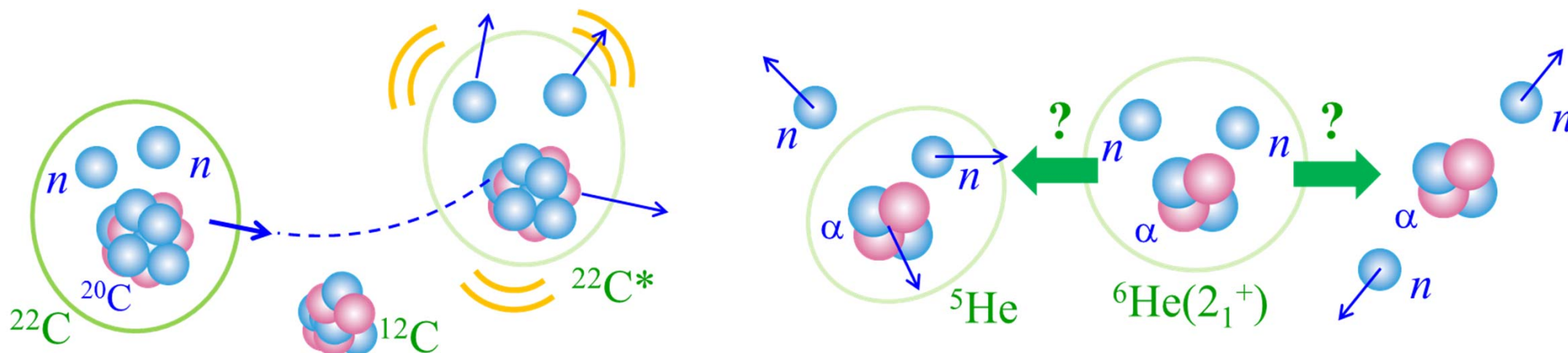


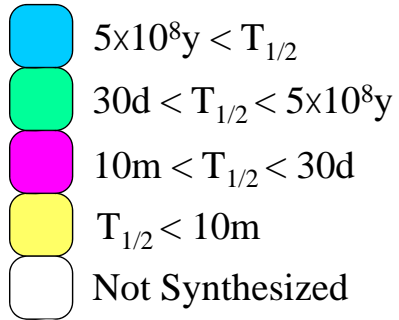
Recent reaction studies on particle-unbound states with CDCC

K. Ogata¹, Y. Kikuchi², T. Myo³, T. Furumoto⁴,
K. Minomo¹, T. Matsumoto⁵, and M. Yahiro⁵

¹*RCNP, Osaka University*, ²*RIKEN Nishina Center*, ³*Osaka Institute of Technology*,
⁴*Ichinoseki National College of Technology*, ⁵*Kyushu University*

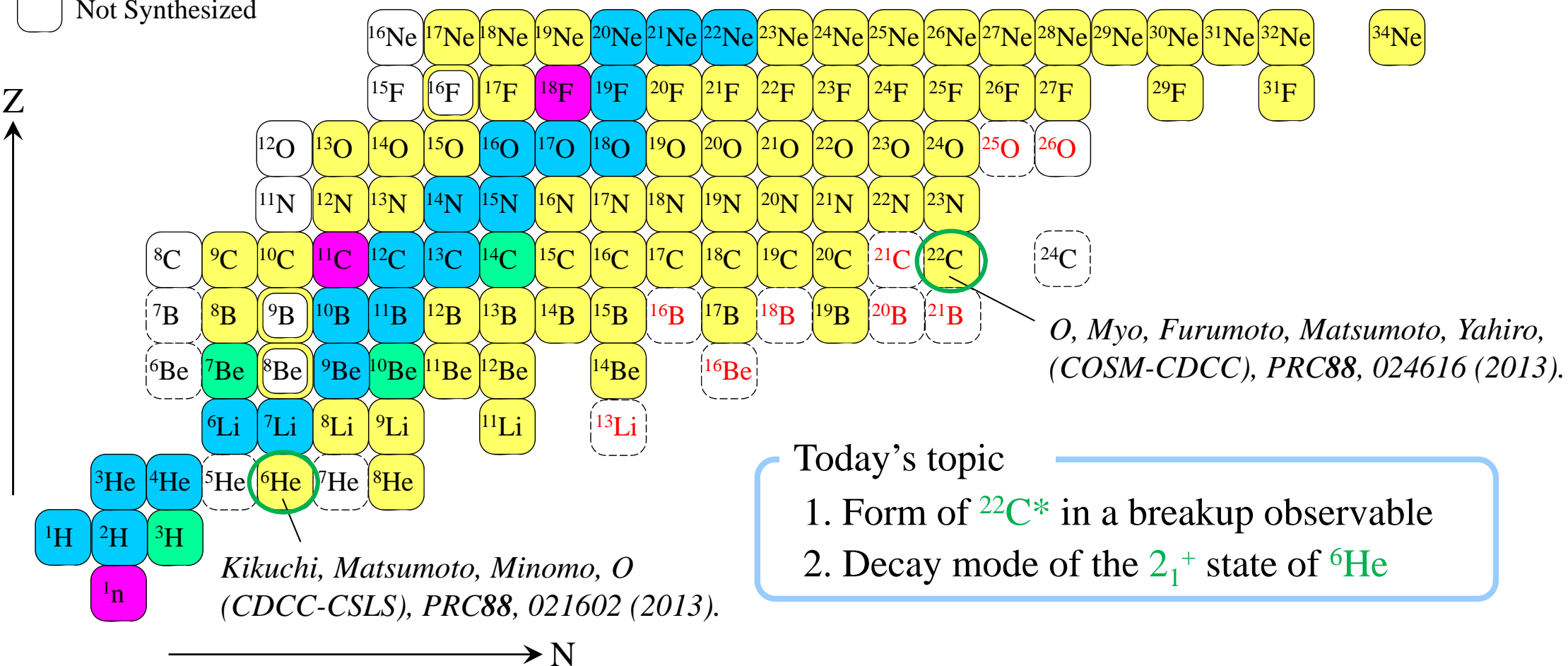


Exploration of unbound (but not free) systems



Our Aim

Dynamical description of Formation and Decay of unbound systems



Today's topic

1. Form of ${}^{22}\text{C}^*$ in a breakup observable
2. Decay mode of the 2_1^+ state of ${}^6\text{He}$

COSM-CDCC for ^{22}C breakup by ^{12}C

Structural part: Cluster Orbital Shell Model (COSM)

