

# Neutron-skin thickness of heavy-ions studied with $\alpha$ -particle scattering

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School of Physics and Nuclear Energy Engineering

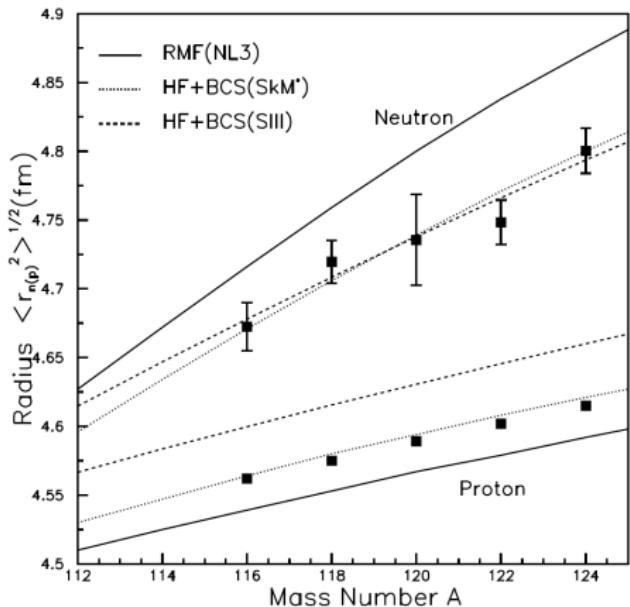
Beihang University

In collaboration with Prof. Jirina Stone (University of Oxford)

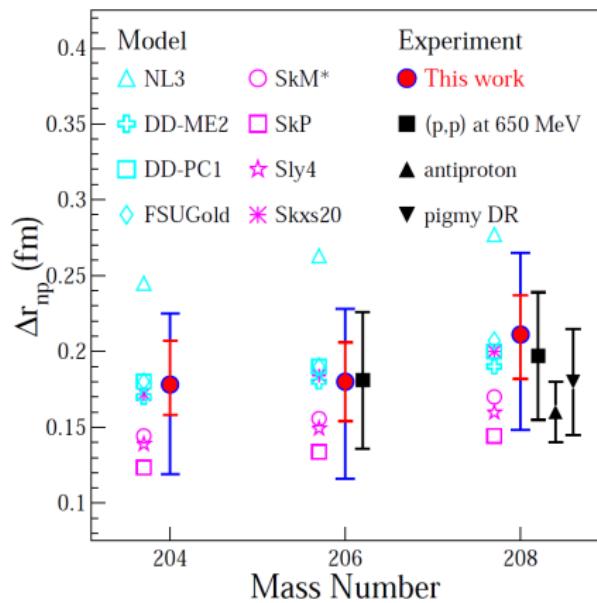
August 14, 2014

# Introduction

S. Terashima, Kyoto U. 2008



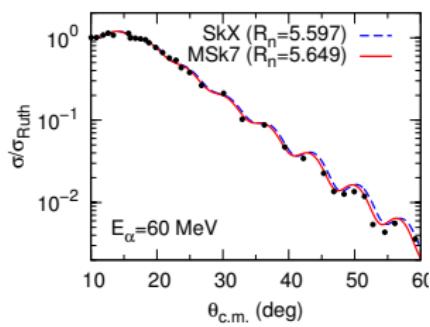
J. Zenihiro, Kyoto U. 2011



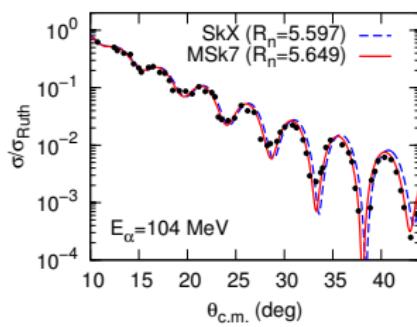
Precision required:  $\Delta R_n / R_n < 1\%$

# Introduction

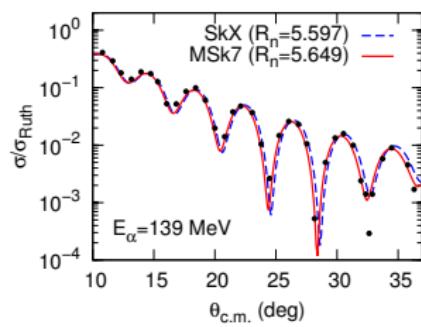
$\alpha + {}^{208}\text{Pb}$  at 60 MeV



$\alpha + {}^{208}\text{Pb}$  at 104 MeV



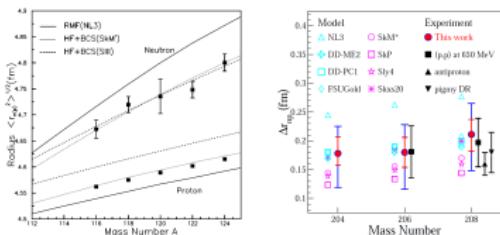
$\alpha + {}^{208}\text{Pb}$  at 139 MeV



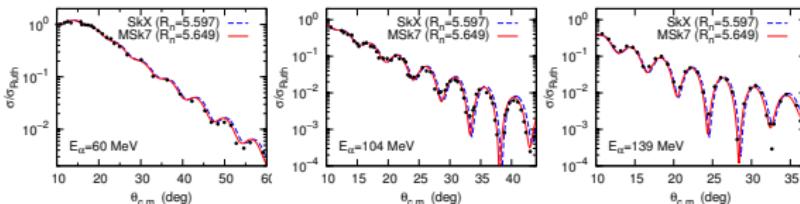
Effect of  $\Delta R_n / R_n \sim 1\%$  is visible in angular distributions

# Introduction

Terashima's and Zenihiro's PhD thesis



Precision required:  
 $\Delta R_n / R_n < 1\%$

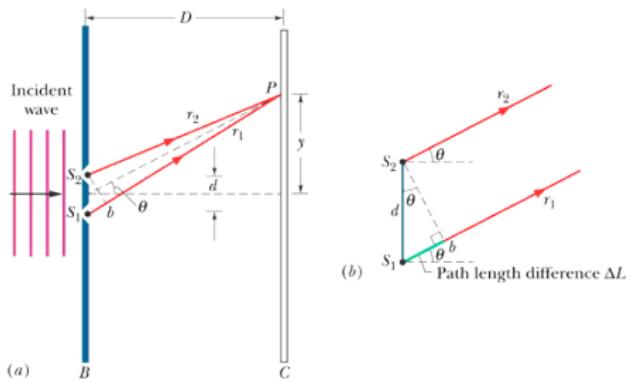
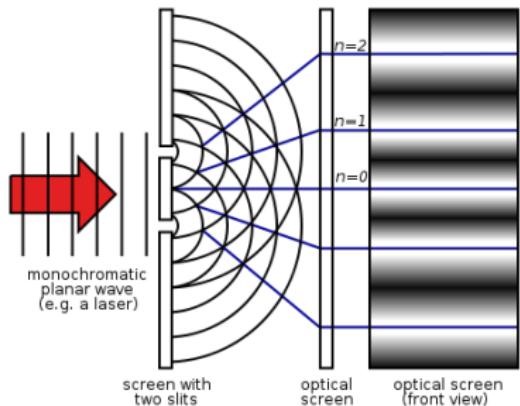


