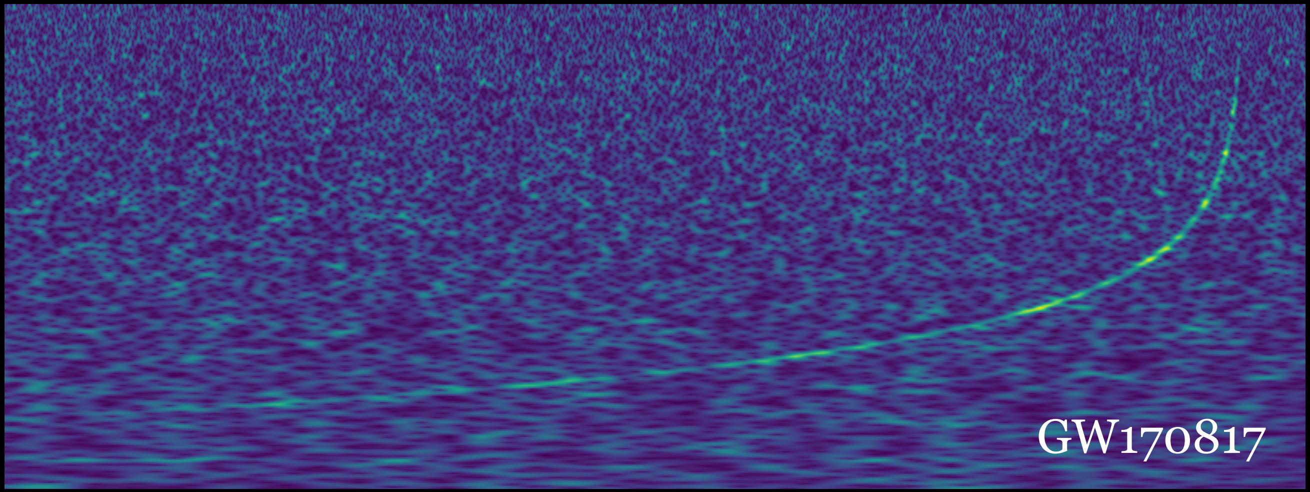
Using gravitational waves to constrain matter at extreme densities



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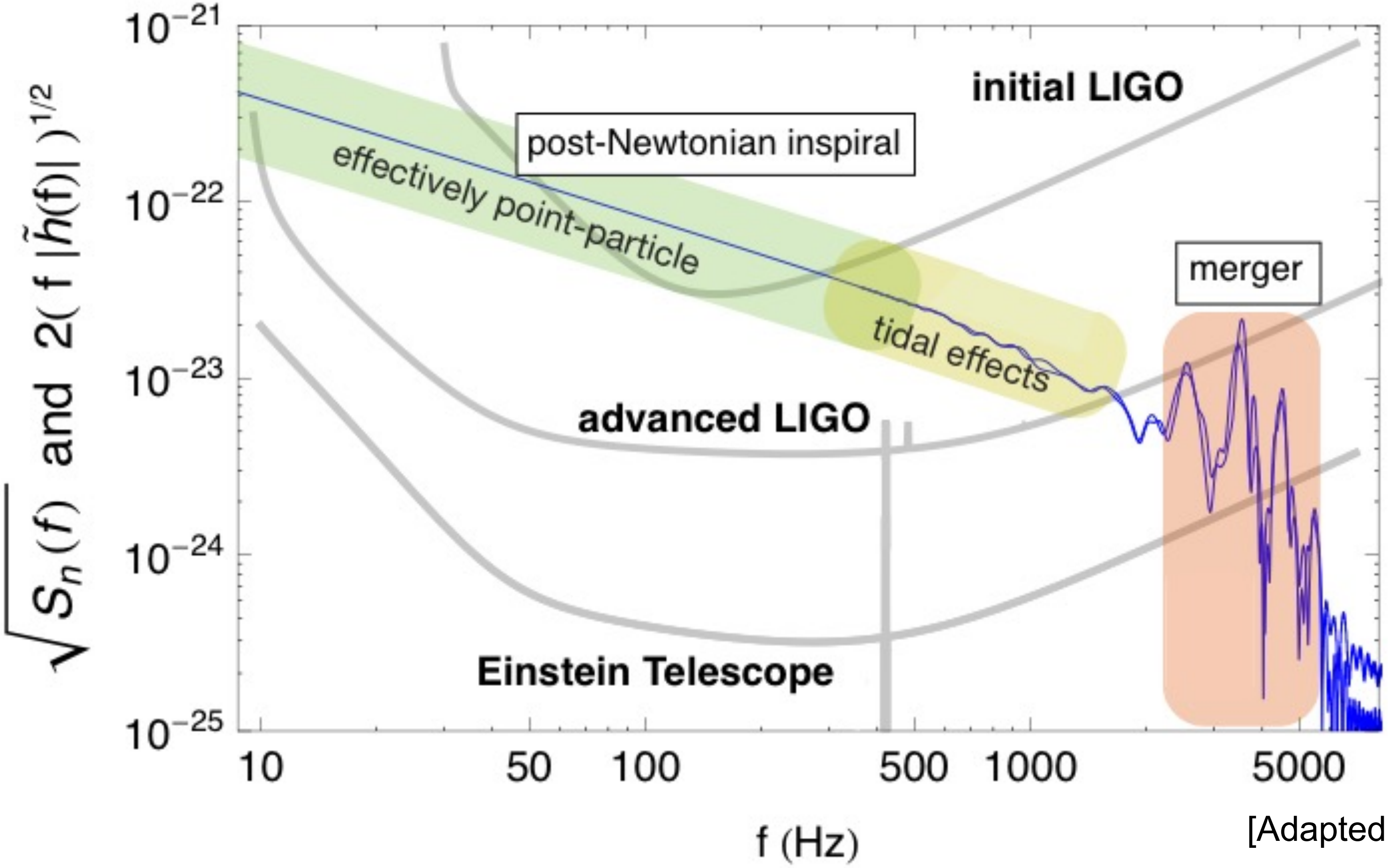
catching the wave