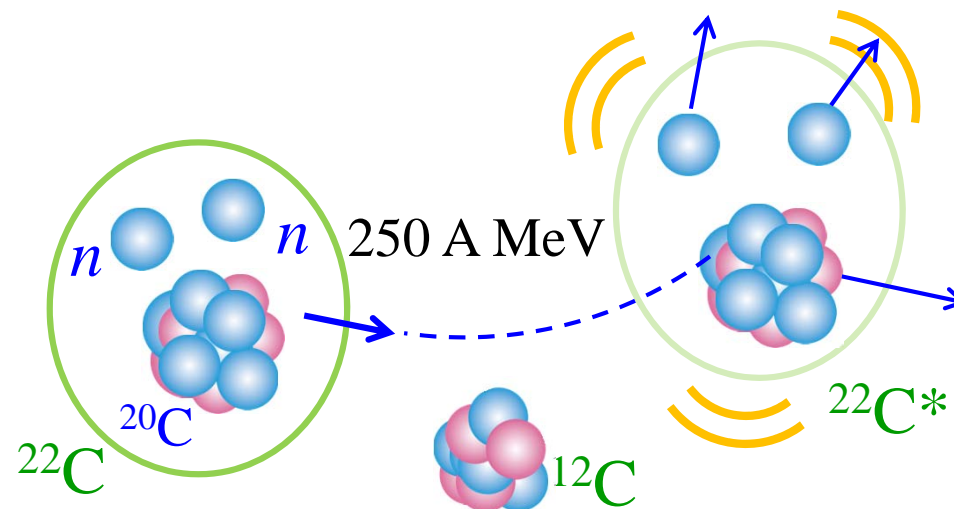
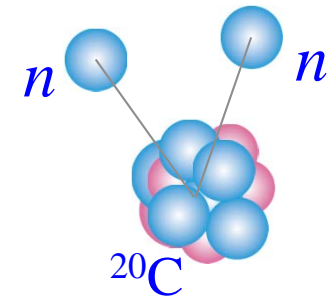
- ✓ Core + valence N system is described well.
- ✓ Pseudo states covering large space are obtained.

Details of COSM:

Y. Suzuki and K. Ikeda, PRC **38**, 410 (1988).

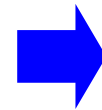
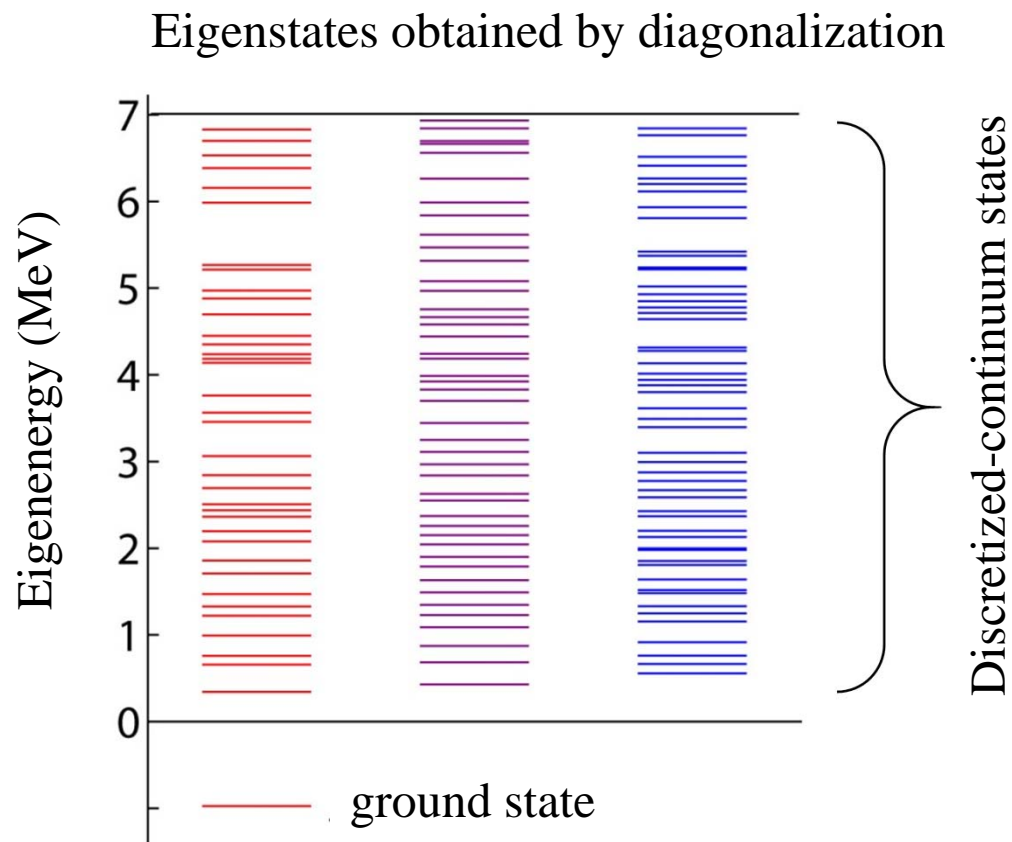
S. Aoyama *et al.*, PTP **116**, 1 (2006) [review].

T. Myo *et al.*, PL **B691**, 150 (2010) and references therein.



COSM-CDCC for ^{22}C breakup by ^{12}C

Reaction part: Four-body CDCC



Set of the ^{22}C internal wave functions
(*basis functions* for the 4-body system)

$$\Psi^{\text{CDCC}} = \sum_n \Phi_n \chi_n$$

Relative motion between ^{22}C and target
(*expansion coefficients*)

Details of four-body CDCC:

T. Matsumoto *et al.*, PRC **70**, 061601(R) (2004); *ibid.* 73, 051602(R) (2006).

M. Rodriguez-Gallardo *et al.*, PRC **80**, 051601 (2009).

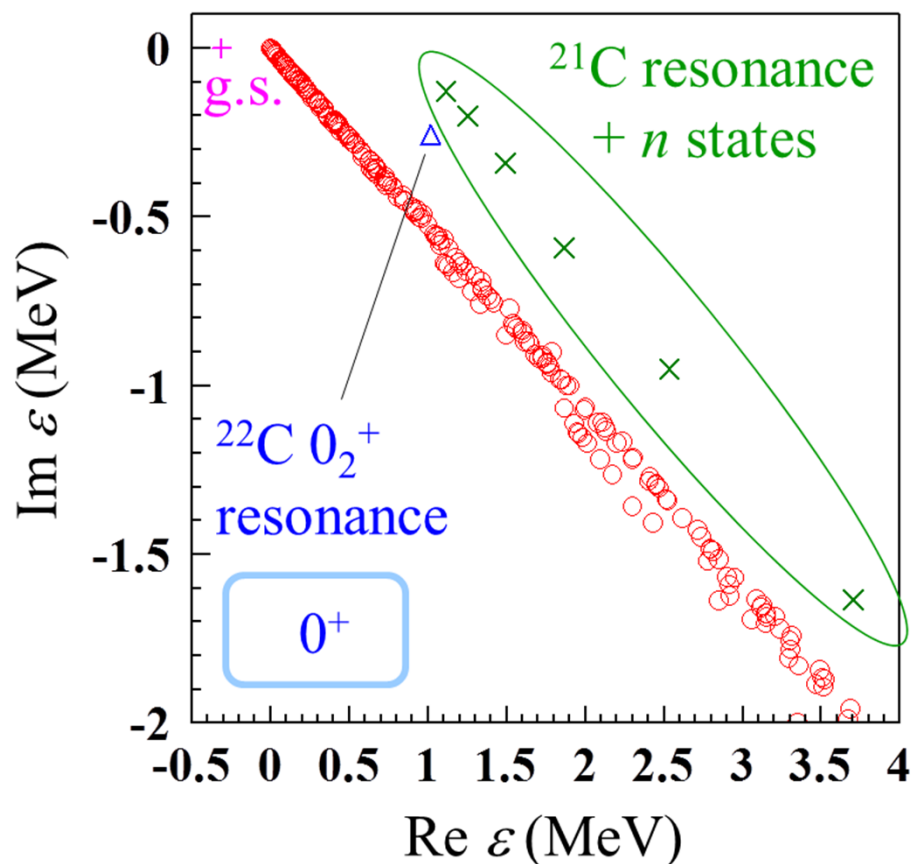
CSM Smoothing

(CSM: Complex-Scaling Method)

