Soft Multipole Collective Modes in $^{40}$Mg

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The drip-line physics

- Dripline at 2007: $^{24}\text{O}$, $^{33}\text{F}$, $^{34}\text{Ne}$, $^{40}\text{Mg}$
- No changes after 10 years, although RIKEN tried experiments with $^{48}\text{Ca}$

- Halo—extended dilute surface, soft modes...
- Nuclear astrophysical interests
- Explain few-body physics by many-body methods
- Extrapolate, verify and improve theoretical modes

Collective excitations of weakly bound nuclei

- Novel relative motion between halo/skin and core: collective or non-collective?
- Enhance astrophysical neutron capture rates
- Related to continuum, neutron halo, deformation, EOS, symmetry energy, incompressibility...

Three-Fluid Hydrodynamical Model of Nuclei*
Radhe Mohan
PRC, 1971

![Diagram of pygmy resonance](image)

Extensive experiments in less exotic nuclei

68Ni

D. Rossi et al., PRL 111 (2013) 242503

(T. Oishi, et.al, Phys.Rev. C 93, 034329 (2016))
What’s the nature of deformed PDR

- Various deformed halos—shape decoupling—How to detect?

Comparative experimental study of splittings in K=0 and K=1 modes in GDR and PDR

S.G. Zhou et al., PRC 2010; J.C.Pei et al., PRC 2013

- The flow pattern of PDR in weakly bound nuclei (a long-standing question)

Can be studied directly by transition currents in deformed QRPA
Deformed QRPA is needed even for spherical nuclei because internal motions is prohibited due to symmetry

Methods: Deformed continuum FAM-QRPA

- A numerical challenge for **deformed continuum QRPA**
- Standard QRPA in the matrix form is extremely expensive for deformed nuclei, even more to include continuum configuration (a huge matrix)
  Finite-Amplitude-Methods-QRPA provides alternative way solving QRPA equation iteratively rather than diagonalization
  (FAM-RPA, T. Nakatsukasa, PRC, 2007)

Several spherical continuum QRPA:
# FAM-QRPA developments

<table>
<thead>
<tr>
<th>Time(year)</th>
<th>Implement</th>
<th>Authors(Group)</th>
</tr>
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<tbody>
<tr>
<td>2011</td>
<td>Spherical FAM-QRPA (based on HFBRAD code)</td>
<td>P.Avogadro, T.Nakatsukasa</td>
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<tr>
<td>2011</td>
<td>Deformed Monopole modes (base on HFBTHO code)</td>
<td>M.Stoitsov, M.Kortelainen et al</td>
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<td>2013</td>
<td>Monopoles modes in relativistic RPA/QRPA</td>
<td>T. Niksic et al Haozhao Liang, et al</td>
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<td>2013</td>
<td>Discrete states and strengths (based on HFB-THO)</td>
<td>N.Hinohara, M.Kortelainen, W. Nazarewicz, E.Olsen</td>
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<td>2014</td>
<td>Beta decay (based on HFB-THO)</td>
<td>M. T. Mustonen and J. Engel</td>
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<td>2014</td>
<td>Soft monopole modes (based on HFB-AX code)</td>
<td>J.C.Pei, M.Kortelainen, Y.N.Zhang, F.R.Xu</td>
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<td>2015</td>
<td>Multipole modes (based on HFB-THO)</td>
<td>M.Kortelainen, N.Hinohara, W.Nazarewicz</td>
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<tr>
<td>2017</td>
<td>Soft Multipole modes (based on HFB-AX code)</td>
<td>K.Wang, M.Kortelainen, J.C.Pei</td>
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</table>
Implement FAM-QRPA in coordinate-space

- HFB-AX output: axial-deformed wavefunctions (17 GB) and energies
- B-spline lattice transformed to Gauss-Legendre lattice
- FAM-QRPA procedure:
  1. Construct full transition densities (including time-odd terms):

\[
\delta \rho (\omega) = U X V^T + V^* Y^T U^\dagger.
\]
\[
\delta \kappa^{(+)} (\omega) = U X U^T + V^* Y^T V^\dagger,
\]
\[
\delta \kappa^{(-)} (\omega) = V^* X^\dagger V^\dagger + U Y^* U^T,
\]

\[
\left\{ s_{\phi}, j_r, j_z, (\nabla \times j)_\phi, (\nabla \times s)_r, (\nabla \times s)_z, (\Delta s)_\phi, T_\phi \right\}
\]

\[
\begin{align*}
\mathcal{E}^{\text{even}}_t &= C_t^\rho [\rho] \rho_t^2 + C_t^T \rho_t \tau_t + C_t^{\Delta \rho} \rho_t \Delta \rho_t + C_t^{\nabla J} \rho_t \nabla J + C_t^J J_t^2, \\
\mathcal{E}^{\text{odd}}_t &= C_t^s [\rho] s_t^2 + C_t^{\Delta s} s_t \cdot \Delta s_t + C_t^T s_t \cdot T_t + C_t^j j_t^2 + C_t^{\nabla j} s_t \cdot \nabla \times j_t \\
&\quad + C_t^{\nabla s} (\nabla s_t).
\end{align*}
\]
Implement FAM-QRPA in coordinate-space