[Adapted from J Read]

Gravitational-wave astronomy provides new opportunities to probe neutron star properties.

Deviations from point-mass dynamics become important during the late stages of binary inspiral, leading through to the (messy) dynamics of the remnant.

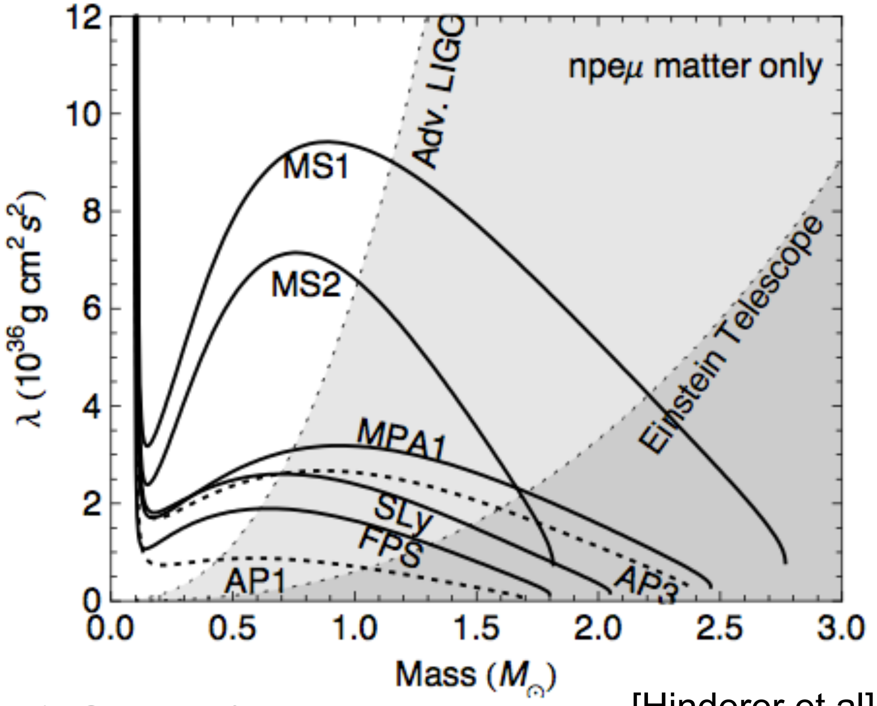
Basically, the tidal interaction affects the number of gravitational-wave cycles

$$\mathcal{N} = \frac{2}{3}\int_{f_a}^{f_b} \frac{E_{\rm orb}}{\dot{E}_{\rm orb}} df \approx \int_{f_a}^{f_b} t_D \left(1 + \frac{E_{\rm r}}{E_{\rm N}} - \frac{\dot{E}_{\rm tide}}{\dot{E}_{\rm gw}}\right) df$$

Template mismatch by (say) half a cycle leads to a significant loss of signal to noise.

