Study of Perey-Buck Nonlocal Optical Potential

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Introduction

What is the effect of the non-locality of OMP?

Perey-Buck nonlocal optical potentials (NLOP)

Prediction on Calcium and Nickel

Conclusion and outlook
Theoretical optical model potential (OMP)

Optical model is one of the essential tools in studying nuclear reactions.

In the nuclear medium:
Nucleon optical potential is equivalent to the nucleon self-energy

Microscopic OMP:
- Nucleon–nucleon interaction
- Brueckner-Hartree-Fock (BHF) theory, Dirac-Brueckner-Hartree-Fock (DBHF) theory
- With local density approximation (LDA): strength and shape of the optical potential

Phenomenological OMP:
- Adopts a suitable analytical form for the potential, usually a Woods–Saxon form
- Determines its depth and geometry parameters by means of parameter adjustment to best fit available experimental data.
Local and nonlocal optical model potential

\[ \frac{\hbar^2}{2m} \nabla^2 \psi(\vec{r}) + \int V(\vec{r}, \vec{r}') \psi(\vec{r}') d\vec{r}' = E \psi(\vec{r}) \]

OMP is nonlocal and energy independence

- Pauli exclusive principle (Hartree-Fock)
- coupling of the inelastic excitations to the ground state and also some more complicated couplings (dynamic polarization potential)

Difficult to qualify

Phenomenological OMP is local and energy dependent

\[ E(k) = \frac{\hbar^2}{2m} k^2 + V(k, E(k)) \]

Local and nonlocal OMP

same phase shifts, i.e. same cross sections

Wave function are different
What is the effect of the non-locality of OMP?

The calculations of 3-body direct nuclear reactions with nonlocal optical potentials are performed for the first time using the framework of Faddeev-type scattering equations.

An important nonlocality effect is found will stimulate the development of new and more precise nonlocal nuclear interaction models.
An effective local $d$-$A$ potential is constructed from local nucleon optical potentials taken at an energy shifted by 40 MeV with respect to the widely used $E_d/2$ value:

$$E_d/2 + \Delta E$$

$$\Delta E = \frac{1}{2} \langle T_{np} \rangle = 40 \text{ MeV}$$

for $^{40}\text{Ca}(d, p)^{41}\text{Ca}$ reaction at $E_d = 11.8\text{ MeV}$

Main feature of the (d,p) reaction amplitude is sensitive only to the short-range (and high relative kinetic energy) n-p components of the three-body wave function.
The Perey correction improves upon the distribution involving local interactions only, it is still unable to fully capture the complex effect of non-locality.

\[ (p,d) \text{ the spectroscopic factors could be affected by approximately 20\%} \]

Perey Correction Factor (PCF).

\[ \psi^{NL}(r) = F(r)\psi^{Loc}(r) \]

\[ F(r) = \left[ 1 - \frac{\mu B^2}{2\hbar^2}(U^{LE}(r) - U_0(r)) \right]^{-1/2} \]

\[ F(r) \to 1 \quad \text{as} \quad r \to \infty \]
Perey and Buck nonlocal optical potential

\[
\left[ \frac{\hbar^2}{2M} \nabla^2 + E \right] \Psi(r) = - \left( U_{so} + i W_{so} \right) S(r) \mathbf{L} \cdot \mathbf{\sigma} \right] \Psi(r) + \int V(r, r') \Psi(r') \, dr'.
\]

\[
V(r, r') = U\left( \frac{1}{2} |r + r'| \right) H(|r - r'|).
\]

\[
H(|r - r'|) = \frac{\exp \left[ - \frac{(r - r')^2}{\beta} \right]}{\sqrt{\pi} \beta^3},
\]

\( \beta \) is the range of the non-locality

\[
-U(p) = [V + i W_I] f_S(p) + i W_D f_D(p)
\]

**TABLE I: Non-local optical potential set PB**

<table>
<thead>
<tr>
<th>V (MeV)</th>
<th>( r_s ) (fm)</th>
<th>( a_s ) (fm)</th>
<th>( W_I ) (MeV)</th>
<th>( r_I ) (fm)</th>
<th>( a_I ) (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>71</td>
<td>1.22</td>
<td>0.65</td>
<td>0</td>
<td>1.22</td>
<td>0.65</td>
</tr>
<tr>
<td>( W_D ) (MeV)</td>
<td>( r_D ) (fm)</td>
<td>( a_D ) (fm)</td>
<td>( U_{so} ) (MeV)</td>
<td>( r_{so} ) (fm)</td>
<td>( a_{so} ) (fm)</td>
</tr>
<tr>
<td>15</td>
<td>1.22</td>
<td>0.47</td>
<td>7.202</td>
<td>1.22</td>
<td>0.65</td>
</tr>
<tr>
<td>( r_C = 0 ) (fm)</td>
<td>( \beta = 0.85 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