2. Calculate $H^{20}$, $H^{02}$ (including time-odd terms), $F^{20}$, etc

\[
\delta H_{\mu\nu}^{20}(\omega) = U^\dagger \delta h V^* - V^\dagger \delta \Delta^{(-)*} V^* + U^\dagger \delta \Delta^{(+)} U^* \\
- V^\dagger \delta h^T U^*, \\
\delta H_{\mu\nu}^{02}(\omega) = -V^T \delta h U + U^T \delta \Delta^{(-)*} U - V^T \delta \Delta^{(+)} V \\
+ U^T \delta h^T V.
\]

3. Calculate $X$, $Y$; and do Broyden non-linear iterations on $X$, $Y$.

4. Finally calculate the strength

\[
X_{\mu\nu} = - \frac{\delta H_{\mu\nu}^{20}(\omega) - F_{\mu\nu}^{20}}{E_\mu + E_\nu - \omega}, \quad Y_{\mu\nu} = - \frac{\delta H_{\mu\nu}^{02}(\omega) - F_{\mu\nu}^{02}}{E_\mu + E_\nu + \omega}.
\]

\[
S(F, \omega) = \frac{1}{2} \sum_{\mu\nu} \left\{ F_{\mu\nu}^{20*} X_{\mu\nu}(\omega) + F_{\mu\nu}^{02*} Y_{\mu\nu}(\omega) \right\},
\]

- Combined MPI+OpenMP parallel calculations in TH-1A,TH-2 supercomputers
Benchmark with HO basis

- SLy4+volume pairing and surface pairing (100Zr, 24Mg)
$^{40}\text{Mg}$: shape coexistence

- Last Mg isotope, weakly bound deformed
- Prolate-oblate coexistence in $^{40}\text{Mg}$ (N=28)
- Experimental interests of spectroscopy (H. Crawford)

Deformed K-splitting

Shape evolution from oblate $^{42}\text{Si}$ to prolate $^{40}\text{Mg}$

A good case to probe excitations based on different shapes
There is no evident core-halo decoupling, however.....
Isovector dipole Strength

- **Box size dependence:**
  Large box is needed for smooth the resonances, otherwise, PDR is fragmented.

- **Self-consistency:**
  Very clear low-energy PDR without spurious states

- **Disproportionate splitting:**
  The splitting is proportional to deformation and centroid energy
  Prolate PDR splitting is $1.4 (0.95) \text{ MeV}$
  Oblate PDR splitting is $0.45 (1.05) \text{ MeV}$

Disproportionate splitting is not due to static core-halo shape decoupling

Kai Wang, M. Kortelainen, J.C. Pei, PRC96,031301 (R) 2017-12-
Related to the excessive neutrons at surfaces and isoscalar dipole modes
The quantum flow topology of the PDR

- A long-sought collective and compressional dipole structures, poles at ±12 fm
- The simplest flow topology with the lowest energy
- Flow patterns characterized by boundary lines

Flows are sensitive to pairing

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Flow pattern in GDR

- More complicated as energies increase

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Flow patterns are robust **physical phenomena**, independent of box sizes.

- Isovector dipole is compressional PDR, not a toroidal mode
- Isoscalar dipole can have pollutions of spurious states
Current flows of isoscalar monopole modes

- Monopole flow patterns are non-trivial and become complicated as $E^*$ increases.
- Toroidal mode is favorable in oblate shape.

Summary

ISM

$^{48}\text{Mg}$, $\beta_2 = 0.39$

(a)

$^{40}\text{Mg}$, $\beta_2 = 0.31$

(b)

ISD

Prolate $^{40}\text{Mg}$ ISD

K = 0, K = 1

IVD

$S(\omega) (\text{e}^2\text{fm}^2/\text{A})$

(c)

(d)

IVD

$\omega$ (MeV)

ISQ

Prolate $^{40}\text{Mg}$ ISQ

K = 0, |K| = 2

Oblate $^{40}\text{Mg}$ ISQ

K = 0, |K| = 2
Summary

- Developed the fully self-consistent deformed continuum FAM-QRPA for multipole excitations in shape-coexisting $^{40}\text{Mg}$
- Monopole mode is dominated, with lowest excitation energy
- Disproportionate pygmy deformation splitting not due to static shape decoupling
- Amazing flow topologies related to energies has been revealed in a large spatial mesh; the long-sought PDR is collective and compressional.
- Toroidal mode is favorable in oblate shape

Thanks for your attention!