However - difficult to alter the wave phasing (e.g. 10^{46} erg at 100 Hz leads to shift of 10^{-3} radians).

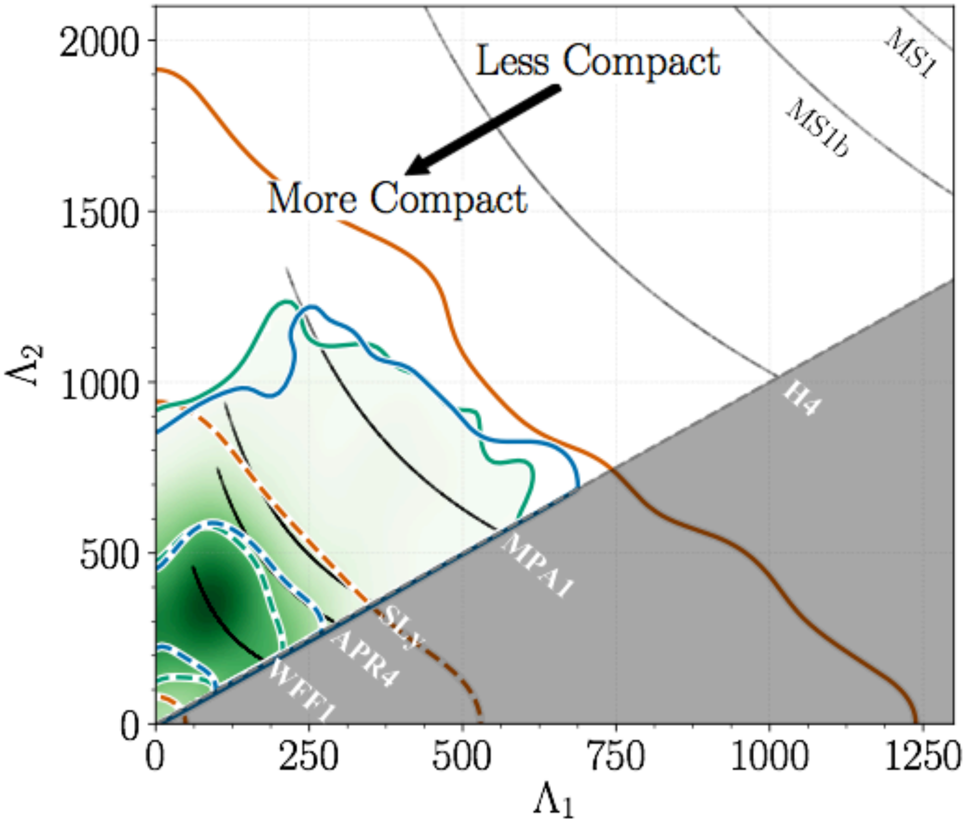
The star's deformability, encoded in the so-called Love number, may lead to a distinguishable secular effect:



[Hinderer et al]

$$\lambda = \frac{2}{3} k_2 R^5 = \frac{Q}{E} = \frac{\text{quadrupole deformation}}{\text{strength of tidal field}}$$

GW170817



[Abbott et al]

Demonstrated by the spectacular GW170817 event.

Best constraints on the tidal deformability for this **single** event (assuming the same EoS, slow spins and max. mass indicated by pulsar data) suggests a neutron star radius in the range

R=10.5-13.3 km

Similar to the x-ray results from accreting neutron stars…

One may also use the data to constrain the allowed maximum mass (of non-rotating stars).

The GW170817 data suggests that the maximum mass is below 2.16$M_\odot$.

Should do better in the future, but may have to wait a while…

beyond equilibrium

The tidal analysis assumes matter in equilibrium, but… the inspiral is much faster than the relevant equilibration rates:

$$\tau_M \sim \frac{2\ \text{months}}{T_9^6}\ , \quad \tau_D \sim \frac{20\ \text{s}}{T_9^4}\ , \quad \tau_H \sim \frac{1\ \text{ms}}{T_9^2}$$

Basically, the matter composition should be "frozen". This should impact on the tidal deformation (at some level).