F. Perey and B. Buck NP 32(1962)353

\[
\chi_{kd}^2 \approx 18.4 \quad \chi_{pb}^2 \approx 106.6
\]
Fitness of nonlocal optical potential

PB nonlocal optical potential

- They solved the Schrödinger equation with nonlocal potential by an iteration method. The iteration often diverges, even if differs very slightly from the optimal potential suggested by PB.

KU method

- In 1990s, Kim and Udagawa (KU) presented a rapid and reliable method for solving the nonlocal optical model Schrödinger equation by utilizing the Lanczos technique.

Minuit

- Minuit is conceived as a tool to find the minimum value of a multi-parameter function and analyze the shape of the function around the minimum.

Openmp for fortran 95

- Parallel calculations have been included in the fitness, 3-4 times faster average

\[ \chi^2 = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{\sigma_{th}(\theta_i) - \sigma_{exp}(\theta_i)}{\Delta\sigma_{exp}(\theta_i)} \right)^2 \]

Physically meaningful parameters.

The parameters must satisfy a numerical optimization criterion.

A good visual fit.
Nonlocal optical potential for neutrons
Nonlocal optical potential for Protons
Parameters of Nonlocal optical potential

### TABLE II: $d\sigma/d\Omega$ database for neutron elastic scattering

<table>
<thead>
<tr>
<th>Target</th>
<th>Ref.</th>
<th>Energy (MeV)</th>
<th>Ref.</th>
<th>Energy (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{120}\text{Sn}$</td>
<td>[10]</td>
<td>13.923, 16.905</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$\chi^2_{\text{LOQ}} = 51.6$
$\chi^2_{\text{Q2203}} = 35.2$

<table>
<thead>
<tr>
<th>V (MeV)</th>
<th>$r_s$ (fm)</th>
<th>$a_s$ (fm)</th>
<th>$W_I$ (MeV)</th>
<th>$r_I$ (fm)</th>
<th>$a_I$ (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>70.00</td>
<td>1.28</td>
<td>0.65</td>
<td>1.39</td>
<td>1.17</td>
<td>0.55</td>
</tr>
</tbody>
</table>

$W_D$ (MeV) | $r_D$ (fm) | $a_D$ (fm) | $U_{so}$ (MeV) | $r_{so}$ (fm) | $a_{so}$ (fm) |
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>21.11</td>
<td>1.15</td>
<td>0.46</td>
<td>9.00</td>
<td>1.10</td>
<td>0.59</td>
</tr>
</tbody>
</table>

$r_C = 0$ (fm) \hspace{2cm} $\beta = 0.90$

### TABLE III: $d\sigma/d\Omega$ database for proton elastic scattering

<table>
<thead>
<tr>
<th>Target</th>
<th>Ref.</th>
<th>Energy (MeV)</th>
<th>Ref.</th>
<th>Energy (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{27}\text{Al}$</td>
<td>[13]</td>
<td>28.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{56}\text{Fe}$</td>
<td>[14]</td>
<td>16.0</td>
<td>[15]</td>
<td>18.6</td>
</tr>
<tr>
<td>$^{90}\text{Zr}$</td>
<td>[16]</td>
<td>9.7</td>
<td>[17]</td>
<td>16.0</td>
</tr>
<tr>
<td>$^{208}\text{Pb}$</td>
<td>[14]</td>
<td>16.0</td>
<td>[18]</td>
<td>24.1, 30.3</td>
</tr>
</tbody>
</table>

$\chi^2_{\text{LOQ}} = 346.8$
$\chi^2_{\text{Q2203}} = 294.2$

<table>
<thead>
<tr>
<th>V (MeV)</th>
<th>$r_s$ (fm)</th>
<th>$a_s$ (fm)</th>
<th>$W_I$ (MeV)</th>
<th>$r_I$ (fm)</th>
<th>$a_I$ (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>70.95</td>
<td>1.29</td>
<td>0.58</td>
<td>9.03</td>
<td>1.24</td>
<td>0.50</td>
</tr>
</tbody>
</table>