T. Matsumoto, Kato, and Yahiro, PRC **82**, 054602(R) (2010).

Eigenstates of H^θ

(complex-scaled Hamiltonian)



$$\tilde{\mathcal{T}}_i^\theta = \sum_n \langle \tilde{\phi}_i^\theta | C(\theta) | \Phi_n \rangle T_n^{\text{CDCC}}$$

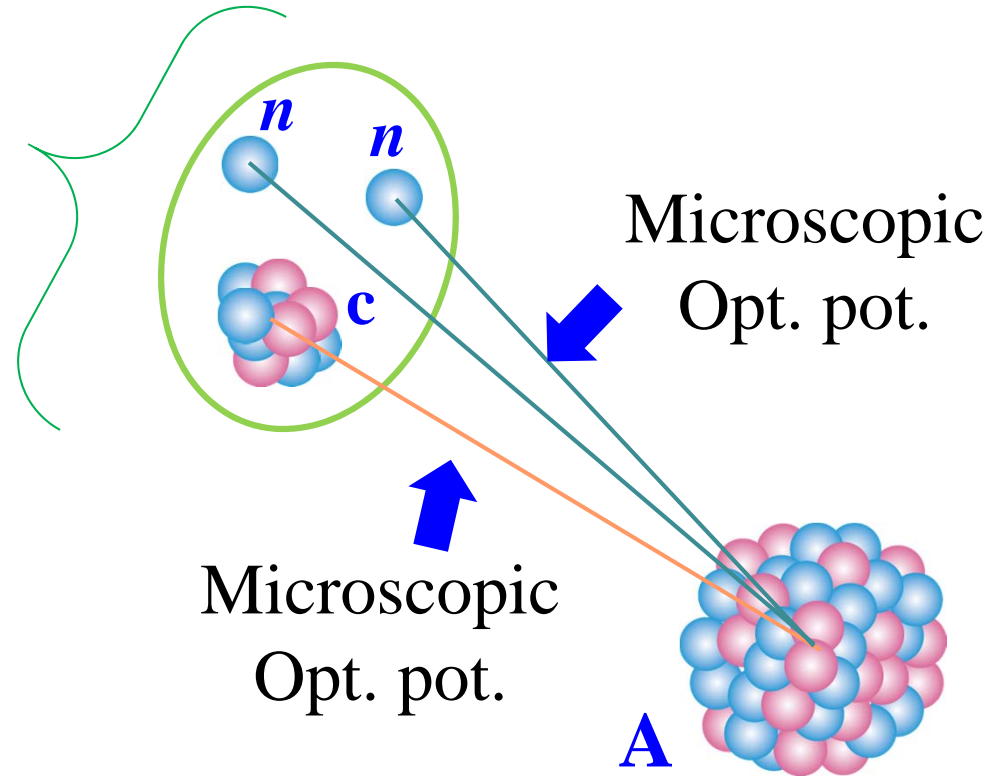
index for the pseudostates Φ_n used in CDCC

$$\frac{d\sigma}{d\epsilon} = \frac{1}{\pi} \text{Im} \sum_i \frac{\mathcal{T}_i^\theta \tilde{\mathcal{T}}_i^\theta}{\epsilon - \epsilon_i}$$

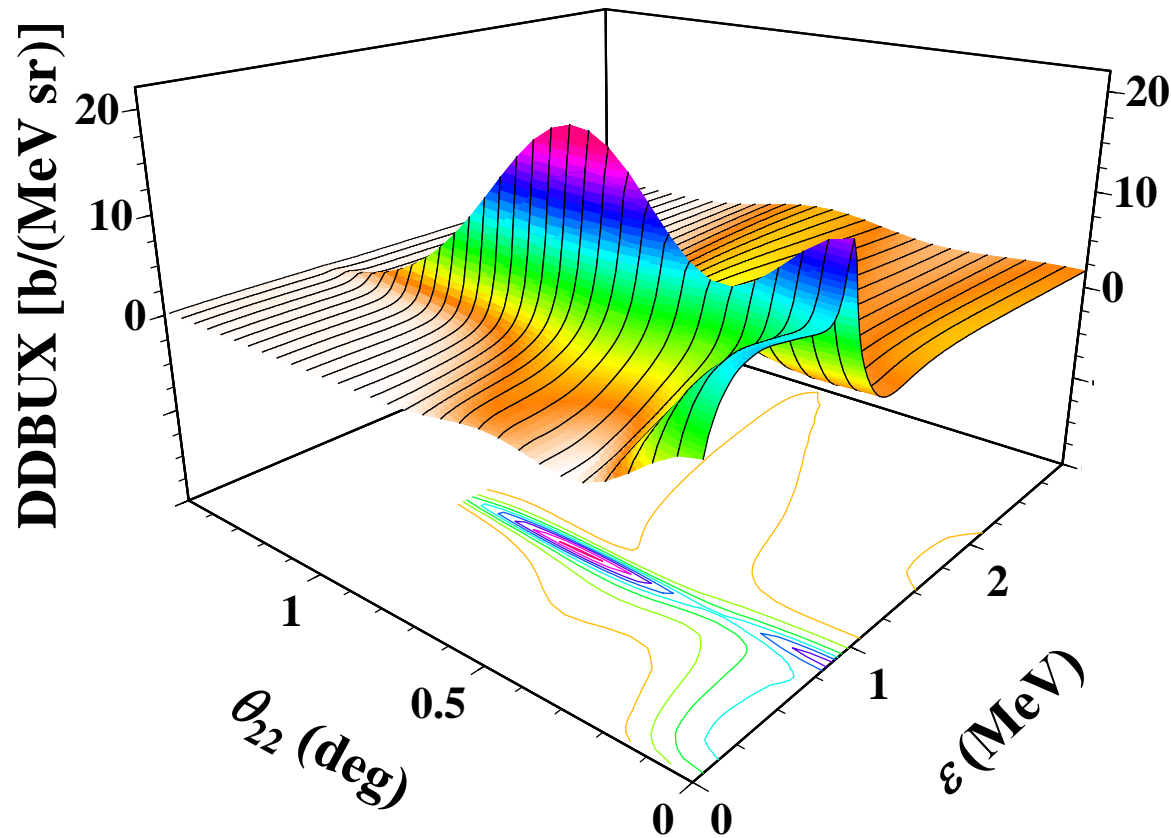
index for the eigenstates ϕ_i^θ of H^θ

Microscopic CDCC

$n + n + c$ dynamics
explicitly described



DDBUX of ^{22}C by ^{12}C



- ✓ The CSM smoothing* is adopted to obtain the BUX.
- ✓ COSM predicts the following resonances:

^{22}C resonance

$$0_2^+: 1.02 - i 0.52/2$$

$$2_1^+: 0.86 - i 0.10/2$$

$$2_2^+: 1.80 - i 0.26/2$$

^{21}C resonance

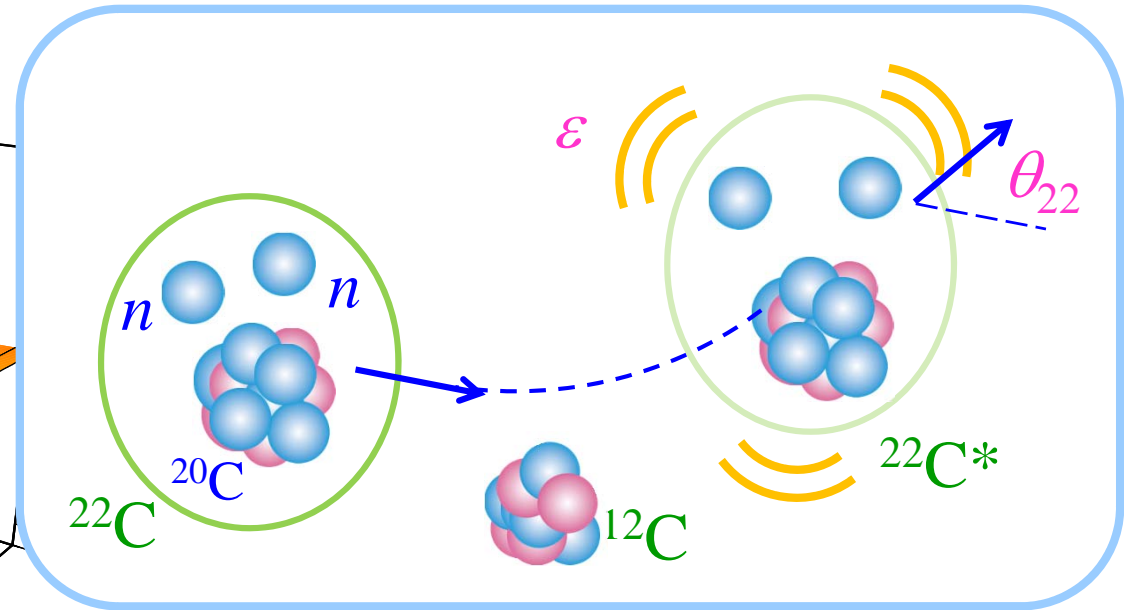
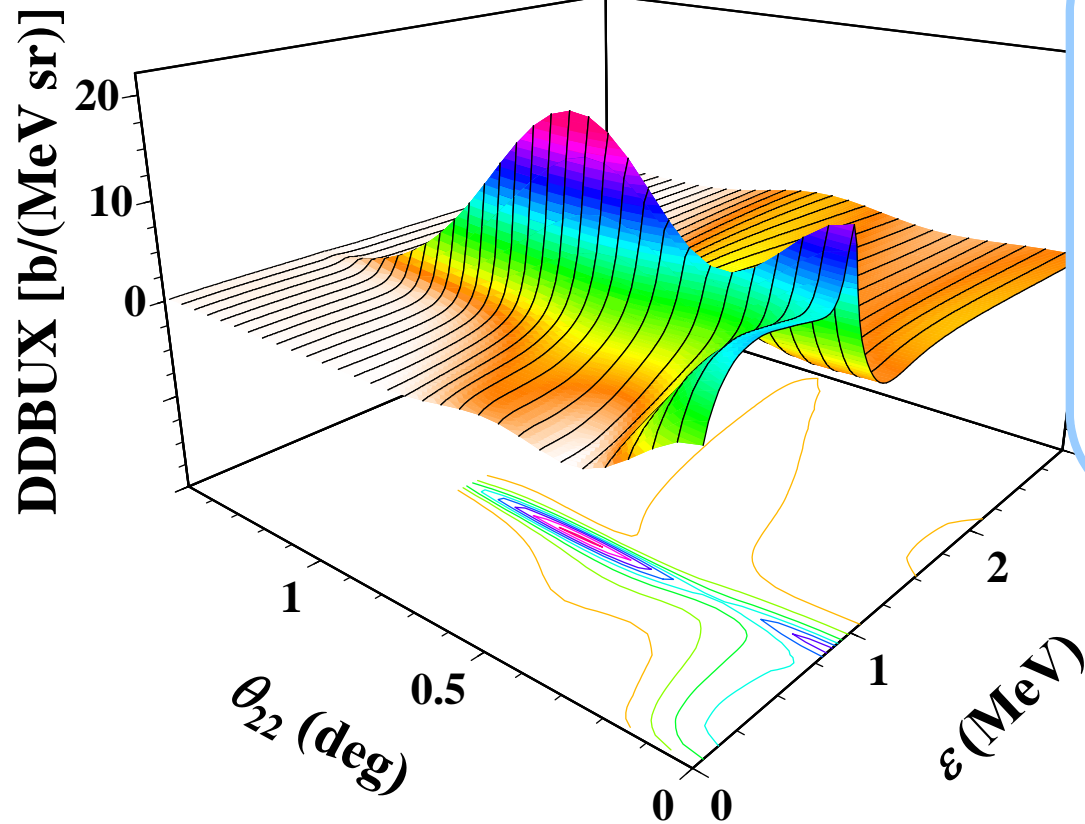
$$d_{3/2}: 1.1 - i 0.10/2$$



How are these resonances observed?

*T. Matsumoto *et al.*, PRC **82**, 054602(R) (2010).