Effect of  
 $\Delta R_n / R_n \sim 1\%$  visible

**Question:** how to get  $R_n$  from  $\alpha$ -elastic scattering?

# Diffraction of light

## Double-slit with light

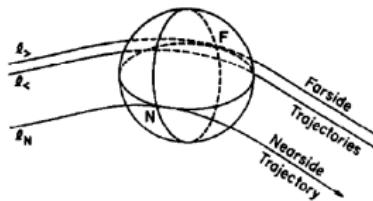


(<http://ipodphysics.com/prop-of-light-youngs-double-slit.php>)

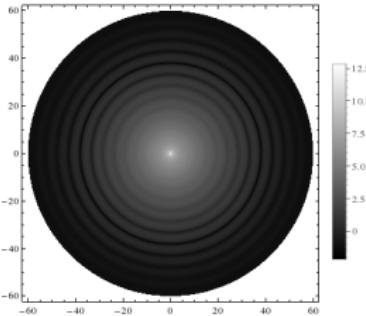
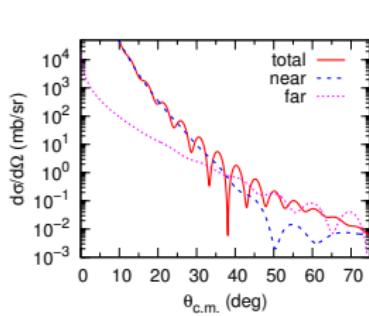
$$\Delta y = \frac{D}{d} \lambda \Rightarrow \Delta \theta = \frac{\Delta y}{D} = \frac{\lambda}{d} = \frac{2\pi}{kd} = \frac{\pi}{k(d/2)}$$

# Diffration of $\alpha$ -particle

## Double-slit with $\alpha$ -particle



Far  $\{ \ell_F, \ell_C \}$   
Near  $\ell_N$

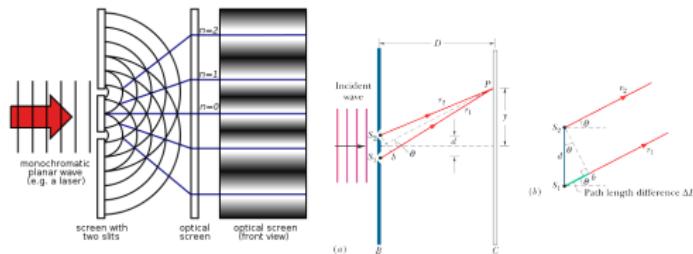


$$\Delta\theta \approx \frac{\pi}{kR}$$

G.R. Satchler, *Direct Nuclear Reactions*, Chapter 11

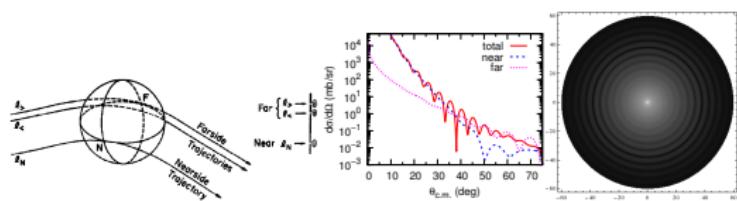
# Diffraction of light

## Double-slit with light



$$\begin{aligned}\Delta x &= \frac{\lambda}{Dd} \Rightarrow \\ \Delta\theta &= \frac{2\pi}{kd} \\ &= \frac{\pi}{k(d/2)}\end{aligned}$$

## Double-slit with $\alpha$ -particle

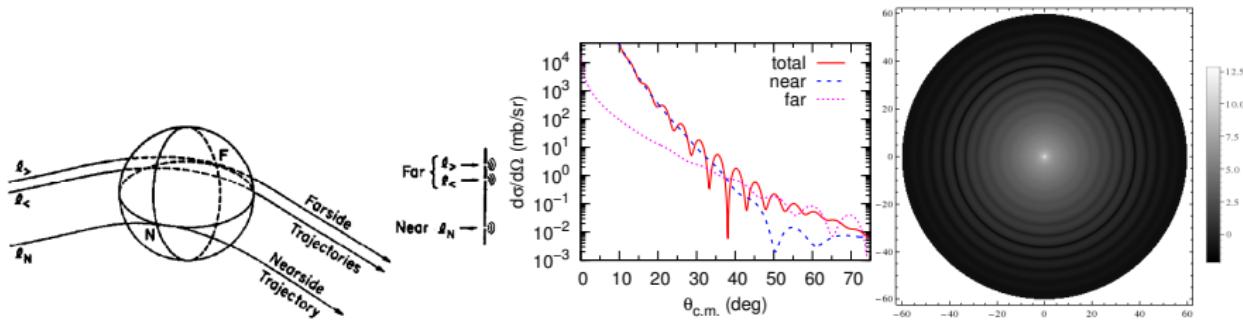


$$\Delta\theta \approx \frac{\pi}{kR}$$

**Question:** how does this  $R$  relate to the radius of the nucleus?

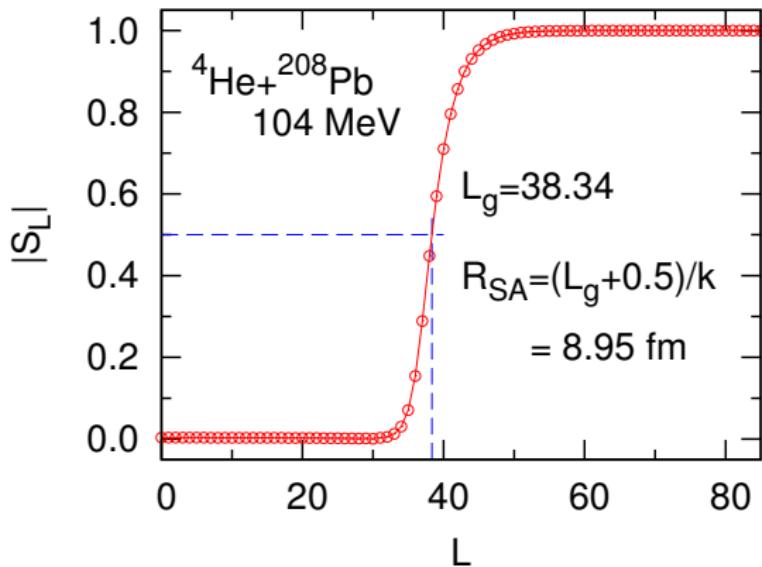
# Logic chain in this work

## Double-slit with $\alpha$ -particle



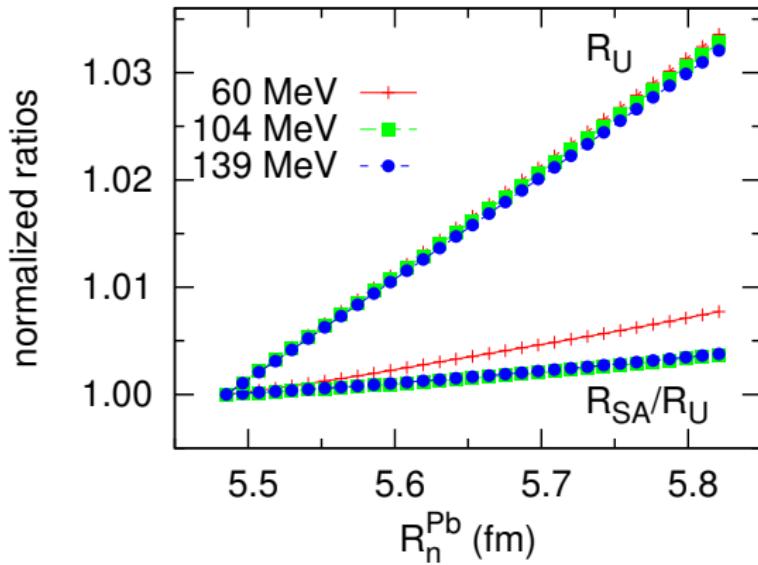
$$\Delta\theta \approx \frac{\pi}{kR} = \frac{\pi}{kR_{SA}}$$

