Recent reaction studies on particle-unbound states with CDCC

K. Ogata¹, Y. Kikuchi², T. Myo³, T. Furumoto⁴, K. Minomo¹, T. Matsumoto⁵, and M. Yahiro⁵

¹RCNP, Osaka University, ²RIKEN Nishina Center, ³Osaka Institute of Technology, ⁴Ichinoseki National College of Technology, ⁵Kyushu University
Exploration of unbound (but not free) systems

Our Aim

Dynamical description of *Formation and Decay* of unbound systems

Today’s topic
1. Form of $^{22}$C* in a breakup observable
2. Decay mode of the $2_1^+$ state of $^6$He
COSM-CDCC for $^{22}$C breakup by $^{12}$C

**Structural part:** Cluster Orbital Shell Model (COSM)

- Core + valence $N$ system is described well.
- Pseudo states covering large space are obtained.

Details of COSM:
COSM-CDCC for $^{22}\text{C}$ breakup by $^{12}\text{C}$

**Reaction part:** Four-body CDCC

![Diagram showing eigenstates obtained by diagonalization, discretized-continuum states, and their relation to $^{22}\text{C}$ internal wave functions and relative motion between $^{22}\text{C}$ and target.]

Details of four-body CDCC:
T. Matsumoto *et al.*, PRC **70**, 061601(R) (2004); ibid. 73, 051602(R) (2006).
CSM Smoothing
(CSM: Complex-Scaling Method)

T. Matsumoto, Kato, and Yahiro, PRC 82, 054602(R) (2010).

Eigenstates of $H^\theta$
(complex-scaled Hamiltonian)

\[ \tilde{T}_i^\theta = \sum_n \left\langle \tilde{\phi}_i^\theta \left| C(\theta) \right| \Phi_n \right\rangle T_{n}^{\text{CDCC}} \]

index for the pseudostates $\Phi_n$ used in CDCC

\[ \frac{d\sigma}{d\epsilon} = \frac{1}{\pi} \text{Im} \sum_i \frac{T_i^\theta \tilde{T}_i^\theta}{\epsilon - \epsilon_i} \]

index for the eigenstates $\phi_i^\theta$ of $H^\theta$
Microscopic CDCC

$n + n + c$ dynamics explicitly described
The CSM smoothing* is adopted to obtain the BUX.

COSM predicts the following resonances:

- **$^{22}$C resonance**
  - $0^+_2$: $1.02 - i 0.52/2$
  - $2^+_1$: $0.86 - i 0.10/2$
  - $2^+_2$: $1.80 - i 0.26/2$

- **$^{21}$C resonance**
  - $d_{3/2}^-$: $1.1 - i 0.10/2$

How are these resonances observed?

*T. Matsumoto et al., PRC 82, 054602(R) (2010).*
A new smoothing method* is adopted to obtain the BUX.

COSM predicts the following resonances:

- $^{22}\text{C}$ resonance $02^+: 1.02 - i 0.52/2$
- $^{22}\text{C}$ resonance $21^+: 0.86 - i 0.10/2$
- $^{22}\text{C}$ resonance $22^+: 1.80 - i 0.26/2$

- $^{21}\text{C}$ resonance $d_{3/2}: 1.1 - i 0.10/2$

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DDBUX of $^{22}$C by $^{12}$C

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✓ COSM predicts the following resonances:

$^{22}$C resonance
- $0^+_2$: $1.02 - i 0.52/2$
- $2^+_1$: $0.86 - i 0.10/2$
- $2^+_2$: $1.80 - i 0.26/2$ negligible

$^{21}$C resonance
- $d_{3/2}^-$: $1.1 - i 0.10/2$ negligible

How are these resonances observed?

*T. Matsumoto et al., PRC 82, 054602(R) (2010).
The narrow peak around 0.8 MeV is due to the $2_1^+$ resonance of $^{22}\text{C}$.

The shape of the $0_2^+$ resonance is due to background phase effect.
BackGround Phase (BGP) effect

- In nuclear physics, we always have $\delta_{bg}$.
- There are many examples of this effect in many research fields.
- In most cases, this effect is observed as small changes in the resonance energy and width.

\[
S(\epsilon) = e^{2i\delta_{bg}(\epsilon) + 2i\delta_{res}(\epsilon)}
\]
\[
= e^{2i\delta_{bg}(\epsilon)} \frac{\epsilon - \epsilon_{res} - i\Gamma/2}{\epsilon - \epsilon_{res} + i\Gamma/2}
\]
The BGP effect is indeed sizable.