DDBUX of ^{22}C by ^{12}C



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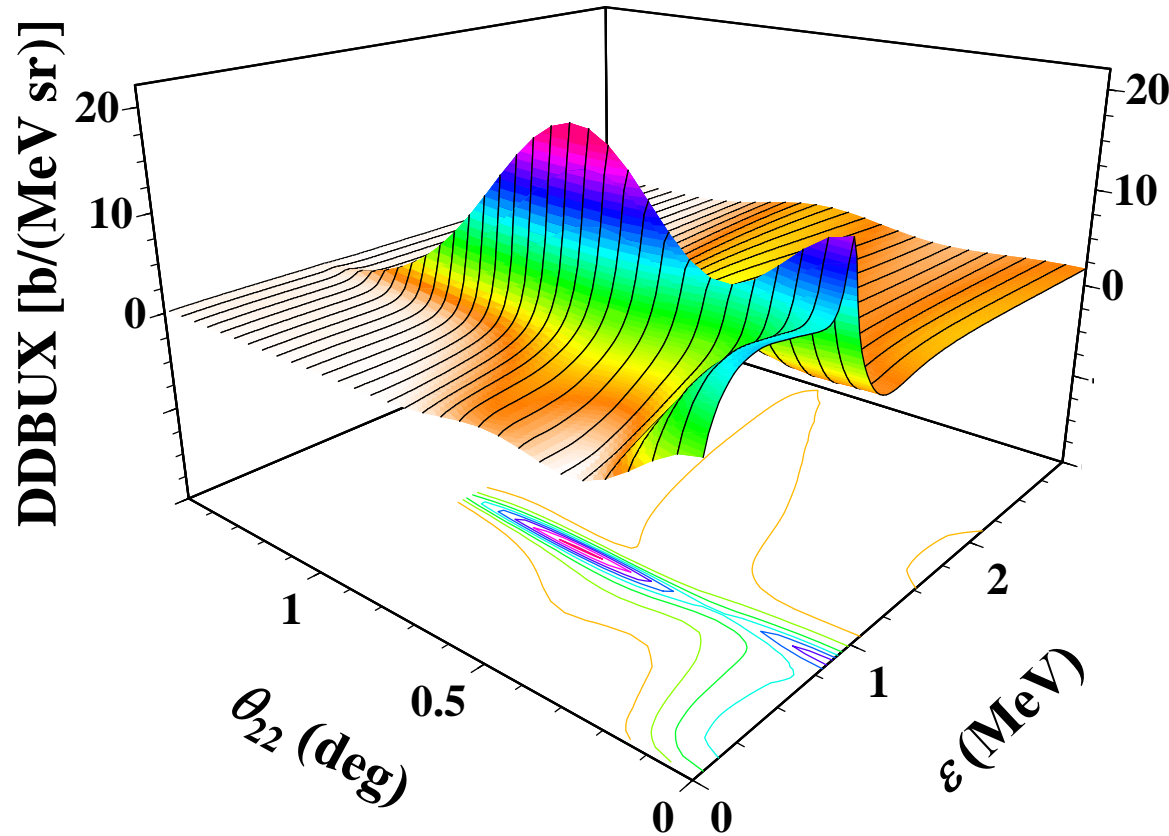
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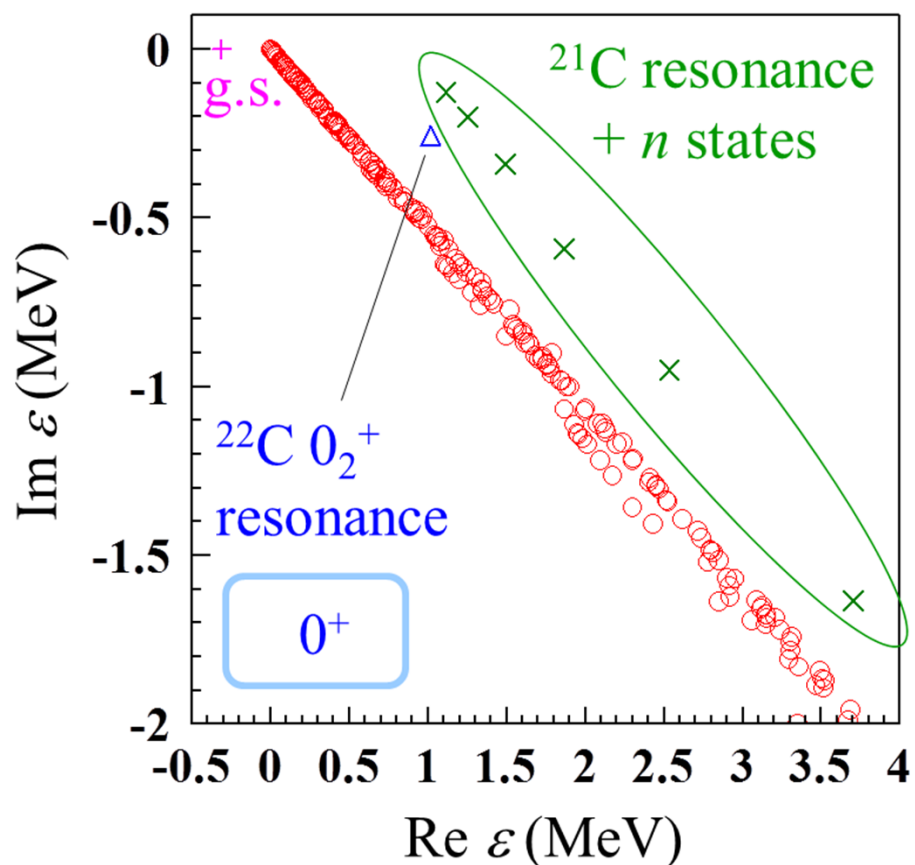
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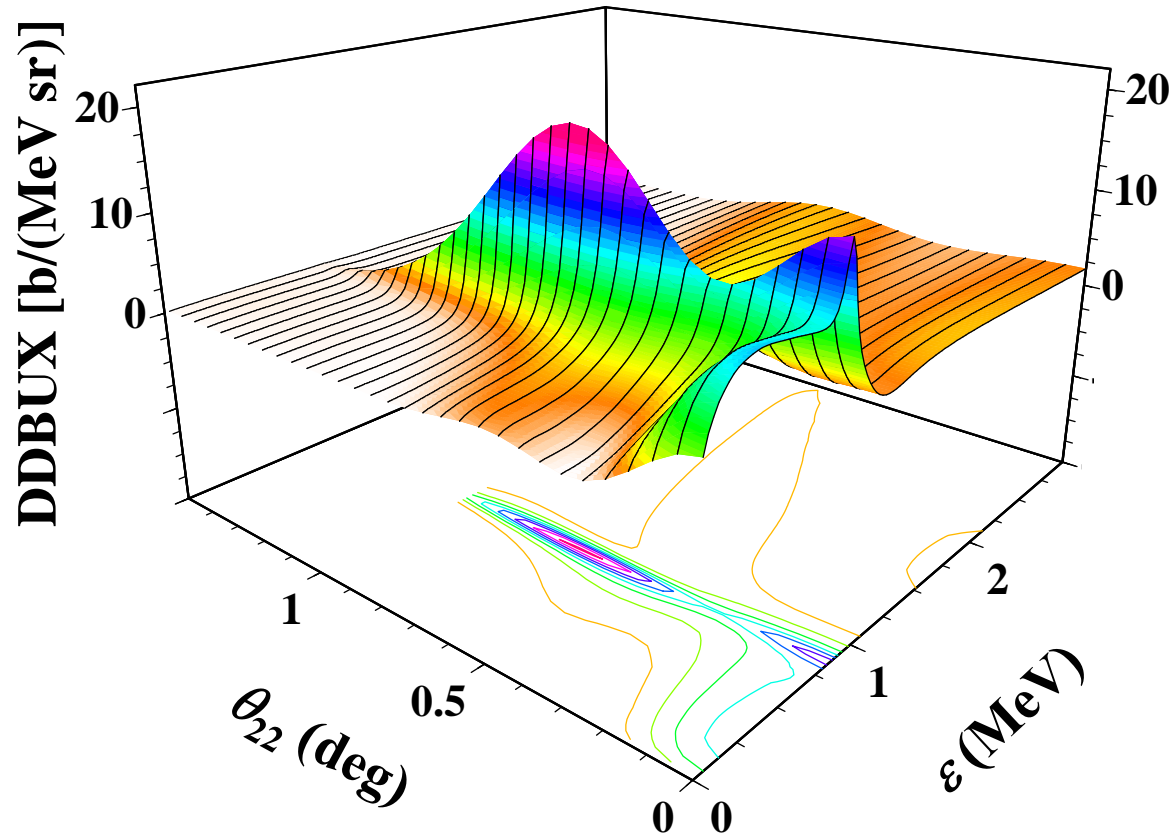
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index for the pseudostates Φ_n used in CDCC