- ① **Angular distribution**  $\Leftrightarrow$  Strong absorption radius  $R_{SA}$
- ②  $R_{SA} \Leftrightarrow$  radius of the optical potential  $R_U$
- ③  $R_U \Leftrightarrow$  neutron density distribution  $\rho_n$
- ④  $\rho_n \Leftrightarrow$  neutron radius  $R_n$

Strong absorption radius  $R_{SA}$ G.R. Satchler, *Direct Nuclear Reactions*, Chapter 11

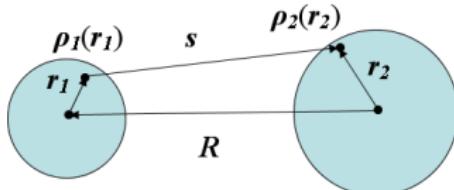
# Strong absorption radius and rms radius of OMP

$R_{SA}$  change closely with the changes of  $R_U$



# Double-folding model of OMP

## The double-folding model



$$U_{DF}(R) = \iint \rho_1(r_1) \rho_2(r_2) V_{NN}(|\mathbf{s}|) dr_1 dr_2.$$

relation of RMS radii in double-folding model

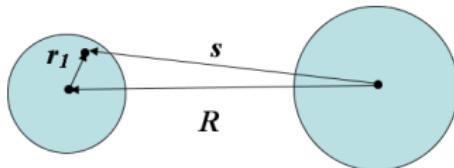
$$\begin{aligned}\langle R_{U_{DF}}^2 \rangle &= \langle R_1^2 \rangle + \langle R_2^2 \rangle + \langle V_{NN}^2 \rangle \\ \Rightarrow \langle R_2^2 \rangle &= \langle R_{U_{DF}}^2 \rangle - (\langle R_1^2 \rangle + \langle R_{NN}^2 \rangle)\end{aligned}$$

$\langle R_{U_{DF}}^2 \rangle$  depends on incident energy  $\Rightarrow$  Need calibration of  $U_{DF}$  for  $\langle R_2^2 \rangle$

M.E. Brandan and G.R. Satchler, Phys. Rep. 285, 143 (1997)

# The systematic single-folding $\alpha$ -nucleus potential

## The single-folding model



$U_{SF}(R) = \iint \rho_1(r_1) U_{NA}(|\mathbf{s}|) dr_1$ ,  $U_{NA}$  being JLMB (JLM+Bruyères) systematics

E. Bauge, J.P. Delaroche, and M. Girod, PRC 63, 024607 (2001)

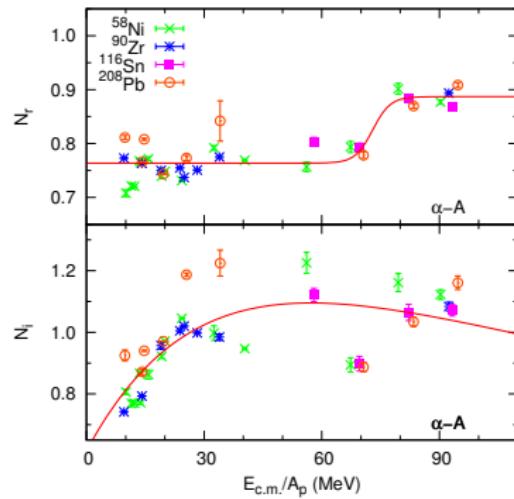
renormalization factors  $N_r$  and  $N_i$

$$U(E, R) = N_r(E) \text{Re}[U_{SF}(E, R)] + N_i(E) \text{Im}[U_{SF}(E, R)] + V_C(R)$$

DYP, Y.L. Ye, F.R. Xu, Phys. Rev. C 83, 064619 (2011)

# The systematic single-folding $\alpha$ -potential

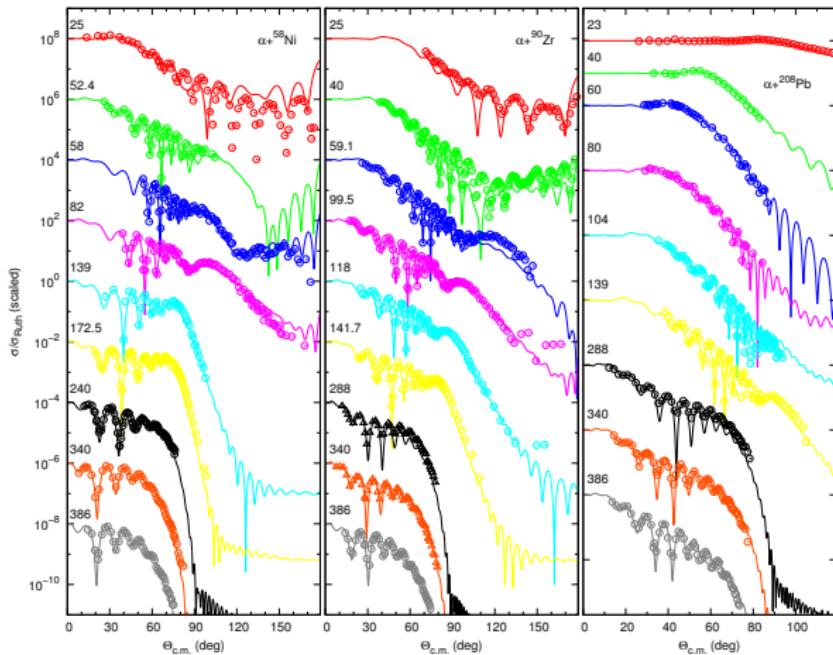
Energy dependence of  $N_r$  and  $N_i$



D.Y. Pang, Y.L. Ye, F.R. Xu, Phys. Rev. C 83, 064619 (2011)

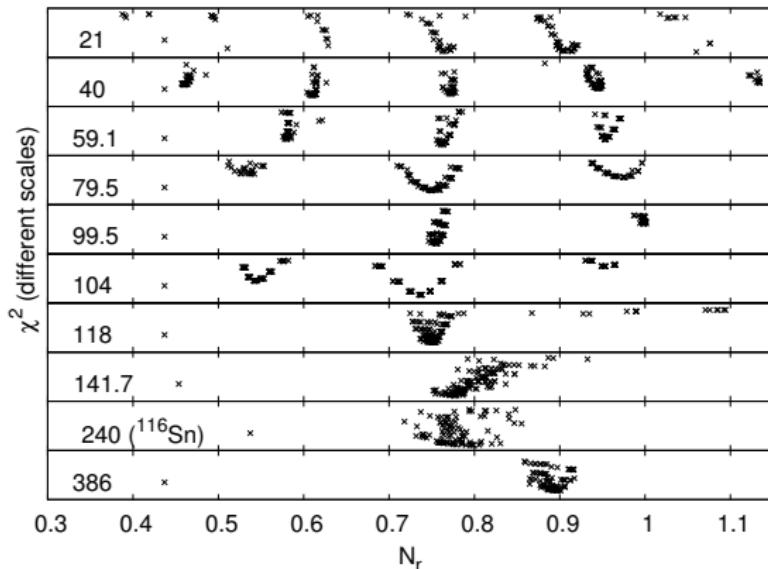
# The systematic single-folding $\alpha$ -potential