Thinking of the deviation from chemical equilibrium $\Delta\beta$ as a function of the density ρ and the proton fraction x_p we have

$$\partial_t \Delta\beta = \left(\frac{\partial\beta}{\partial\rho}\right)_{x_\mathrm{p}} \partial_t \Delta\rho + \left(\frac{\partial\beta}{\partial x_\mathrm{p}}\right)_\rho \frac{\gamma}{n}\Delta\beta$$

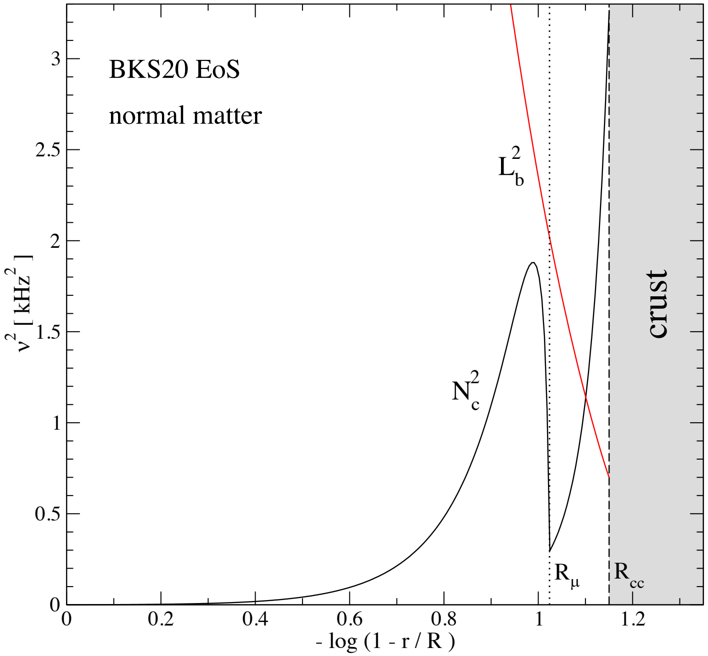
For slow reactions, we need to solve the time dependent problem – which is a bit tricky.

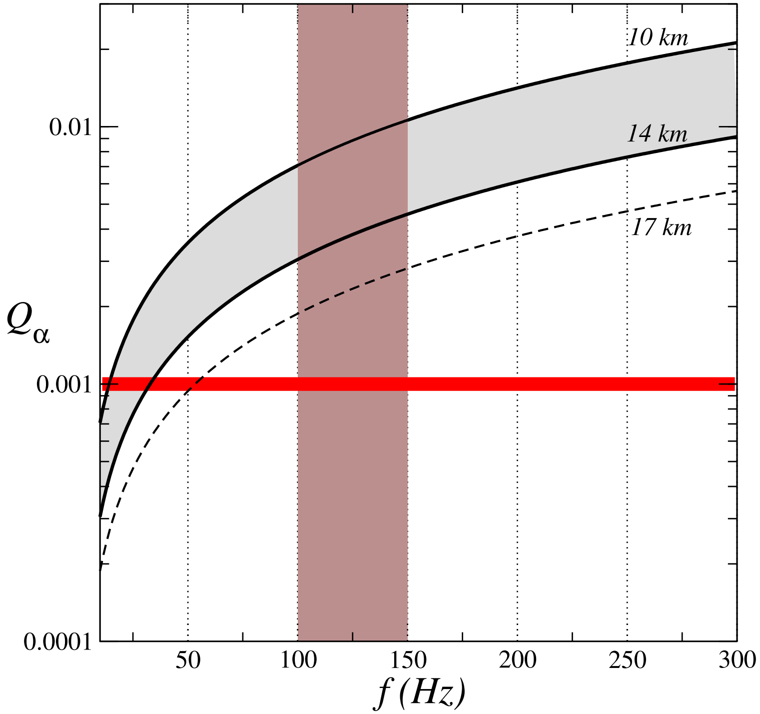
However, it is interesting to ask to what extent we can also use observations to constrain the **matter composition**. Also, we need to quantify systematic effects that impact on parameter extraction.

The composition also affects the star's oscillation modes – in particular, the g-modes.

The **state of matter** comes into play:

- in superfluid npe matter there are no g-modes, but…
- as the muons appear, matter is stratified, leading to a set of (higher frequency) g-modes.





