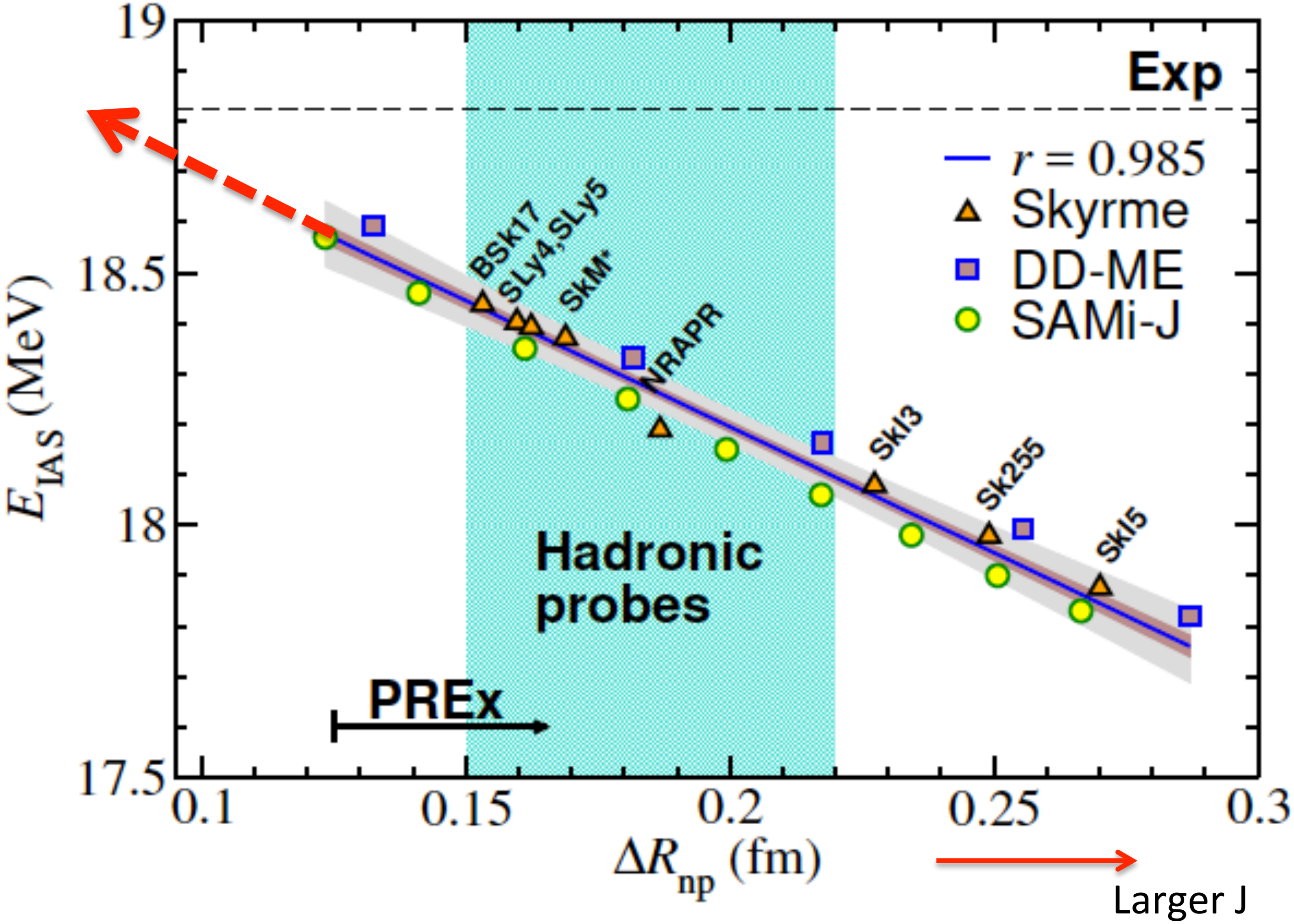
One may expect: **the larger the Δr_{np} the smallest E_{IAS}**

COULOMB ENERGIES AND THE EXCESS NEUTRON DISTRIBUTION FROM THE STUDY OF ISOBARIC ANALOG RESONANCES†

Naftali Auerbach, Jörg Hüfner, A. K. Kerman, and C. M. Shakin

	$\|\pi\rangle$ Parent State	Ca^{49}	Sr^{89}	Ba^{139}	Pb^{209}
E_R-E_A	Contin.-Comp. Mixing	-0.06	-0.10	-0.17	-0.48
	Dyn. p-n Mass Effect	0.04	0.04	0.04	0.04
	El.Magn. Spin Orbit	-0.07	-0.08	-0.01	-0.02
$\Delta E_d^{C.D.}$	Estimate Eq. (5)	-0.20	-0.16	-0.23	-0.25
	Phenomen. Force	-0.02	-0.16	—	—
ΔE_d^{Coul}	Direct Term	7.60	12.10	15.46	19.95
	Exchange Term	-0.31	-0.35	-0.35	-0.35
$\Delta E_d^{F.S.}$	Finite Proton Size	-0.10	-0.11	-0.11	-0.11
ΔE_d^{CORR}	Short Range Correlat.	~0.1	~0.1	~0.1	~0.1
ΔE_d^{T-IMP}	Collective Model	-0.01	-0.04	-0.06	-0.09
E_R-E_π	Theory	7.08±.20	11.40±.25	14.67±.25	18.79±.25
	Experiment	7.083±.015 (a)	11.40±.02 (a)	14.67±.02 (a)	18.790±.013 (b)
c_o [fm]	Charge Distribution	1.03	1.08	1.09	1.12
t [fm]	Charge Distribution	2.3	2.3	2.3	2.2
r_o [fm]	Neutron Potential	1.06±.08	1.10±.05	1.11±.05	1.12±.04
R_{rms} [fm]	Excess Neutrons	3.71±.18	4.36±.15	4.99±.15	5.63±.15
	Protons	3.42	4.10	4.75	5.42
	All Neutrons	3.51±.04	4.17±.05	4.83±.05	5.50±.05

E_{IAS} in Energy Density Functionals (No Corr.)



EDFs derived from Hartree-(Fock) + Random Phase approximations using relativistic (and non-relativistic) interactions where the nuclear part is isospin symmetric and U_{ch} is calculated from the ρ_p

How can we reconcile this contradiction between IAS energy and neutron skin?