Comparison with experimental data



# Reduction of potential ambiguities

## ① Discrete ambiguities



D.G. Perkin, A.M. Kobos, and J.R. Rook, NPA 245, 343 (1975), D.F. Jackson and R.C. Johnson, PLB 49, 249 (1974)

# Reduction of potential ambiguities

## ① Discrete ambiguities

removed by selection of  $N_r$  families, guided by theoretical models

D.G. Perkin, A.M. Kobos, and J.R. Rook, NPA 245, 343 (1975)

D.F. Jackson and R.C. Johnson, PLB 49, 249 (1974)

## ② continuum ambiguities

a well-known one is  $VR_U^n = \text{constant}$ , in folding model  $R_U$  is uniquely determined by  $R_p$ ,  $R_t$  and  $R_{V_{NN}}$

See, e.g., G.R. Satchler, *Direct Nuclear reactions*, Chap.12

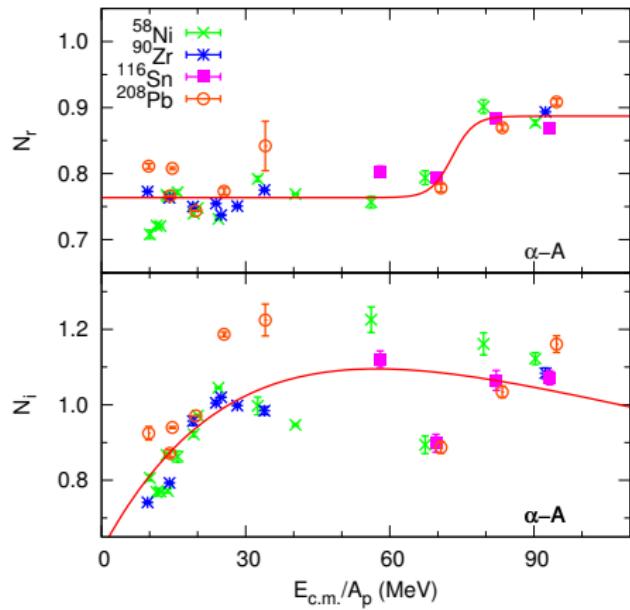
## ③ correlations of real and imaginary parts

reduced in systematically fitting many sets of data

See, e.g., L. Freindl *et al.*, Acta Phys. Pol. B11, 405 (1980)

# Uncertainties in $N_r$ and $N_i$

## Energy dependence of $N_r$ and $N_i$



$$\overline{\Delta N_r / N_r} = 0.04$$

$$\overline{\Delta N_i / N_i} = 0.08$$

# The SkX interaction in HF calculations

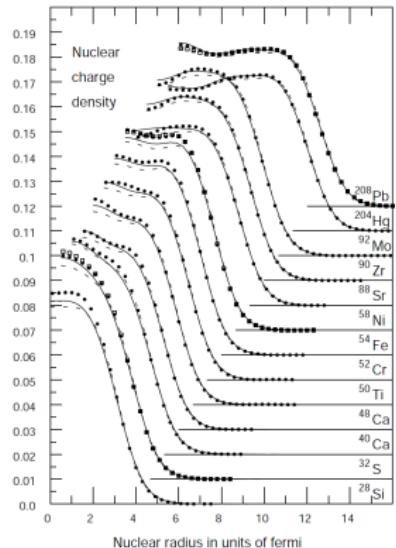
SkX was obtained by fitting:

- binding energies for  $^{16,24}\text{O}$ ,  $^{34}\text{Si}$ ,  $^{40,48}\text{Ca}$ ,  $^{48,68}\text{Ni}$ ,  $^{88}\text{Sr}$ ,  $^{100,132}\text{Sn}$  and  $^{208}\text{Pb}$  with “errors” ranging from 1.0 MeV for  $^{16}\text{O}$  to 0.5 MeV for  $^{208}\text{Pb}$
- rms charge radii for  $^{16}\text{O}$ ,  $^{40,48}\text{Ca}$ ,  $^{88}\text{Sr}$  and  $^{208}\text{Pb}$  with “errors” ranging from 0.03 fm for  $^{16}\text{O}$  to 0.01 fm for  $^{208}\text{Pb}$
- About 50 Single particle energies with “errors” ranging from 2.0 MeV for  $^{16}\text{O}$  to 0.5 MeV for  $^{208}\text{Pb}$

B.A. Brown, PRC 58, 220 (1998), PREX Aug. 17 (2008)

# The SkX interaction in HF calculations

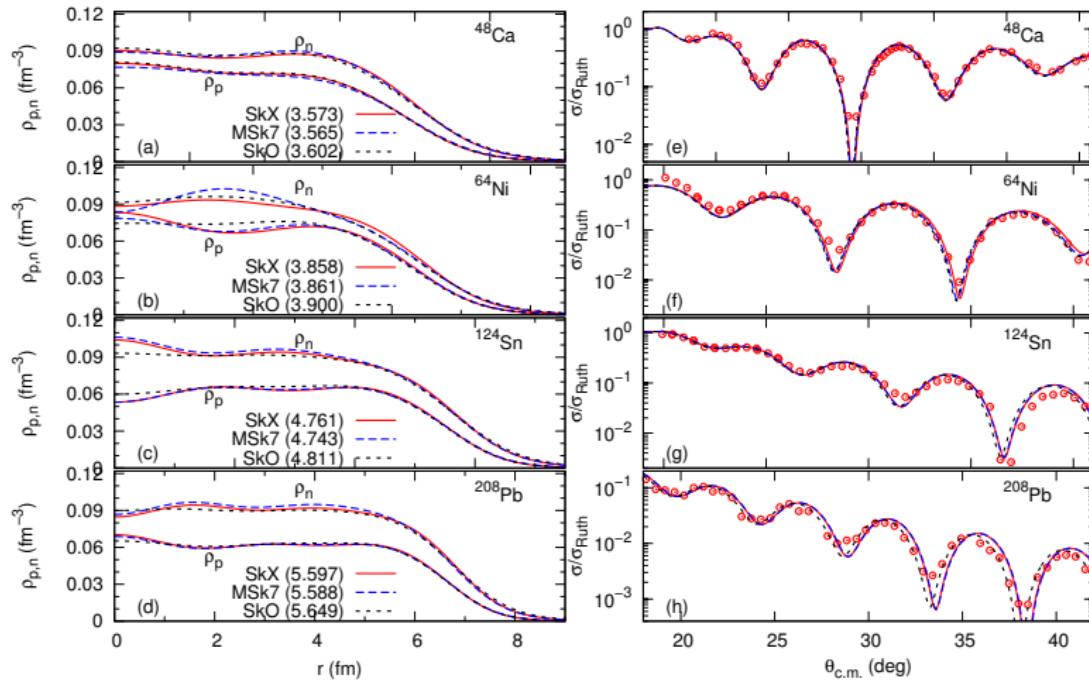
## SkX in reproduction of charge density distributions