- We have a variety of patterns of the resonant (and $0^+$) cross section.
- Appear in only the $0^+$ state
The BGP effect is indeed sizable.
We have a variety of patterns of the resonant (and $0^+$) cross section.
Appear in only the $0^+$ state
Summary of the 1st topic

What is the form of $^{22}\text{C}^*$ in a breakup observable?

KO, Myo, Furumoto, Matsumoto, Yahiro, PRC 88, 024616 (2013).

✓ The $2_1^+$ state: Breit-Wigner form
✓ The $0_2^+$ state: peculiar form due to the BGP effect (coexistence of the $0^+$ resonant and nonresonant waves)
✓ The BGP has a strong scattering-angle dependence.
✓ We should be careful to identify the $0_2^+$ state of $^{22}\text{C}$ in the observables.
What is the decay mode of the $2_{1}^{+}$ state of $^{6}$He?


Sequential decay  di-neutron decay  democratic decay
CDCC-CSLS

✓ The method of Complex-Scaled solutions of the Lippmann-Schwinger Eq.


$$T (p, k) = \left\langle \Phi^{(-)} (p, k) e^{iK \cdot R} \left| U \right| \Psi^{CDCC} \right\rangle$$

$$= \sum_{n} \left\langle \Phi_{n} \right\rangle \left\langle \Phi_{n} \right| \approx 1$$

$$\approx \sum_{n} \left\langle \Phi^{(-)} (p, k) \left| \Phi_{n} \right\rangle T_{n}^{CDCC} \right|$$

$$\equiv f_{n} (p, k)$$

$$f_{n} (p, k) = \left\langle \varphi_{\text{free}} (p, k) \left| \Phi_{n} \right\rangle + \sum_{i} \left\langle \varphi_{\text{free}} (p, k) \left| V_{\alpha nn} C^{-1} (\theta) \right| \phi_{i} \right\rangle$$

$$\times \frac{1}{\varepsilon - \varepsilon_{i}^{\theta}} \left\langle \tilde{\phi}_{i}^{\theta} \left| C (\theta) \right| \Phi_{n} \right\rangle$$
Sequential decay quenched

When \( \varepsilon \sim 1 \text{ MeV} \) and \( \varepsilon_{\alpha-n} \sim 0.7 \text{ MeV} \), the other neutron \( (\sim 0.3 \text{ MeV}) \) hardly penetrates the centrifugal barrier (\( p \)-wave).

The peak of the green line suggests the di-neutron decay or the democratic decay.
Coexistence of two decay modes

\[ ^6\text{He}(2_1^+) \]

The lower peak suggests the di-neutron decay due to the Fin. State Int. (FSI).

The higher peak indicates the democratic decay.

✓ The lower peak suggests the di-neutron decay due to the Fin. State Int. (FSI).
✓ The higher peak indicates the democratic decay.

→ Decay of a di-neutron in the \( 2_1^+ \) state not due to the FSI.
Summary of the 2nd topic

What is the decay mode of the $2_1^+$ state of $^6$He?


Sequential decay  di-neutron decay  democratic decay
What is the decay mode of the $2_1^+$ state of $^6\text{He}$?


- **Sequential decay**
- **Di-neutron decay** (due to FSI)
- **Democratic decay** (not due to FSI)
Exploration of unbound (but not free) systems

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Numerical inputs

**22C wave function**

- Minnesota force for $n-n$, Woods-Saxon potential for $n-^{20}C$.
- $s_{1/2}, p_{3/2}, p_{1/2}, d_{5/2}, d_{3/2}, f_{7/2}, f_{5/2}, g_{9/2}, g_{7/2}, h_{11/2},$ and $h_{9/2}$ for the $n$ s.p. orbit.
- Each orbit is described by 10 Gaussian basis functions.