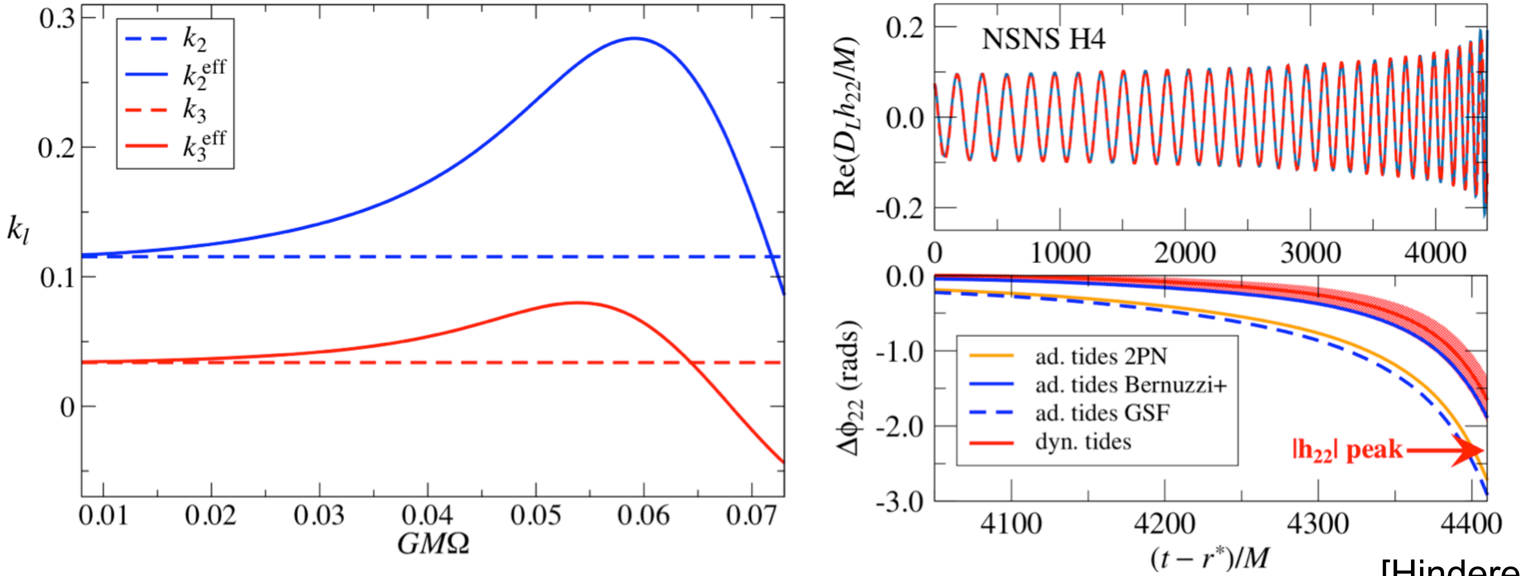
The tidal interaction leads to resonances.

In principle, observations constrain the "overlap" between the tide and a mode's eigenfunction

$$\Delta\mathcal{N} \approx -4\times 10^{-4}\hat{f}_\alpha^{-2} Q_\alpha^2 \left(\frac{c^2 R_1}{GM_1}\right)^5 \frac{1}{q(1+q)}$$

Tidal resonances may not have major impact, but…

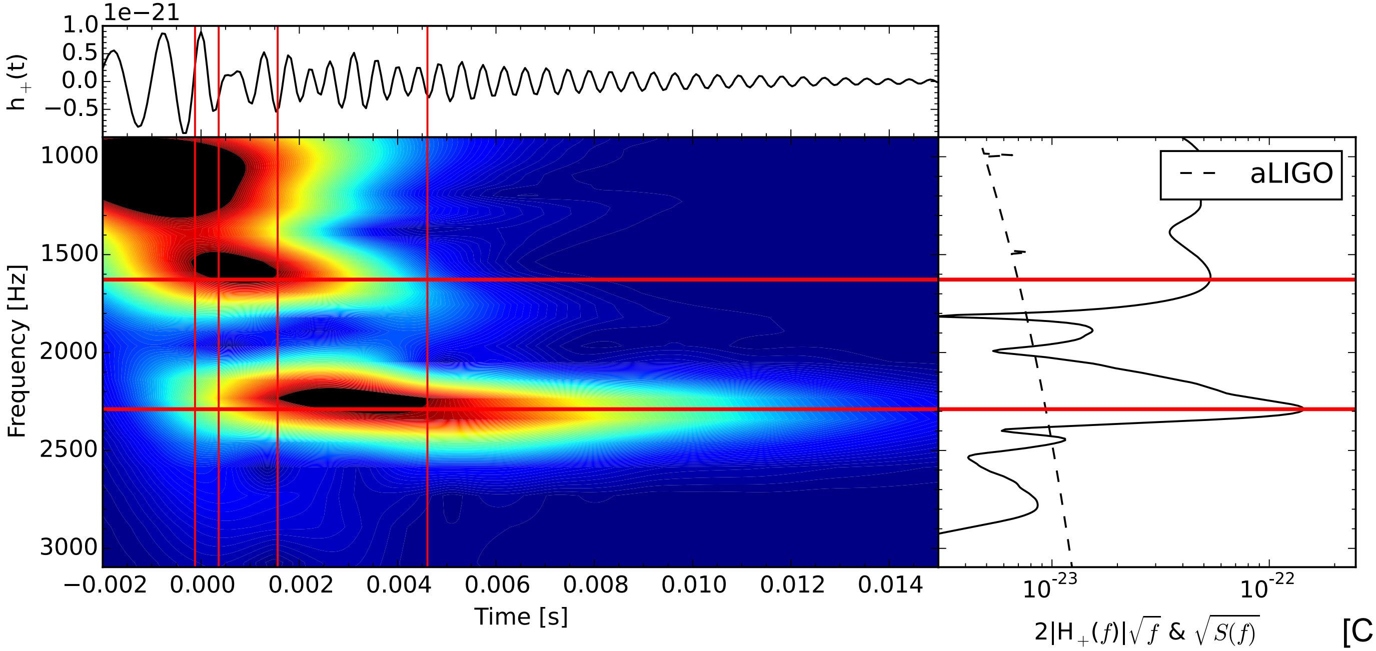
- the excitation of the star’s f-mode could leave an imprint close to merger,