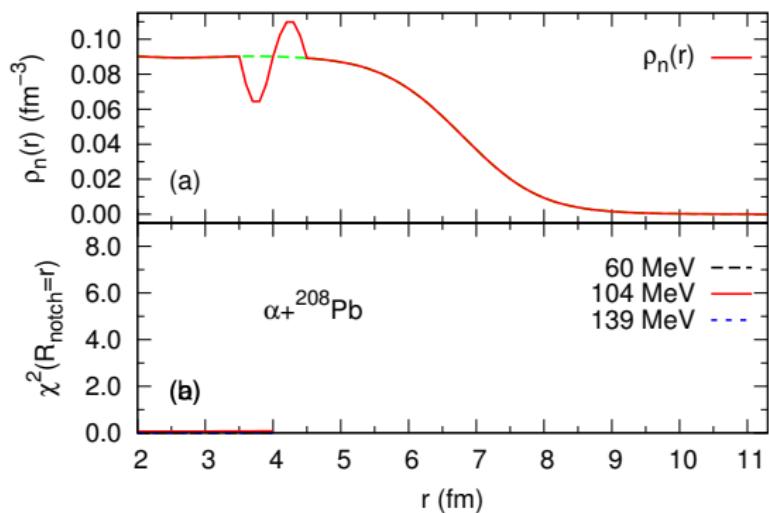


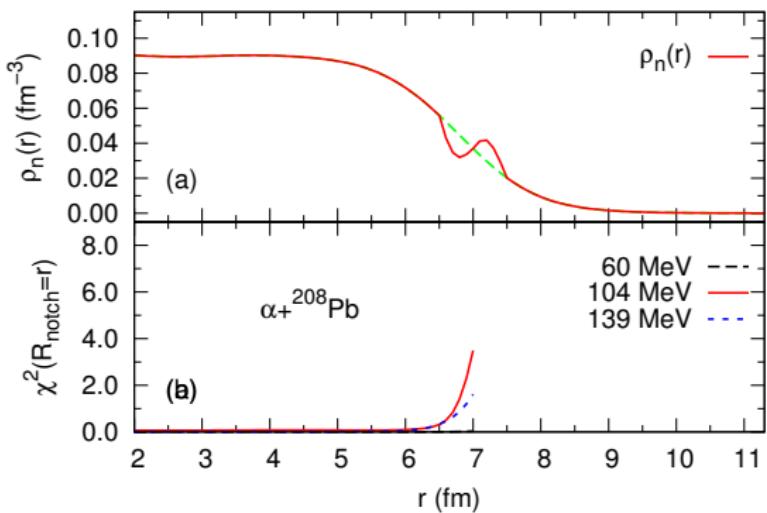
B.A. Brown, PREX Aug. 17 (2008)