$0^+$ ground state with $S_{2n} = 289$ keV, 604 $0^+$ and 1,385 $2^+$ PS

**22C-$^{12}C$ breakup reaction**

- 77 ($0^+$) + 164 ($2^+$) PS below 10 MeV are included as breakup states of $^{22}C$.
- Distorting potentials are calculated by a microscopic folding model with CEG07 nucleon-nucleon $g$ matrix.
- We adopt the so-called no-recoil approximation for the $^{20}C$ core nucleus.

p-12C scattering at 250 MeV

Nuclear density: L. C. Chamon et al., PRC 66, 014610 (2002) [Sao Paulo group]
Complex Scaling Method (CSM)

S. Aoyama, T. Myo, K. Kato, and K. Ikeda,

Complex-scaling operator: \( U^\theta \)

\[
U^\theta f(r) = e^{i3/2\theta} f(re^{i\theta})
\]

Coordinate: \( r \rightarrow re^{i\theta} \)

Momentum: \( k \rightarrow ke^{-i\theta} \)

Asymptotic form

\[
e^{ikr} \rightarrow e^{ikr} \cos \theta e^{-kr \sin \theta}
\]

Useful for searching many-body resonances

Green’s function with Complex-Scaling Method (CDCS Green’s function)

\[
\mathcal{G}^{(-)}(E, \xi, \xi') = \frac{1}{E - H - i\epsilon} \approx \sum_{\nu} U^{-\theta} \frac{\langle \Phi^\theta_{\nu} \rangle \langle \tilde{\Phi}^\theta_{\nu} \rangle}{E - E^\theta_{\nu}} U^\theta
\]
New Smoothing Procedure with \textit{CSM}


\[ \frac{d\sigma}{dE} = \int T^\dagger(E') T(E') \delta(E - E') dE' = \frac{1}{\pi} \text{Im} \mathcal{R}(E) \]

\[ T(E) = \langle \Psi^(-)(E, \xi) \chi_C^(-)(R) | V | \Psi^+(\xi, R) \rangle \]

\textbf{Response function}

\[ \mathcal{R}(E) = \int d\xi d\xi' \langle \Psi^+(\xi, R) | V^* | \chi_C^(-)(R) \rangle_{R} \mathcal{G}^(-)(E, \xi, \xi') \langle \chi_C^(-)(R) | V | \Psi^+(\xi, R) \rangle_{R} \]

\textbf{Green’s function with Complex-Scaling Method (CDCS Green’s function)}

\[ \mathcal{G}^(-)(E, \xi, \xi') = U^{-\theta} \frac{1}{E - H^\theta - i\epsilon} U^\theta \approx \sum_\nu U^{-\theta} \frac{\langle \Phi_{\nu}^\theta | \tilde{\Phi}_{\nu}^\theta \rangle}{E - E_{\nu}^\theta} U^\theta \]

\[ \mathcal{G}^(-)(E, \xi, \xi') \approx \sum_\nu \sum_{i,j} \langle \Phi_i | U^{-\theta} | \Phi_{\nu}^\theta \rangle \langle \tilde{\Phi}_{\nu}^\theta | U^\theta | \Phi_j \rangle \frac{\langle \Phi_{\nu}^\theta | \tilde{\Phi}_{\nu}^\theta \rangle}{E - E_{\nu}^\theta} \langle \Phi_j | \}

\[ \mathcal{R}(E) = \sum_\nu \sum_{i,j} \langle \Psi^+(\xi, R) | V^* | \chi_C^(-)(R) \Phi_i \rangle \frac{\langle \Phi_i | U^{-\theta} | \Phi_{\nu}^\theta \rangle \langle \tilde{\Phi}_{\nu}^\theta | U^\theta | \Phi_j \rangle}{E - E_{\nu}^\theta} \langle \Phi_j \chi_C^(-)| V | \Psi^+(\xi, R) \rangle \]

\textbf{T-matrix calculated by CDCC}

Courtesy of Matsumoto
The complex-scaling method classifies the continuum states of $^{22}$C.

Why so large BGP effect?

s-wave neutrons have no barriers

Nonresonant $0^+$

Monopole transition

$80\%$ $13\%$

$0^+$ ground state

$2$ d-wave neutrons form $0^+$ resonance

Coexistence in low-energy continuum state

$\checkmark$ In a core $+n$ system, this will hardly be realized.

$\checkmark$ This resonant-nonresonant $0^+$ coexistence is expected for (s-wave) two-neutron halo nuclei generally.

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This resonant-nonresonant $0^+$ coexistence is expected for (s-wave) two-neutron halo nuclei generally.
平滑化関数（PS法）

固有値に対応する$k$にピークを持つが、かなりの拡がりを持つ。
平滑化の実例(Av法 vs PS法)


平滑化した遷移強度は両者で極めて良く一致。