Xavi Roca-Maza, Gianluca Colo and HS

PHYSICAL REVIEW LETTERS **120**, 202501 (2018)

Isospin proposed by J. Heisenberg

Isospin conservation $[H,T]=0$

$$[H,T]=[V_C+V_{CSB}+V_{CIB},T]\neq 0$$

Scattering Length

$$a^{pp}_{(S=0)} = -17.3\pm 0.4\text{fm},$$
$$a^{nn}_{(S=0)} = -18.7\pm 0.6\text{fm},$$
$$a^{pn}_{(S=0)} = -23.70\pm 0.03\text{fm}.$$

The difference between a_0^{pp} and a_0^{nn} is an evidence of CSB (charge symmetry breaking) nuclear force, while the difference between a_0^{pn} and the average $(a_0^{pp}+a_0^{nn})/2$ is due to CIB (charge invariance breaking) force.

Proton=(uud) m_uc^2~2.3MeV
Neutron=(udd) m_dc^2~4.8MeV
QCD dynamics of strong interaction
Explicit Chiral symmetry breaking

- **Isospin symmetry breaking** (Skyrme-like): **two parts**

H. Sagawa, N. V. Giai, and T. Suzuki, Phys. Lett. B **353**, 7 (1995).

charge symmetry breaking + **charge independence breaking***

$$V_{CSB} = V_{nn} - V_{pp}$$

$$V_{CIB} = \frac{1}{2}(V_{nn} + V_{pp}) - V_{pn}$$

$$V_{CSB}(\vec{r}_1, \vec{r}_2) \equiv \frac{1}{4}[\tau_z(1) + \tau_z(2)]\Big\{s_0(1 + y_0 P_\sigma)\,\delta(\vec{r}_1 - \vec{r}_2) + \frac{1}{2}s_1(1 + y_1 P_\sigma)\left[P'^2\delta(\vec{r}_1 - \vec{r}_2) + \delta(\vec{r}_1 - \vec{r}_2)P^2\right] + s_2(1 + y_2 P_\sigma)\vec{P}' \cdot \delta(\vec{r}_1 - \vec{r}_2)\vec{P}\Big\}$$

where $\vec{P} \equiv \frac{1}{2\imath}(\vec{\nabla}_1 - \vec{\nabla}_2)$ acts on the right and P' is its complex conjugate acting on the left and $P_{\tau/\sigma}$ are the usual projector operators in isospin and spin spaces.

$$V_{CIB}(\vec{r}_1, \vec{r}_2) \equiv \frac{1}{2}\tau_z(1)\tau_z(2)\Big\{u_0(1 + z_0 P_\sigma)\,\delta(\vec{r}_1 - \vec{r}_2) + \frac{1}{2}u_1(1 + z_1 P_\sigma)\left[P'^2\delta(\vec{r}_1 - \vec{r}_2) + \delta(\vec{r}_1 - \vec{r}_2)P^2\right] + u_2(1 + z_2 P_\sigma)\vec{P}' \cdot \delta(\vec{r}_1 - \vec{r}_2)\vec{P}\Big\}$$

* general operator form $\tau_z(1)\tau_z(2) - \frac{1}{3}\vec{\tau}(1)\cdot\vec{\tau}(2)$. Our prescription $\tau_z(1)\tau_z(2)$ not change structure of HF+RPA.

- Opposite to the other corrections, **ISB contributions depends on new parameters that need to be fitted!**

Isospin symmetry breaking in the medium:

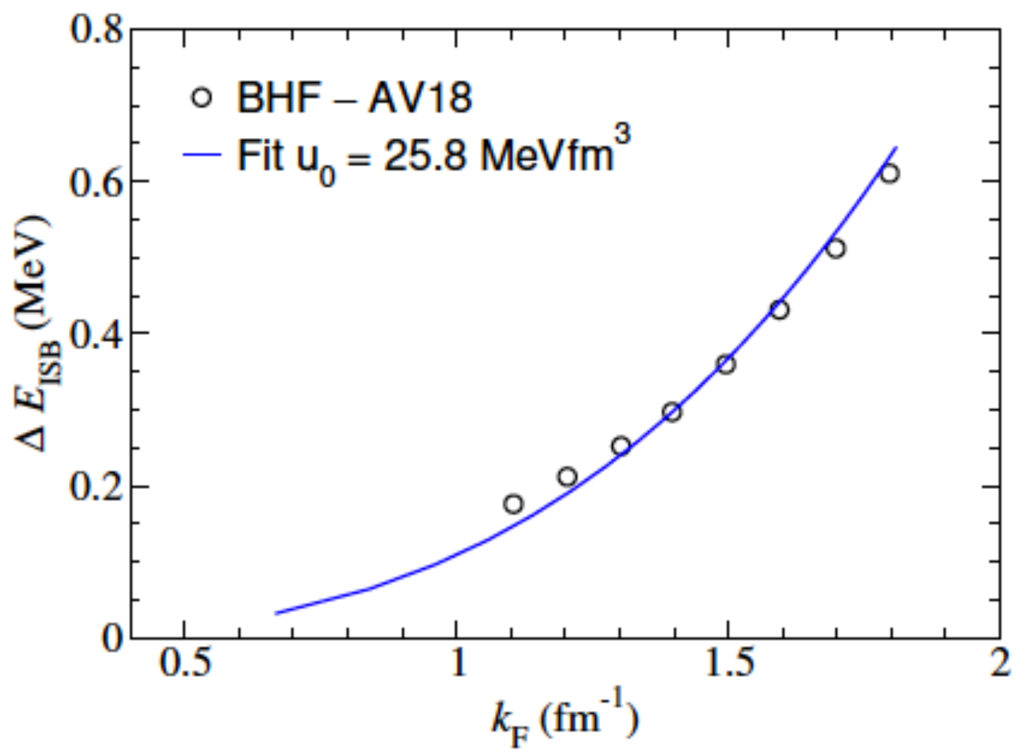
- **keeping** things **simple**: **CSB and CIB** interaction just **delta function** depending on s_0 **and** u_0. **Different possibilities:**

→ **Fitting** to (two) experimentally known **IAS energies**

→ **Derive from theory**

→ **our option:** u_0 **to reproduce BHF** (symmetric nuclear matter) and s_0 **to reproduce** E_{IAS} **in** ^{208}**Pb**

$$\Delta E_{ISB} \propto u_0 \rho^2$$



Physics Letters B 445, 259 (1999)

SAMi-ISB finite nuclei properties

SAMI is refitted with the protocol

El.	N	B [MeV]	B^{exp} [MeV]	r_c [fm]	r_c^{exp} [fm]	ΔR_{np} [fm]
Ca	28	417.67	415.99	3.49	3.47	0.214
Zr	50	783.60	783.89	4.26	4.27	0.097
Sn	82	1102.75	1102.85	4.73	–	0.217
Pb	126	1635.78	1636.43	5.50	5.50	0.151