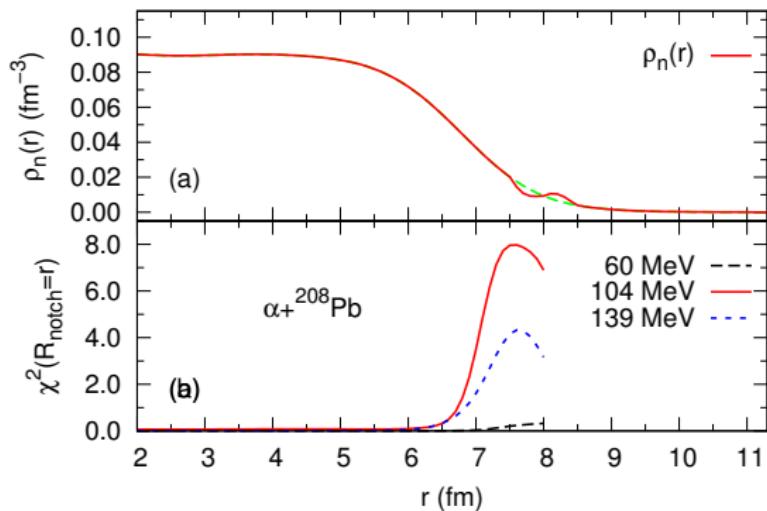
# Test of the systematic potential

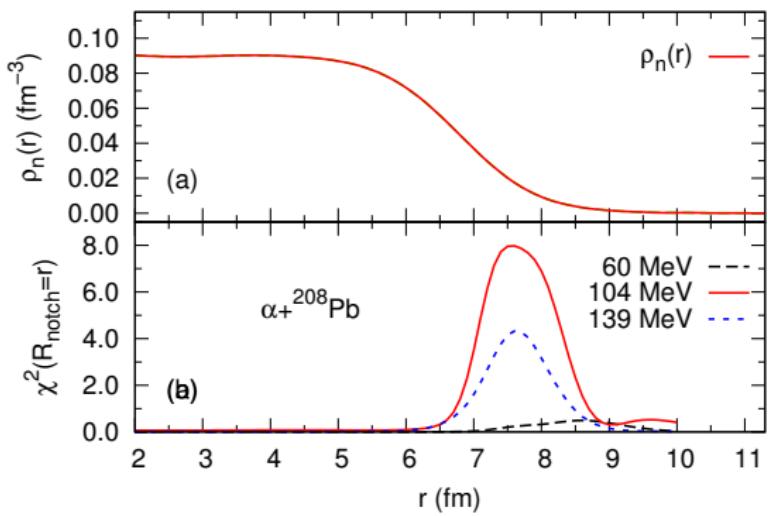
$\alpha$  elastic scattering at 104 MeV

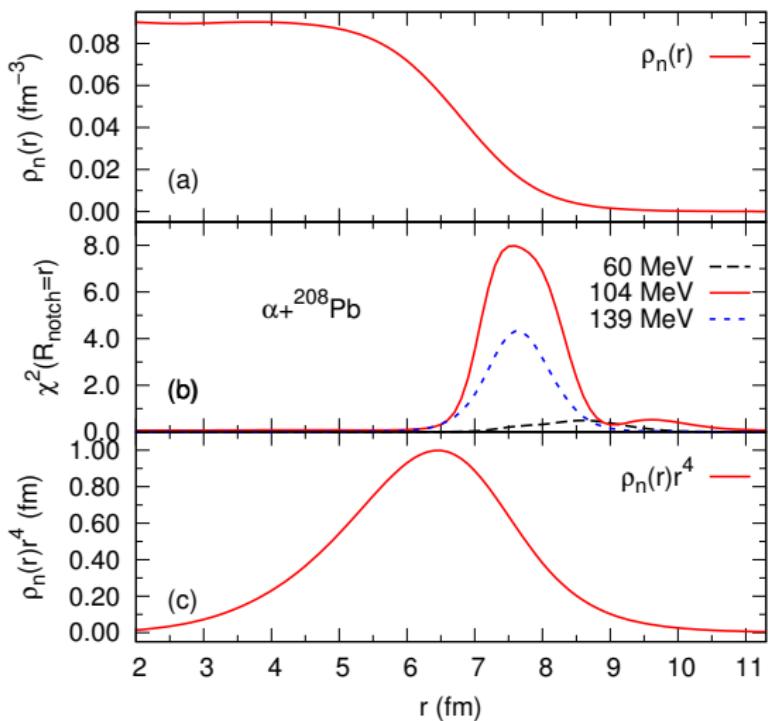


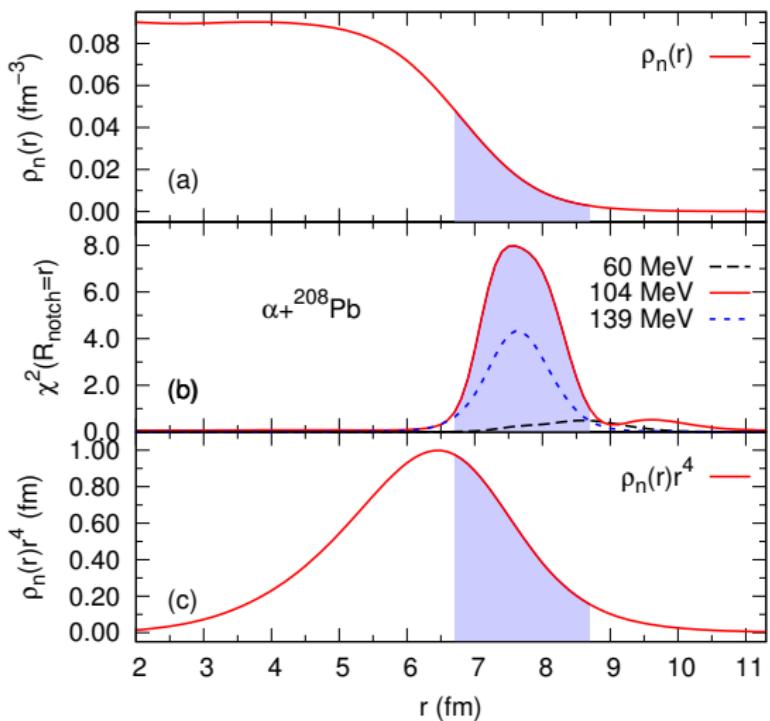
Probed region in  $\rho_n$ : a notch-test

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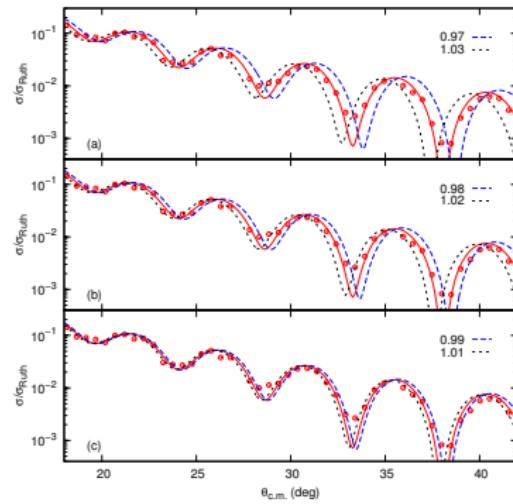
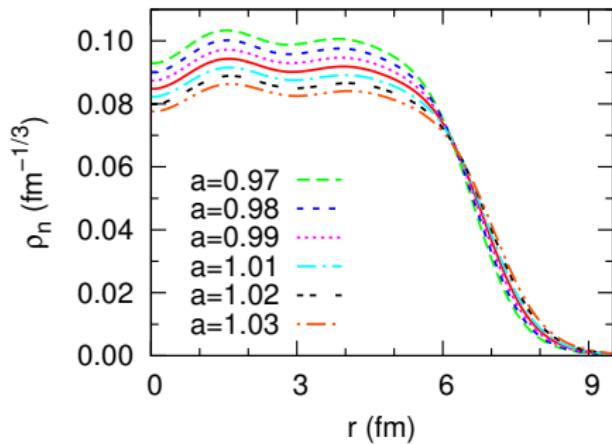
Probed region in  $\rho_n$ : a notch-test

Probed region in  $\rho_n$ : a notch-test

Probed region in  $\rho_n$ : a notch-test

# Determination of $R_n$ from $\alpha$ -scattering

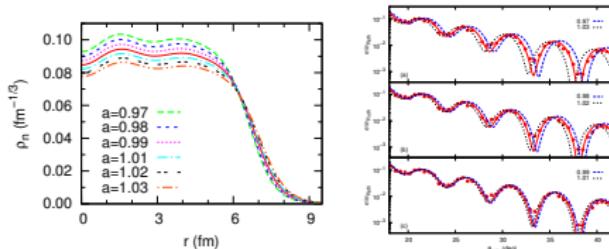
Shift of scattering angles wrt  $\Delta R_n/R_n$  ( $\alpha + {}^{208}\text{Pb}$  104 MeV)



$$\rho(r) \rightarrow \rho(r) = \frac{1}{a^3} \rho_0(r/a)$$

# Determination of $R_n$ from $\alpha$ -scattering

Shift of scattering angles wrt  $\Delta R_n/R_n$  ( $\alpha + {}^{208}\text{Pb}$  104 MeV)



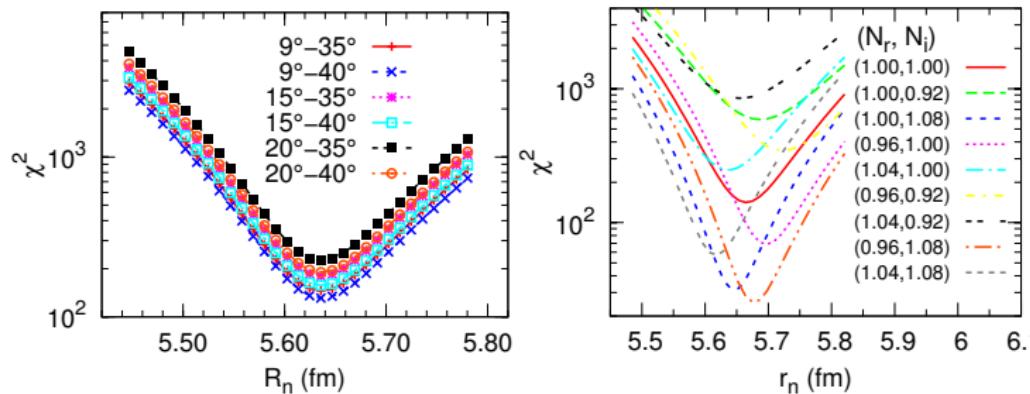
$$\rho(r) \rightarrow \rho(r) = \frac{1}{a^3} \rho_0(r/a)$$

Shift of scattering angles wrt  $E_{lab}$  (A MeV) with  $\Delta R_n/R_n = 0.01$

$E_{lab}$ (A MeV)	${}^{58}\text{Ni}$	${}^{116}\text{Sn}$	${}^{208}\text{Pb}$
20	0.22	0.32	0.45
40	0.15	0.16	0.24
80	0.06	0.07	0.08

# Determination of $R_n$ from $\alpha$ -scattering

Determine  $R_n$  of  $^{208}\text{Pb}$  with SkX at  $E_\alpha = 139$  MeV

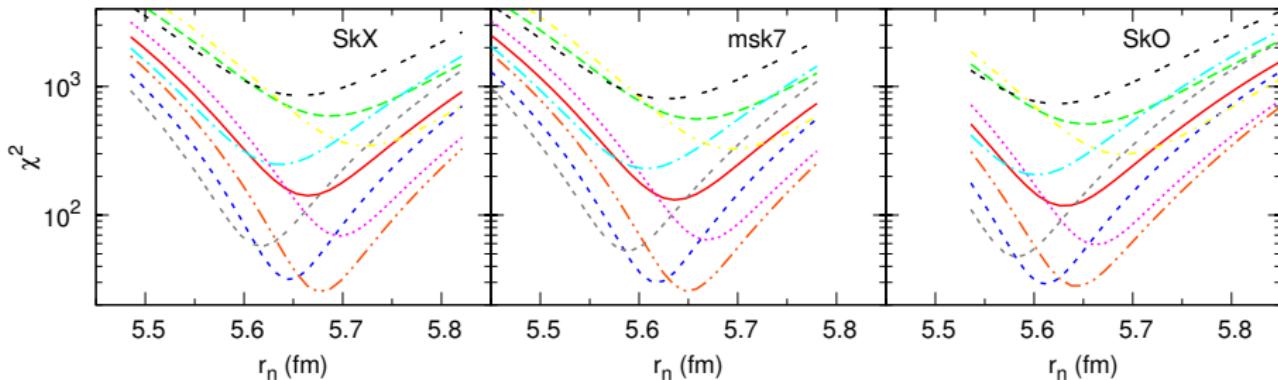


$$R_n = 5.661 \pm 0.024 \text{ fm}$$

$$\Delta r_{np} = 0.220 \pm 0.024 \text{ fm}$$

# Determination of $R_n$ from $\alpha$ -scattering

Determine  $R_n$  of  $^{208}\text{Pb}$  with SkX, SkO and MSk7 at  $E_\alpha = 139$  MeV