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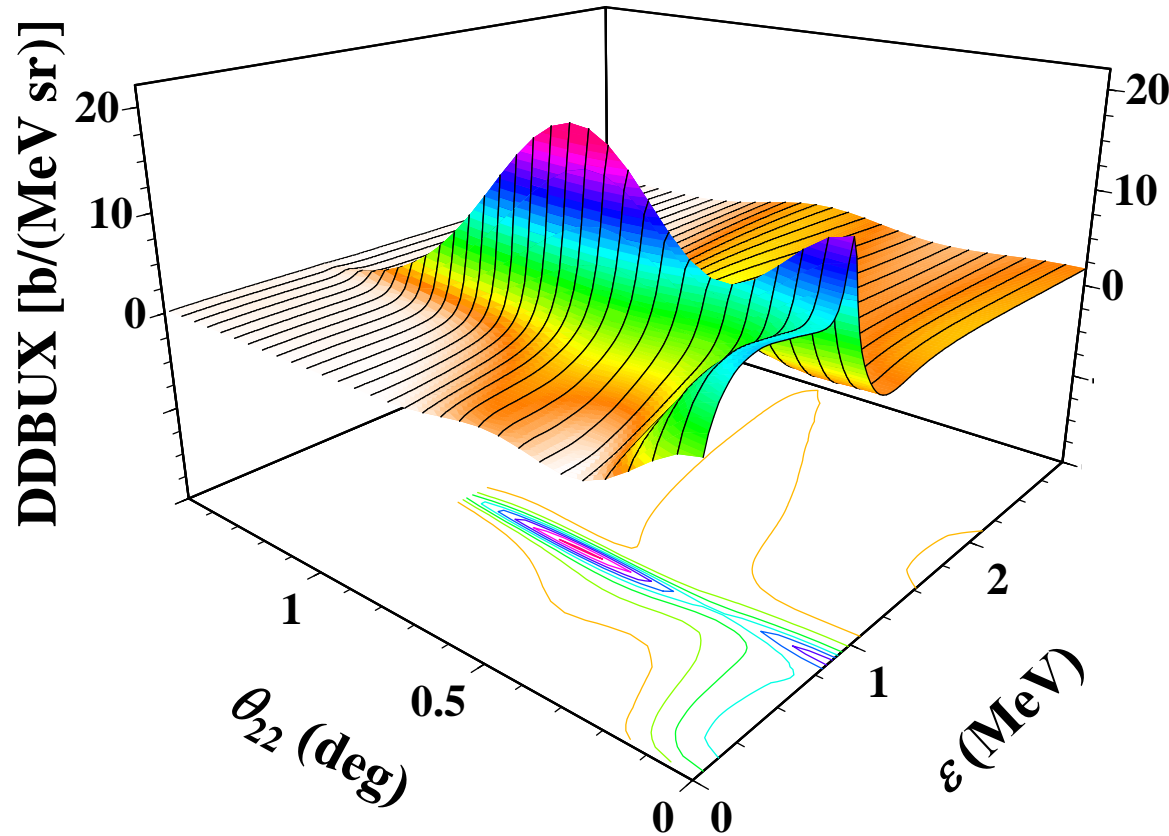
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$$0_2^+: 1.02 - i 0.52/2$$

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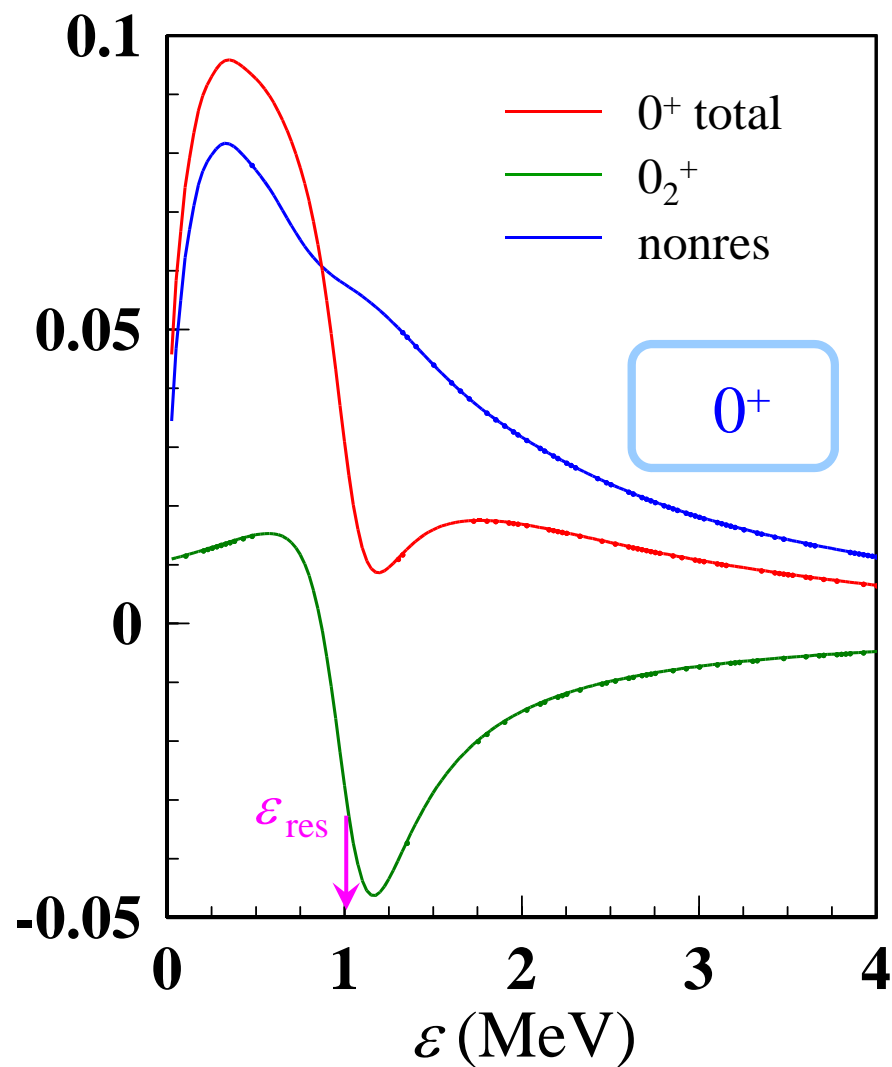