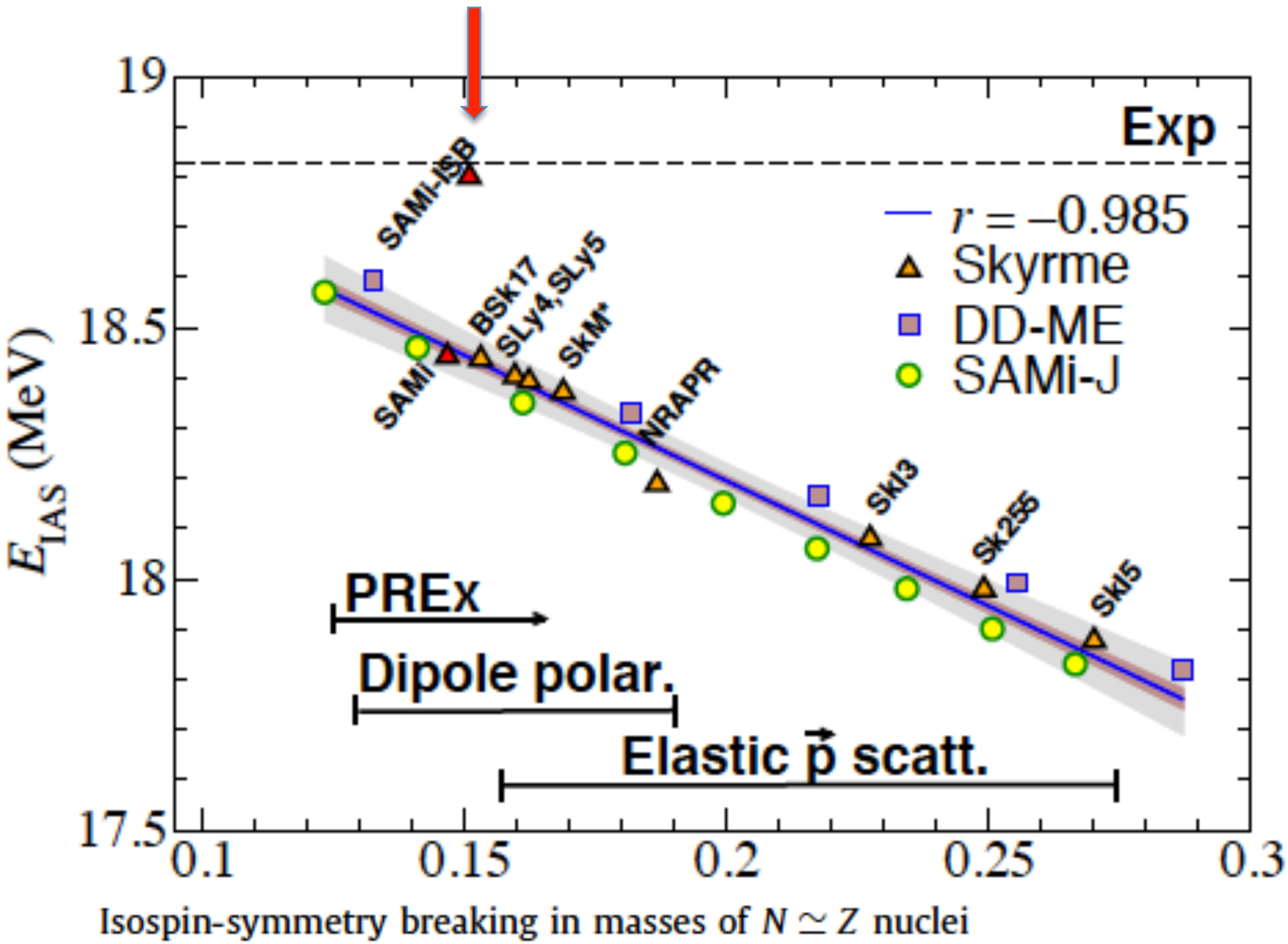
Corrections on E_{IAS} for ^{208}Pb one by one

	E_{IAS} [MeV]	Correction [keV]
No corrections[a]	18.31	
Exact Coulomb exchange	18.41	+100
n/p mass difference	18.44	+30
Electromagnetic spin-orbit	18.45	+10
Finite size effects	18.40	-50
Vacuum polarization (V_{ch})	18.53	+130
Isospin symmetry breaking	18.80	+270

[a] From Skyrme Hamiltonian where the nuclear part is isospin symmetric and V_{ch} is calculated from the ρ_p

$E_{IAS}^{exp} = 18.826 \pm 0.01$ MeV. *Nuclear Data Sheets 108, 1583 (2007).*

E_{IAS} with SAMi-ISB



P. Bączyk [a,*], J. Dobaczewski [a,b,c,d], M. Konieczka [a], W. Satuła [a,d], T. Nakatsukasa [e], K. Sato [f]

Physics Letters B 778 (2018) 178–183

Direct determination of the neutron skin thicknesses in ^{40,48}Ca from proton elastic scattering at E_p = 295 MeV

J. Zenihiro,[1,*] H. Sakaguchi,[2] S. Terashima,[3] T. Uesaka,[1] G. Hagen,[4,5] M. Itoh,[6] T. Murakami,[7] Y. Nakatsugawa,[8] T. Ohnishi,[1] H. Sagawa,[1,9] H. Takeda,[1] M. Uchida,[10] H.P. Yoshida,[2] S. Yoshida,[11] and M. Yosoi[2]