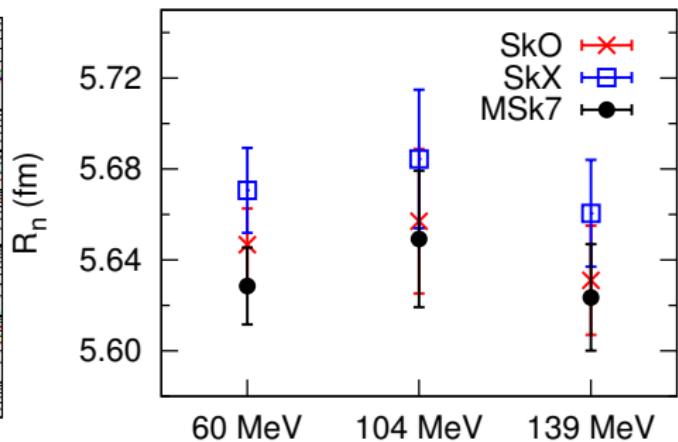
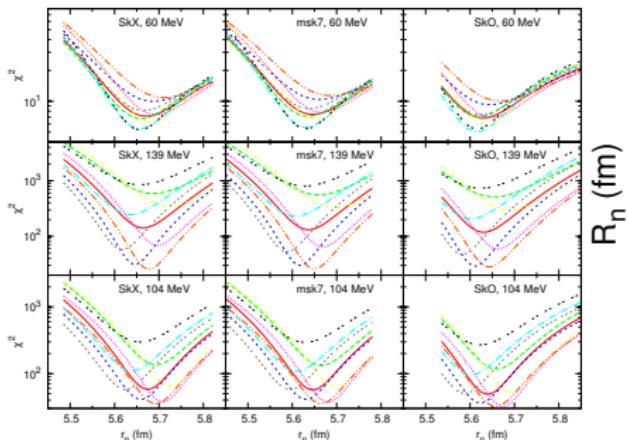


$$R_n = 5.638 \pm 0.020 \text{ fm}$$

$$\Delta r_{np} = 0.198 \pm 0.022 \text{ fm}$$

# Determination of $R_n$ from $\alpha$ -scattering

Determine  $R_n$  of  $^{208}\text{Pb}$  at  $E_\alpha = 60, 104$  and  $139$  MeV

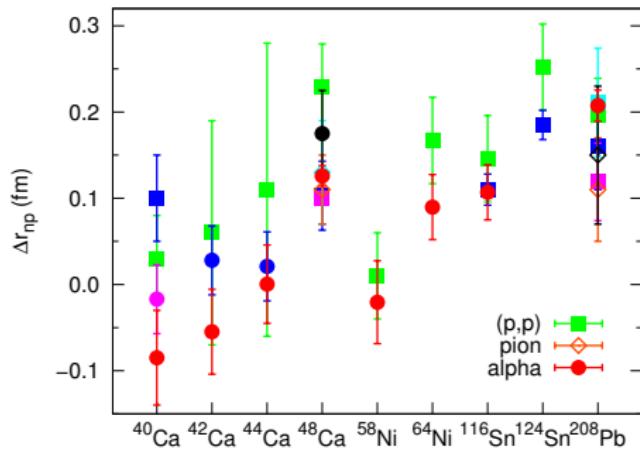


$$R_n = 5.647 \pm 0.019 \text{ fm}$$

$$\Delta r_{np} = 0.207 \pm 0.021 \text{ fm}$$

# Results for $^{48}\text{Ca}$ , $^{64}\text{Ni}$ , $^{124}\text{Sn}$ , and $^{208}\text{Pb}$

## Neutron-skin thickness: a compilation



R.H. McCamis et al., PRC 33, 1624 (1984); C.J. Batty et al., Advances in Nuclear Physics, Vol. 19 (1989); E. Friedman, NPA 896, 46 (2012); H.J. Gils et al., PRC 29, 1295 (1984); L. Ray, PRC 19, 1855 (1979); B.C. Clark et al., PRC 67, 054605 (2003); S. Shlomo, Rep. Prog. Phys. 41, 957 (1978); S. Terashima and J. Zenihiro PhD thesis

# The EoS parameters

The nuclear symmetry energy  $E_{\text{sym}}(\rho)$  at nuclear density  $\rho$  can be expanded around the nuclear matter saturation density  $\rho_0$  as

$$E_{\text{sym}}(\rho) = E_{\text{sym}}(\rho_0) + \frac{L}{3} \left( \frac{\rho - \rho_0}{\rho_0} \right) + \frac{K_{\text{sym}}}{18} \left( \frac{\rho - \rho_0}{\rho_0} \right)^2 \quad (1)$$

where  $L$  and  $K_{\text{sym}}$  are the slope and curvature of the nuclear symmetry energy at  $\rho_0$ , i.e.,

$$L = 3\rho_0 \frac{\partial E_{\text{sym}}(\rho)}{\partial \rho} \Big|_{\rho=\rho_0} \quad (2)$$

Lie-Wen Chen et al., Phys. Rev. C 72, 064309 (2005).

# Constraints of $R_n$ on EoS parameters

## Correlation between $\Delta r_{np}$ and Nuclear matter properties

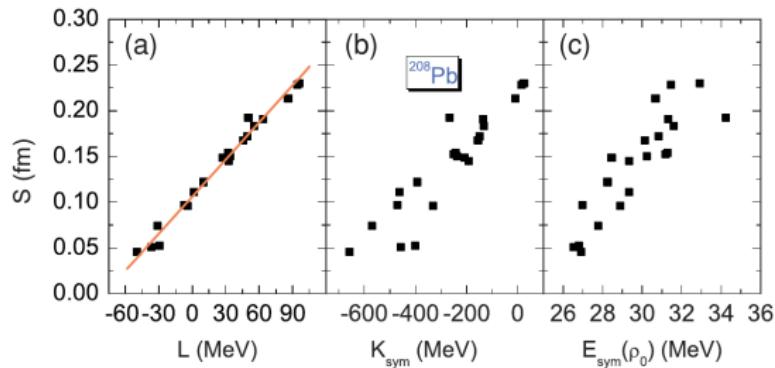
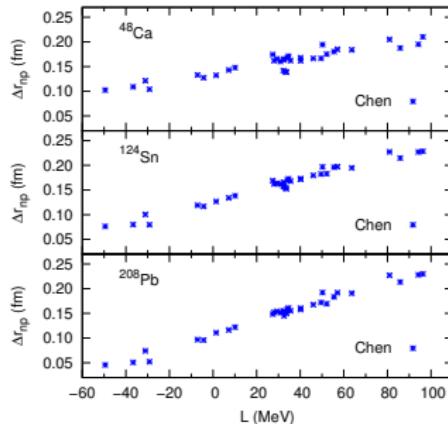


FIG. 2. (Color online) Neutron skin thickness  $S$  of  $^{208}\text{Pb}$  as a function of (a)  $L$ , (b)  $K_{\text{sym}}$ , and (c)  $E_{\text{sym}}(\rho_0)$  for 21 sets of Skyrme interaction parameters. The line in panel (a) represents a linear fit.