[Hinderer et al]

- there may an instability due to nonlinear p-g mode coupling, supposedly a non-resonant effect that relies on a strong overlap between the tide and high overtone p/g modes (high/low frequencies),
- coupling to the interface mode at the crust-core transition may shatter the crust (this is another story…)
- the impact of the crust elasticity is weak, but the effect of a solid core (e.g. crystalline CFL) could be significant (also… another story)

The final merger was not seen in the GW170817 signal – need detectors to be more sensitive at high frequencies – although the launch of the GRB may require a fairly prompt collapse to a black hole.



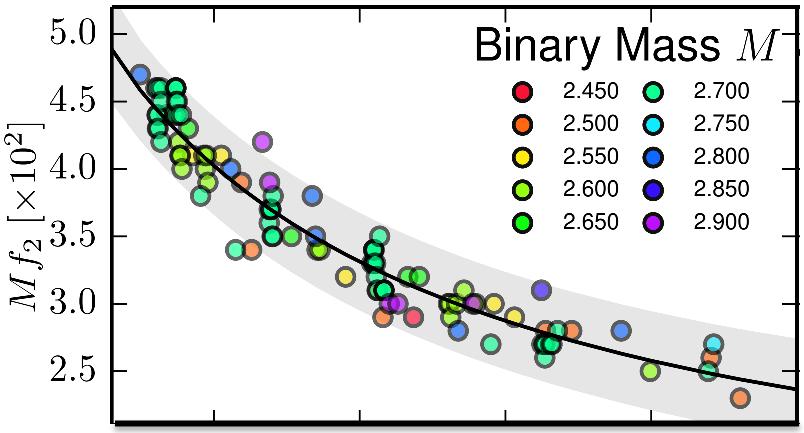
[Clark et al]

Eventually… we should be able to detect the oscillations of the merger remnant.

This should enable **hot** neutron star seismology.

Simulations suggest a strong correlation between the tidal deformability and the main peak in the spectrum of the oscillating remnant (the f-mode).

This is “useful” as it would help parameter extraction.



[Bernuzzi et al]

In principle, the correlation is “expected” from results for single stars:

$$M f_2 \sim C^{3/2} \sim \left(\kappa_2^t\right)^{-3/10}$$

However, the result is “intriguing” as it suggests thermal effects and differential rotation do not significantly affect the oscillations. How do we understand this?

- simulations suggest the core rotates (roughly) uniformly and relatively slowly,
- thermal effects may be (largely) equation of state independent and affect the f-mode frequency as an overall factor.

take home message

As we enter the era of gravitational-wave astronomy we can expect to probe neutron star physics in new ways.

Analysis of the GW170817 event demonstrates how gravitational-wave data can be used to constrain uncertain matter properties.

Expect to do better in the future. There will be more detections – although we should keep in mind that GW170817 was “special”.

In order to ”dig deeper”, we need to improve the models:

- account for composition variation (reactions?) during late inspiral,
- explore role of state of matter (crust/superfluidity),
- build numerical simulations based on actual thermodynamics (heat),
- account for dynamical role of neutrinos (bulk viscosity),
- implement electromagnetism ”beyond ideal MHD”.