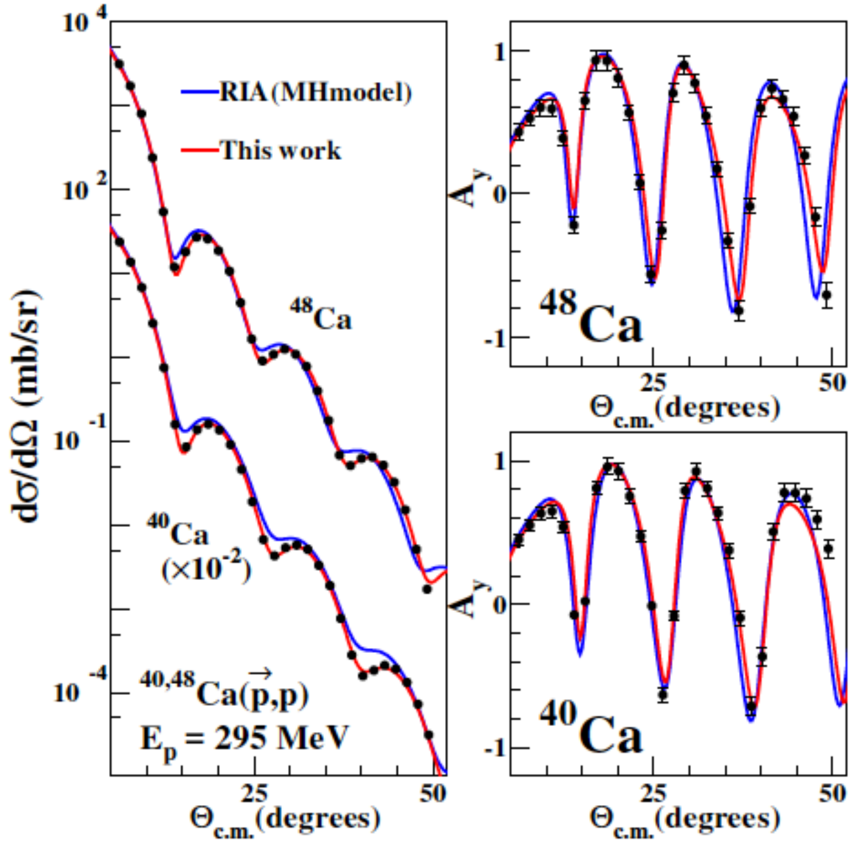


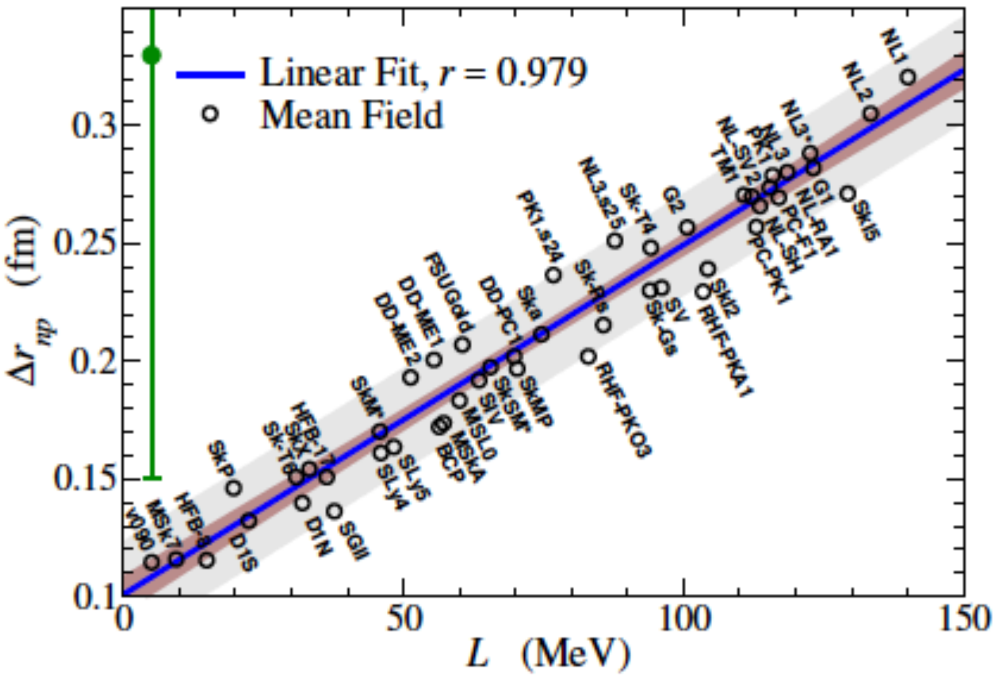
FIG. 1. Differential cross sections and analyzing powers for polarized proton elastic scattering from ^{40,48}Ca at 295 MeV. Blue and red lines are from the MH model and the result of the best-fit in this analysis, respectively.

To be published

Examples: EoS parameters from nuclear observables

Isovector properties (e.g. $S(\rho)$) are thought to be well determined by the **neutron skin thickness** ($\Delta r_{np} \equiv \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2}$) **of a heavy nucleus such as ^{208}Pb)**:

Macroscopic model: $\Delta r_{np} \sim \frac{1}{12}\frac{(N-Z)}{A}\frac{R}{J}L \quad (L \propto p_0^{neut})$



Micorscopic models (EDFs) confirm such a relation

However the experimental precision and accuracy needed in the measurment of this property is very challenging nowadays.

Physical Review Letters **106**, 252501 (2011)

[Exp. from strongly interacting probes: $\sim 0.15 - 0.22$ fm (*Physical Review C* **86** 015803 (2012))].