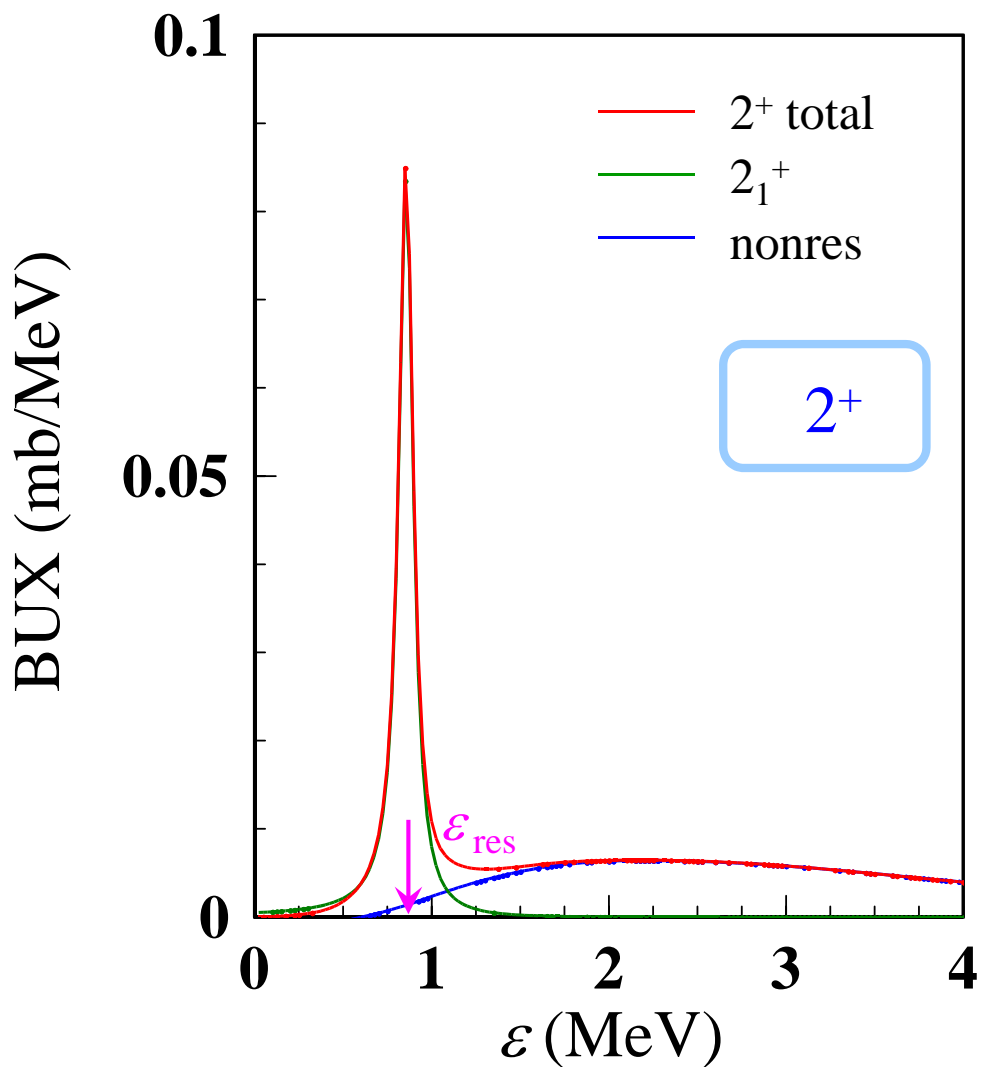
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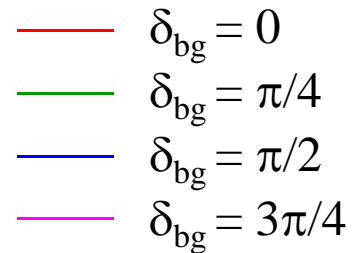
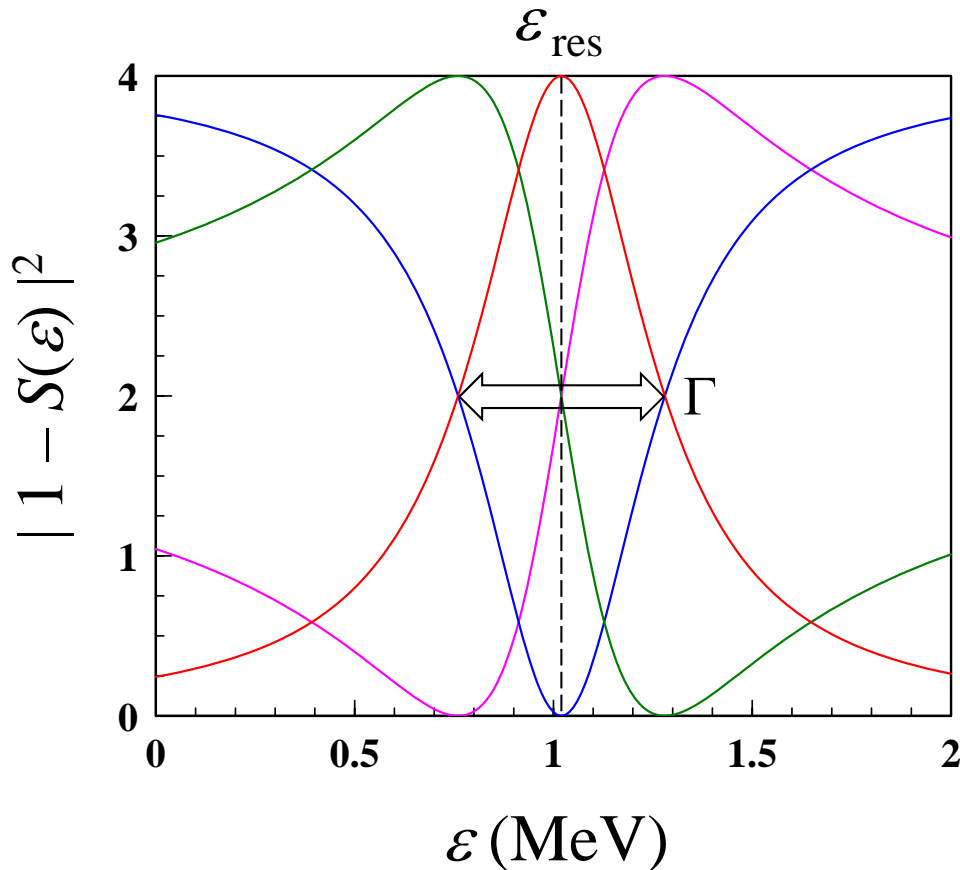
*T. Matsumoto *et al.*, PRC **82**, 054602(R) (2010).