Lie-Wen Chen et al., PRC 72, 064309 (2005); B. Alex Brown, Phys. PRL 85, 5296 (2000); M. Centelles, et al., PRL 102, 122502 (2009).

# Correlation between $\Delta r_{np}$ and $L$

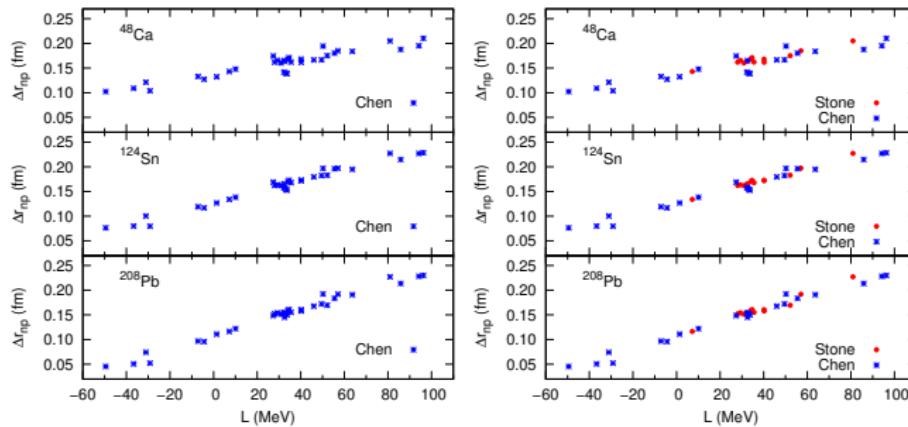
HF calculations with Skyrme parameters



Blue points: 21 sets of parameters used Lie-Wen Chen et al., PRC 72, 064309 (2005);

# Correlation between $\Delta r_{np}$ and $L$

## HF calculations with Skyrme parameters



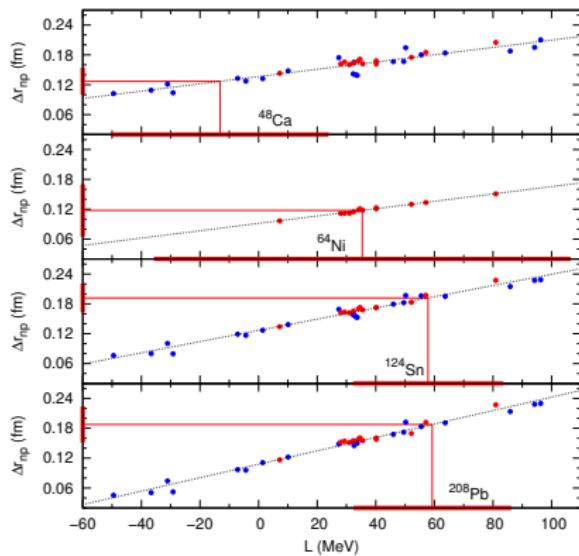
Blue points: 21 sets of parameters used Lie-Wen Chen et al., PRC 72, 064309 (2005);

Red points: 13 sets of parameters in P. Klüpfel et al., PRC 79, 034310 (2009);

The correlations seems not depend on the choice of Skyrme parameters.

# $\Delta r_{np}$ constraints on the $L$ -value

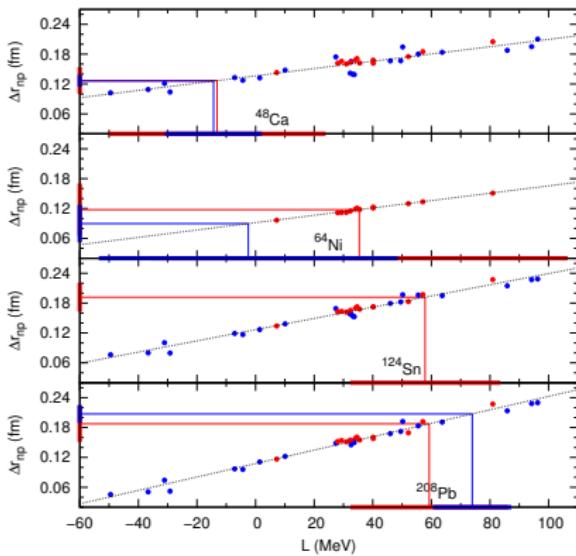
Constraints on  $L$ -value from  $\Delta r_{np}$  of  $^{48}\text{Ca}$ ,  $^{64}\text{Ni}$ ,  $^{124}\text{Sn}$  and  $^{208}\text{Pb}$



Average  $\Delta r_{np}$  of existing results weighted with their error bars

# $\Delta r_{np}$ constraints on the $L$ -value

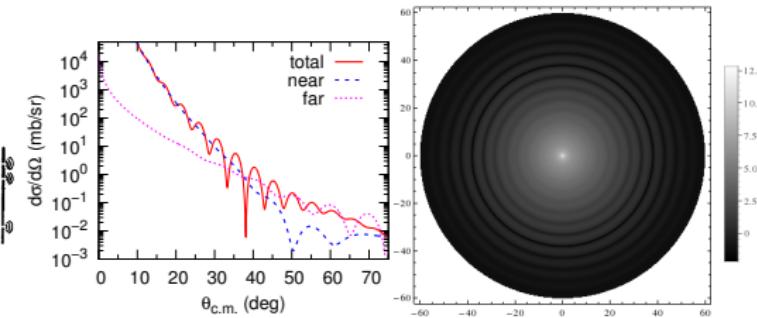
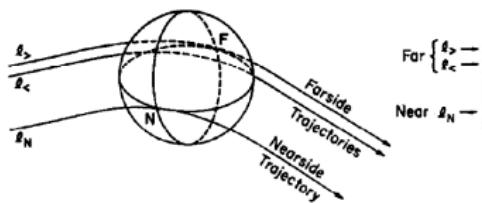
Constraints on  $L$ -value from  $\Delta r_{np}$  of  $^{48}\text{Ca}$ ,  $^{64}\text{Ni}$ ,  $^{124}\text{Sn}$  and  $^{208}\text{Pb}$



Average  $\Delta r_{np}$  of existing results weighted with their error bars

results of the present work

# Summary



$$\Delta\theta \approx \frac{\pi}{kR_{SA}}$$

- ① Angular distribution  $\Leftrightarrow$  Strong absorption radius  $R_{SA}$
- ②  $R_{SA} \Leftrightarrow$  radius of the optical potential  $R_U$
- ③  $R_U \Leftrightarrow$  neutron density distribution  $\rho_n$  with **systematic folding potential**
- ④  $\rho_n \Leftrightarrow$  neutron radius  $R_n$

thanks to

Collaborator: Prof. Jirina Stone

Prof. P.-G. Reinhard for providing the HF code and instructions

Prof. LieWen Chen for providing his HF results and

Prof. Peter Mohr for providing some elastic  $\alpha$  scattering data