Neutron density

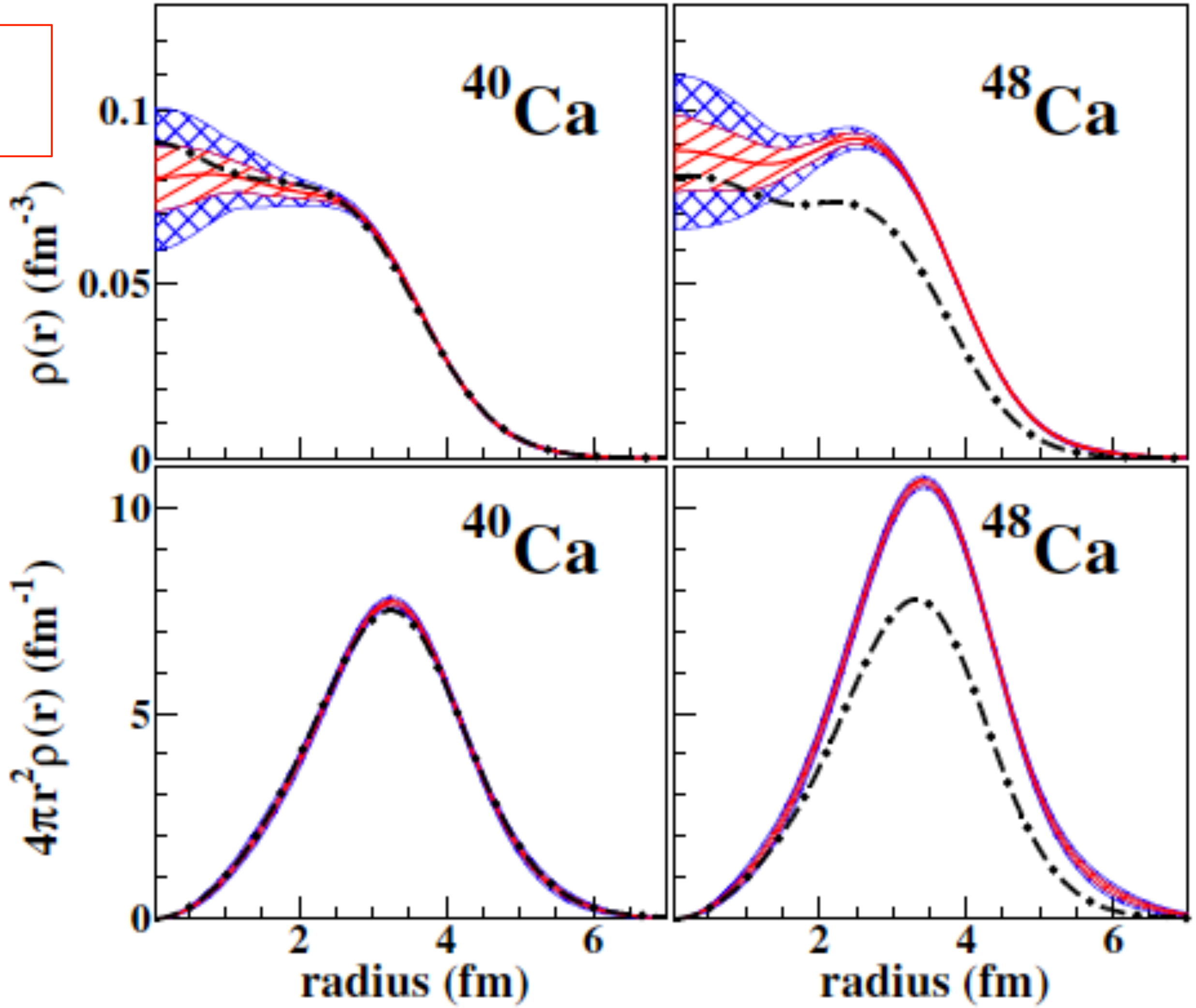
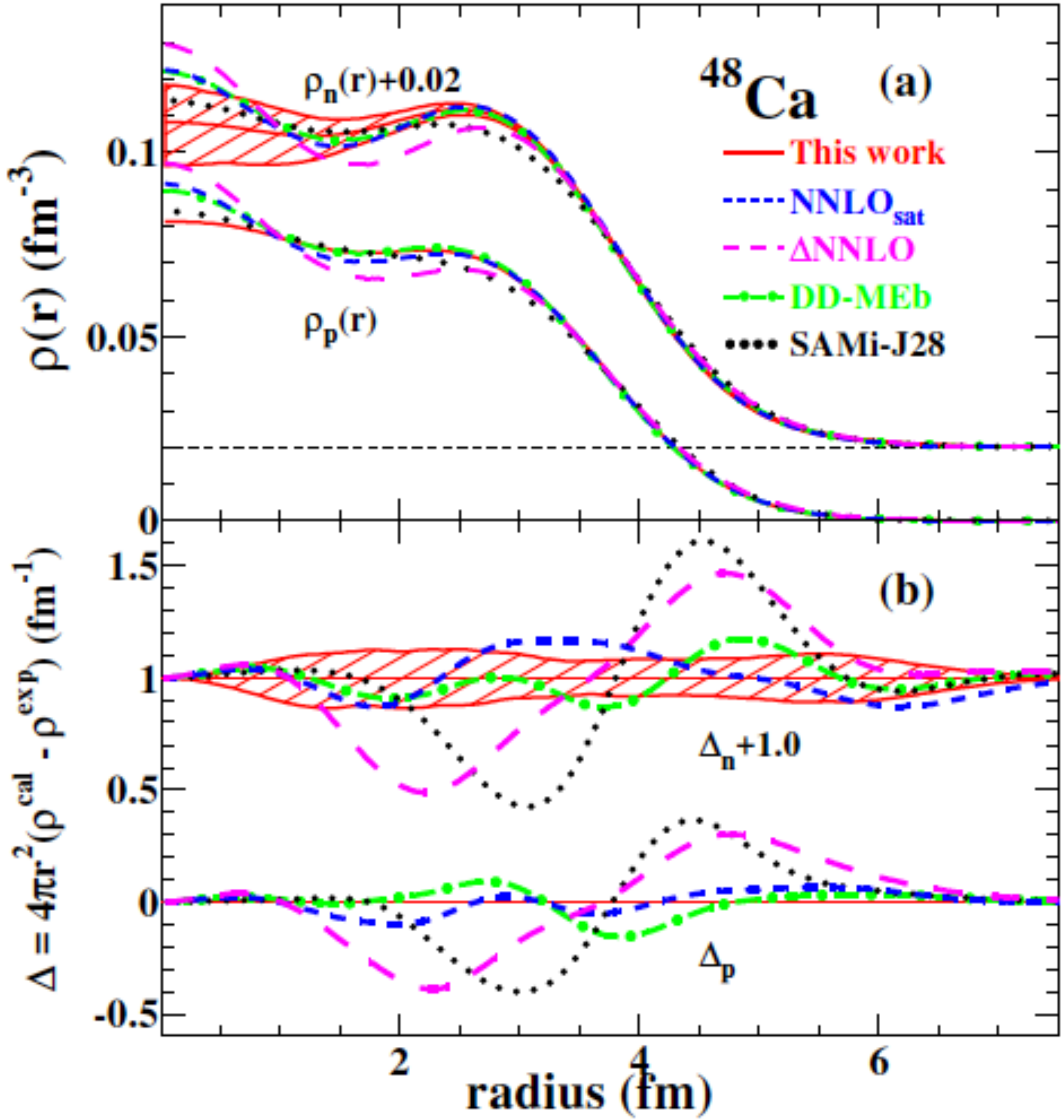
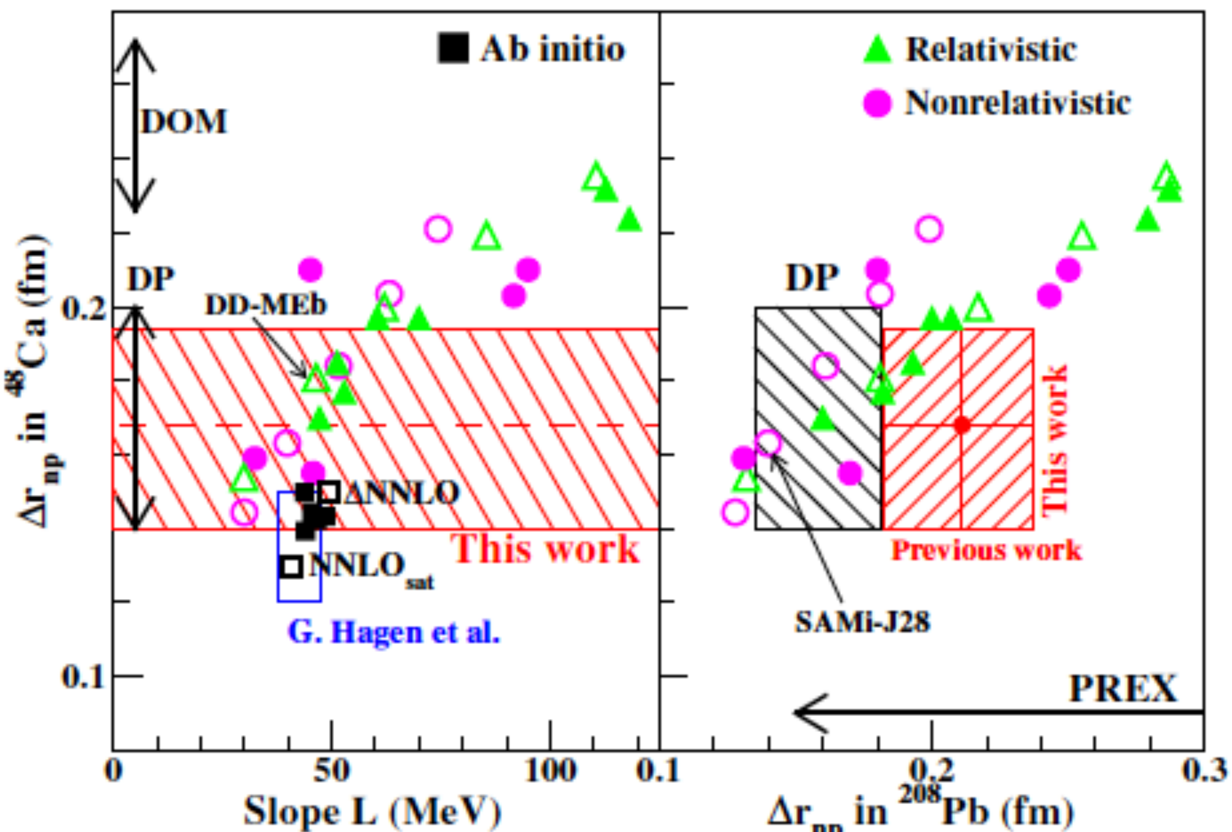