$W_D$ (MeV) | $r_D$ (fm) | $a_D$ (fm) | $U_{so}$ (MeV) | $r_{so}$ (fm) | $a_{so}$ (fm) |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>15.74</td>
<td>1.20</td>
<td>0.45</td>
<td>8.13</td>
<td>1.02</td>
<td>0.59</td>
</tr>
</tbody>
</table>

$r_C = 1.34$ (fm) \hspace{2cm} $\beta = 0.88$
Prediction on Nickel

\[ ^{60}\text{Ni}(n,n)^{60}\text{Ni} \]

\[ ^{60}\text{Ni}(p,p)^{60}\text{Ni} \]

\[ \frac{d\sigma}{d\Omega} \text{ [mb/sr]} \]

\[ \Theta_{cm} \text{ [deg]} \]

- Exp.
- NLOP
- KD03
Prediction on Calcium
Prediction on Analyzing power
An important nonlocality effect is found in the d,p stripping reaction. Based on Perey-Buck nonlocal optical potential, we constructed neutron and proton nonlocal optical potentials (NLOPs). Parameters of the NLOPs are introduced by fitting nucleon-nucleus elastic scattering angular distribution on some elements ranging from Al to Pb in the energy range 10 to 30 MeV. Comparing with the experimental data and local and globe optical model KD03, these NLOPs are remarkable good.

Study the d,p and p,d reaction with fully nonlocal optical potential.
Thank you 谢谢!!!
In principle, the optical potential is nonlocality.

- Pauli exclusion principle
- Feshbach potential

Local optical potential

- Local equivalent potential
- KD local potential
How to get a new nonlocal optical potential

(i) Physically meaningful parameters.
(ii) The parameters must satisfy a numerical optimization criterion.
(iii) A good visual fit.

\[ \chi^2 = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{\sigma_{\text{th}}(\theta_i) - \sigma_{\text{exp}}(\theta_i)}{\Delta \sigma_{\text{exp}}(\theta_i)} \right)^2, \]

\[ (d\sigma/d\Omega)_{\text{unpol}} = |A|^2 + |B|^2 \]

\[ \varepsilon = \frac{|A(\theta) + B(\theta)|^2 - |A(\theta) - B(\theta)|^2}{|A(\theta) + B(\theta)|^2 + |A(\theta) - B(\theta)|^2} = \frac{B^*(\theta)A(\theta) + A^*(\theta)B(\theta)}{|A(\theta)|^2 + |B(\theta)|^2} \]
What is the effect of the non-locality of OMP?

Stripping reaction $A(d,p)B$
- Exploring shell evolution
- Changing magic numbers
- The evolution of the element abundances in the Universe.

DWBA (Distorted Wave Bonn Approximation)
- Deuterons are treated as point particles,
- Not account for the breakup effect of deuterons
- N and p are loosely bound in deuteron

3-body model of deuteron-nucleus collisions:
- Deuterons are loosely bound systems ($\approx -2.226$ MeV)
- Elastic deuteron scattering + elastic deuteron breakup + target in its ground state all included in a unified way.
- Excited states of the target $A$ do not appear explicitly.

ADWA (the adiabatic distorted wave approximation)
- The adiabatic approximation separate the variables into “slow” and “fast”, and fix the slow one
- 3-body problem becomes a 2-body problem for fixed $r$
THE OPTICAL MODEL AND ITS JUSTIFICATION

BY HERMAN FESHBACH

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1. INTRODUCTION

This article will deal with the optical model description of the scattering of a nuclear particle by a nucleus. In this model the many-body problem arising from the interactions of the nucleons in the target nucleus with the incident particle is approximated by a two-body problem. The various interactions are replaced by a potential \( V \) between the incident particle and the nucleus. In other words, in the center-of-mass system the motion of the particle is given by the Schroedinger equation:

\[
\nabla \psi + \frac{2\mu}{J^2} (E - V)\psi = 0
\]

where \( \mu \) is the reduced mass. This approximation is referred to as the “optical model,” because it is in many ways analogous to the index of refraction approximation which is employed to describe the propagation of light in a medium. There the many-body problem, the interaction of light with each particle in the medium, is approximated by a propagation problem in which the effect of the medium is represented by an index of refraction. Of course, this analogy should not be taken too literally, as will be seen in Section 4.

Optical potential:

\[ V = U + iW \]