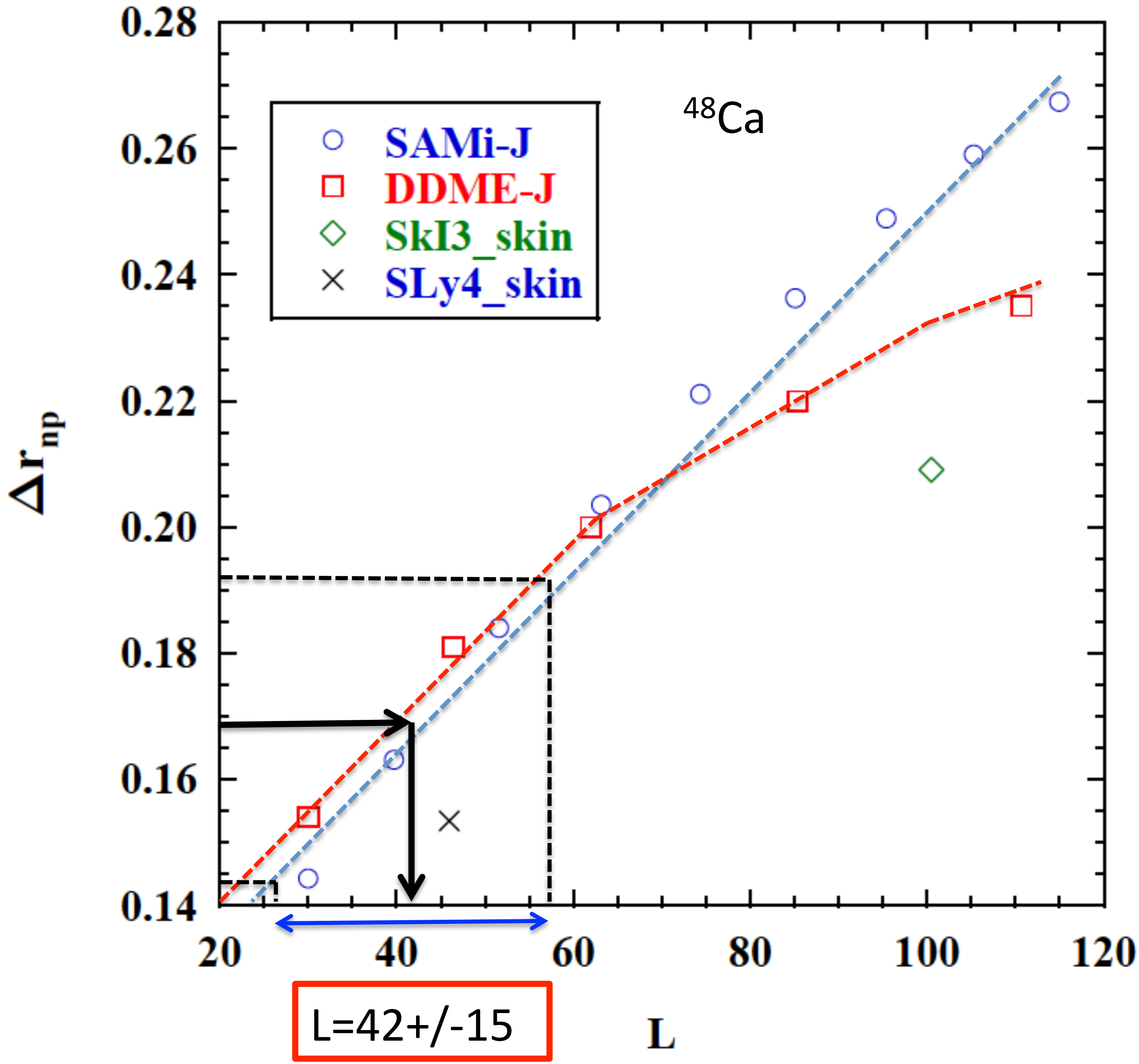
FIG. 2. Extracted ρ_n with error-envelopes of ^{40,48}Ca (red hatched and blue cross-hatched) and ρ_p derived from ρ_{ch} (black dash-dotted). Upper panels are the density distribution ($\rho(r)$), and the lower panels are the nucleon number distribution ($4\pi r^2 \rho(r)$).



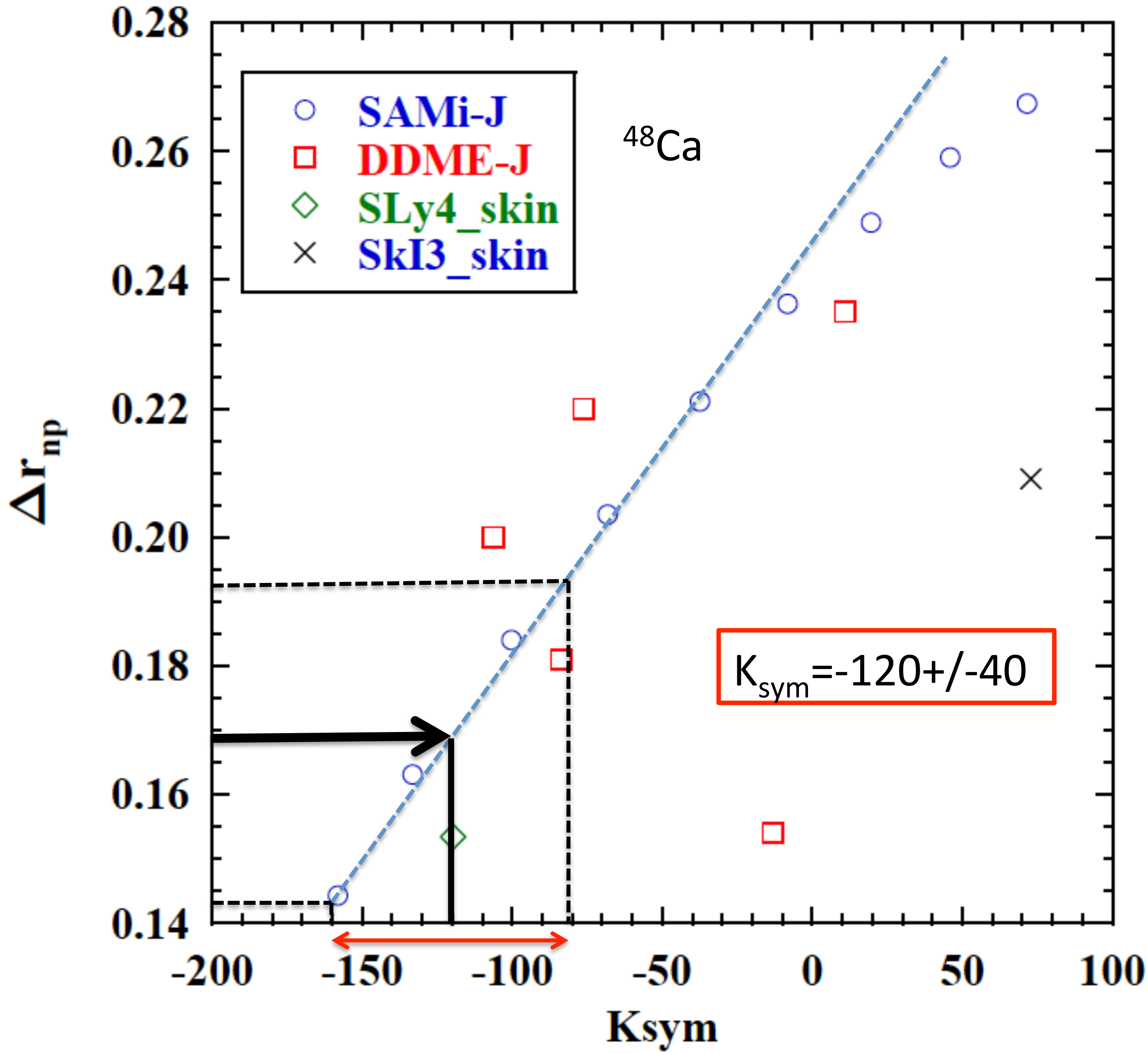
48Ca
(a)
This work
NNLOsat
ΔNNLO
DD-MEb
SAMi-J28
ρn(r)+0.02
ρp(r)
ρ(r) (fm-3)
0.1
0.05
0
(b)
Δn+1.0
Δp
Δ = 4πr2(ρcal - ρexp) (fm-1)
1.5
1
0.5
0
-0.5
2
4
6
radius (fm)

		r_{ch}	r_p	r_n	Δr_{np}	δ^{exp}	$\delta^{\mathrm{exp+mdl}}$
^{40}Ca	This work	3.480	3.385	3.375	−0.010	$^{+0.022}_{-0.023}$	$^{+0.049}_{-0.048}$
^{48}Ca	This work	3.460	3.387	3.555	0.168	$^{+0.025}_{-0.028}$	$^{+0.052}_{-0.055}$
	DD-MEb	-	3.39	3.57	0.18	-	-
	SAMi-J28	-	3.44	3.60	0.16	-	-
	NNLO$_{\mathrm{sat}}$	-	3.41	3.54	0.13	-	-
	ΔNNLO	-	3.47	3.62	0.15	-	-

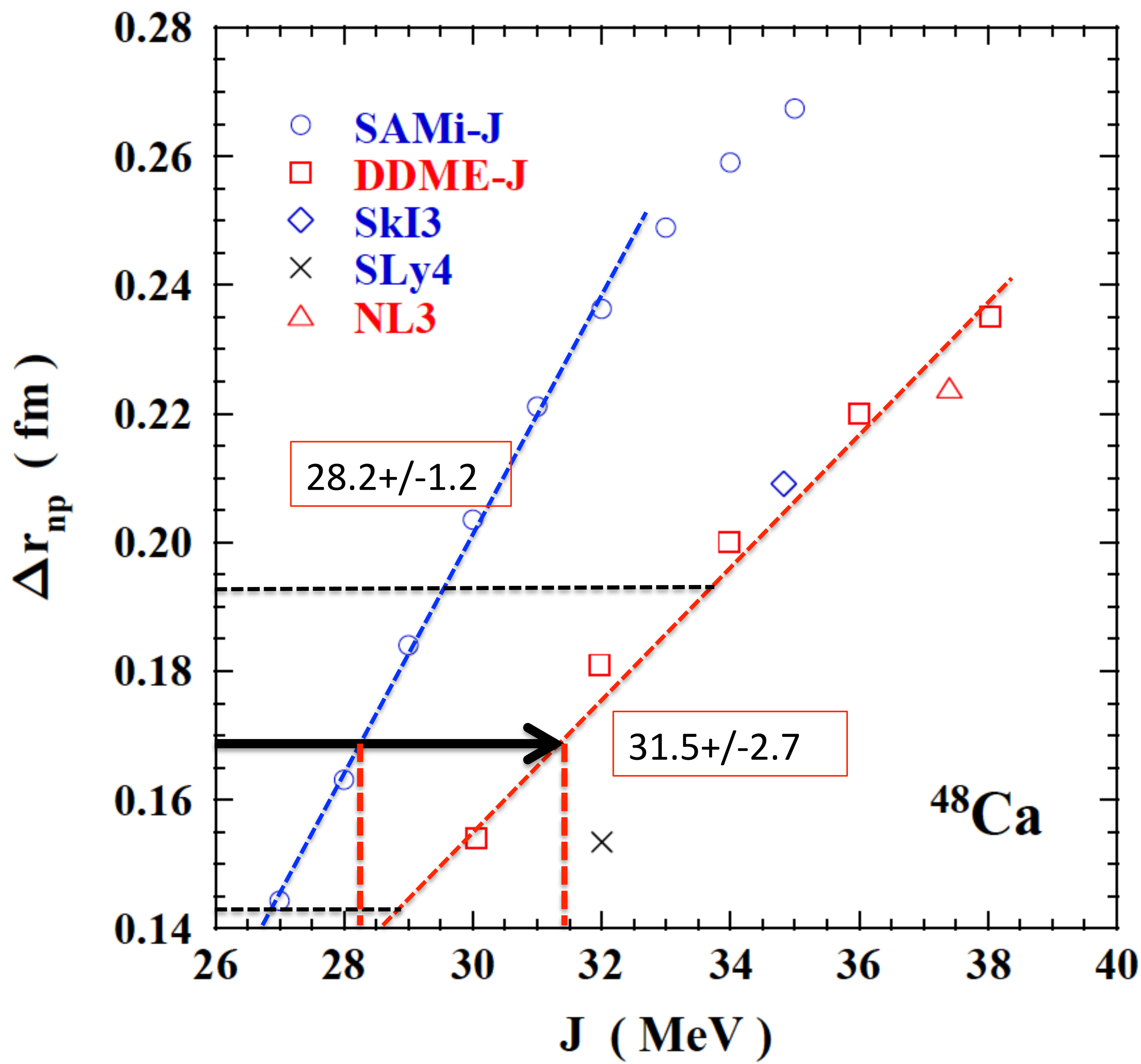




0.28
0.26
0.24
0.22
0.20
0.18
0.16
0.14
Δr_{np}
SAMi-J
DDME-J
SkI3_skin
SLy4_skin
^{48}Ca
20
40
60
80
100
120
L
L=42+/-15

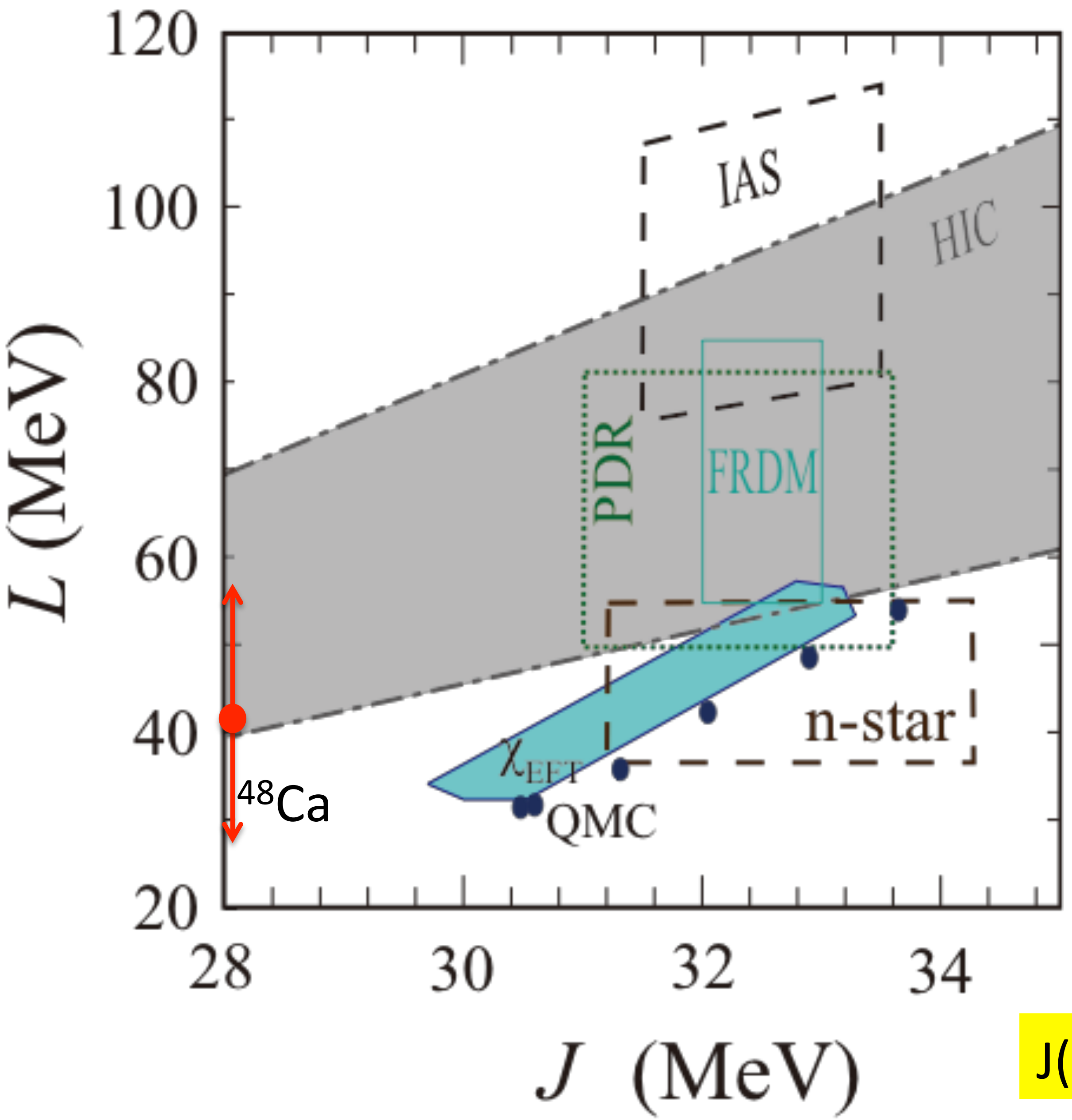


SAMi-J
DDME-J
SLy4_skin
SkI3_skin
48Ca
Ksym=-120+/-40
Δr np
Ksym
0.28
0.26
0.24
0.22
0.20
0.18
0.16
0.14
-200
-150
-100
-50
0
50
100



0.28
0.26
0.24
0.22
0.20
0.18
0.16
0.14
Δr_np (fm)
26
28
30
32
34
36
38
40
J (MeV)
SAMi-J
DDME-J
SkI3
SLy4
NL3
28.2+/-1.2
31.5+/-2.7
48Ca

Constraints on J and L



Tsang PRC2012

HIC: Heavy Ion Collision Analysis
Tsang PRL2009

IAS: Isobaric Analog State Energy
Danielewicz&Lee NPA2009

PDR: Pygmy Dipole Resonance in
^{132}Sn, ^{68}Ni, Carbone PRC2010

FRDM: Finite Range Droplet Model
Moeller PRL2012

n-star: Quiescent Low-Mass X-ray
Binaries, Stainer PRL2012

χ_{EFT}: Chiral Effective Field Theory,
Tews PRL2013

QMC: Quantum Monte-Carlo Calc.
Gandolfi, EPJA50, 10(2014).

J(FRDM)=32.5/-0.5

SUMMARY

1. Neutron skin of ^{48}Ca is determined by the polarized proton elastic scattering to be

   delta r_{np}=0.169 +/- 0.025 fm

2. This value constrains the symmetry energy coefficients

   L=42 +/- 15
   K_{sym}= -120 +/-40