Integrated BUX (0 – 0.1 deg)



- ✓ The narrow peak around 0.8 MeV is due to the 2_1^+ resonance of ^{22}C .
- ✓ The shape of the 0_2^+ resonance is due to **background phase effect**.

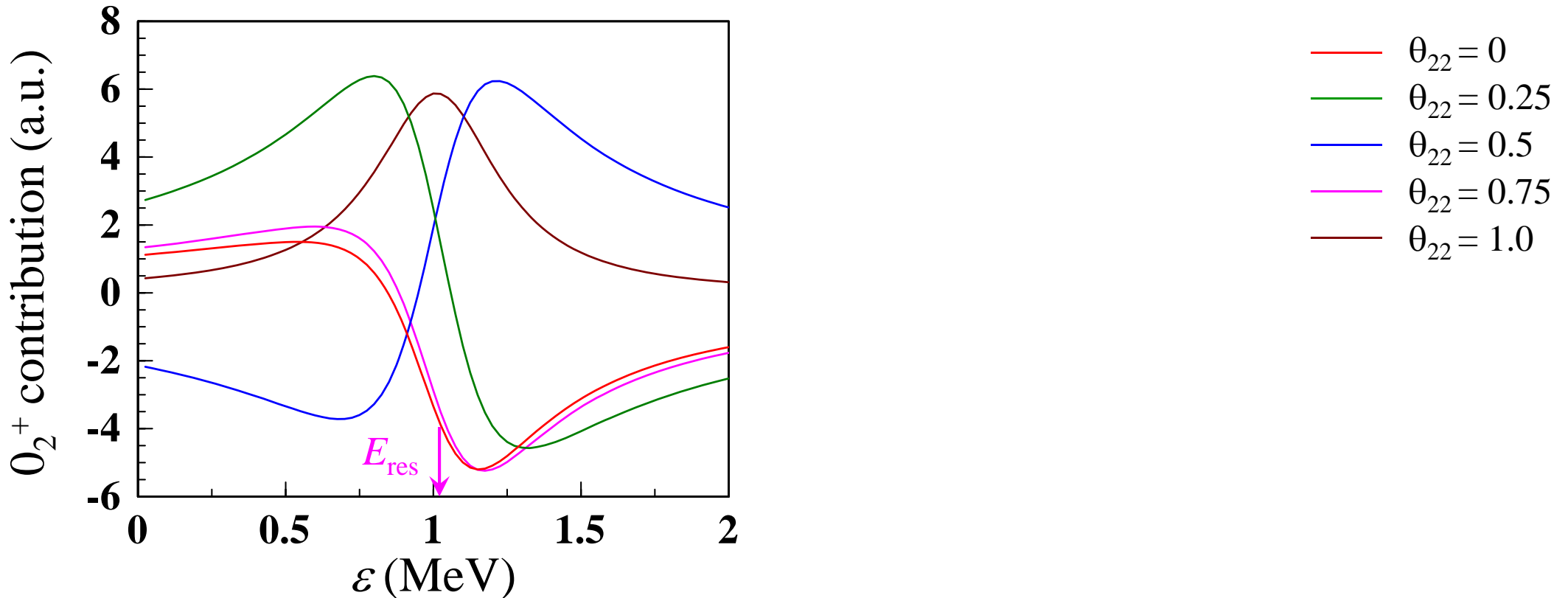
BackGround Phase (BGP) effect



$$\begin{aligned}
 S(\epsilon) &= e^{2i\delta_{\text{bg}}(\epsilon) + 2i\delta_{\text{res}}(\epsilon)} \\
 &= e^{2i\delta_{\text{bg}}(\epsilon)} \frac{\epsilon - \epsilon_{\text{res}} - i\Gamma/2}{\epsilon - \epsilon_{\text{res}} + i\Gamma/2}
 \end{aligned}$$

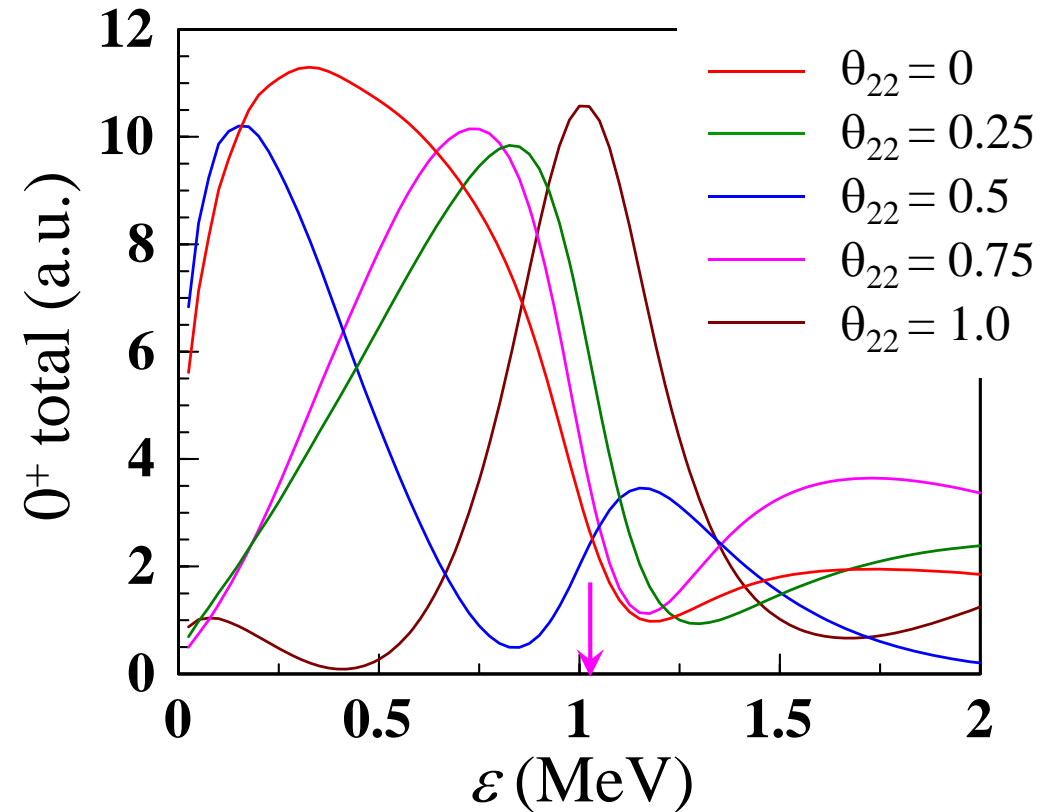
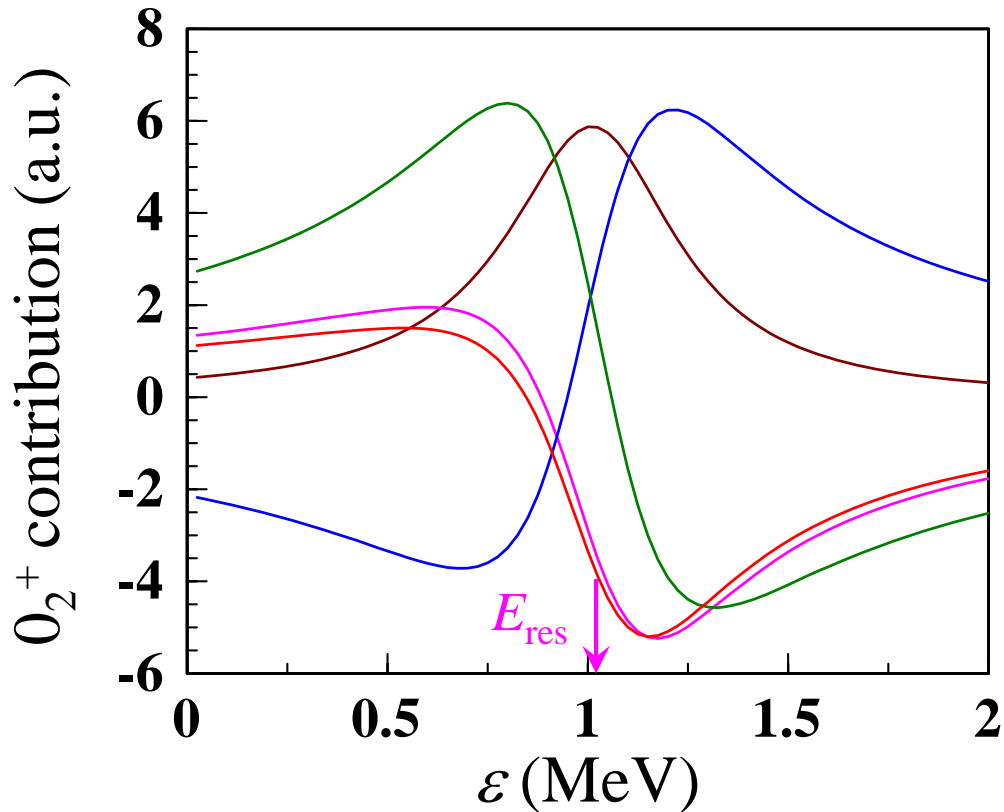
- ✓ In nuclear physics, we **always** have δ_{bg} .
- ✓ There are many examples of this effect in many research fields.
- ✓ In most cases, this effect is observed as **small changes** in the **resonance energy** and **width**.

BGP effect on the DDBUX



- ✓ The BGP effect is indeed **sizable**.
- ✓ We have a **variety of patterns** of the resonant (and 0^+) cross section.
- ✓ Appear in only the 0^+ state

BGP effect on the DDBUX



- ✓ The BGP effect is indeed **sizable**.
- ✓ We have a **variety of patterns** of the resonant (and O^+) cross section.
- ✓ Appear in only the O^+ state

Summary of the 1st topic

What is the form of $^{22}\text{C}^*$ in a breakup observable?

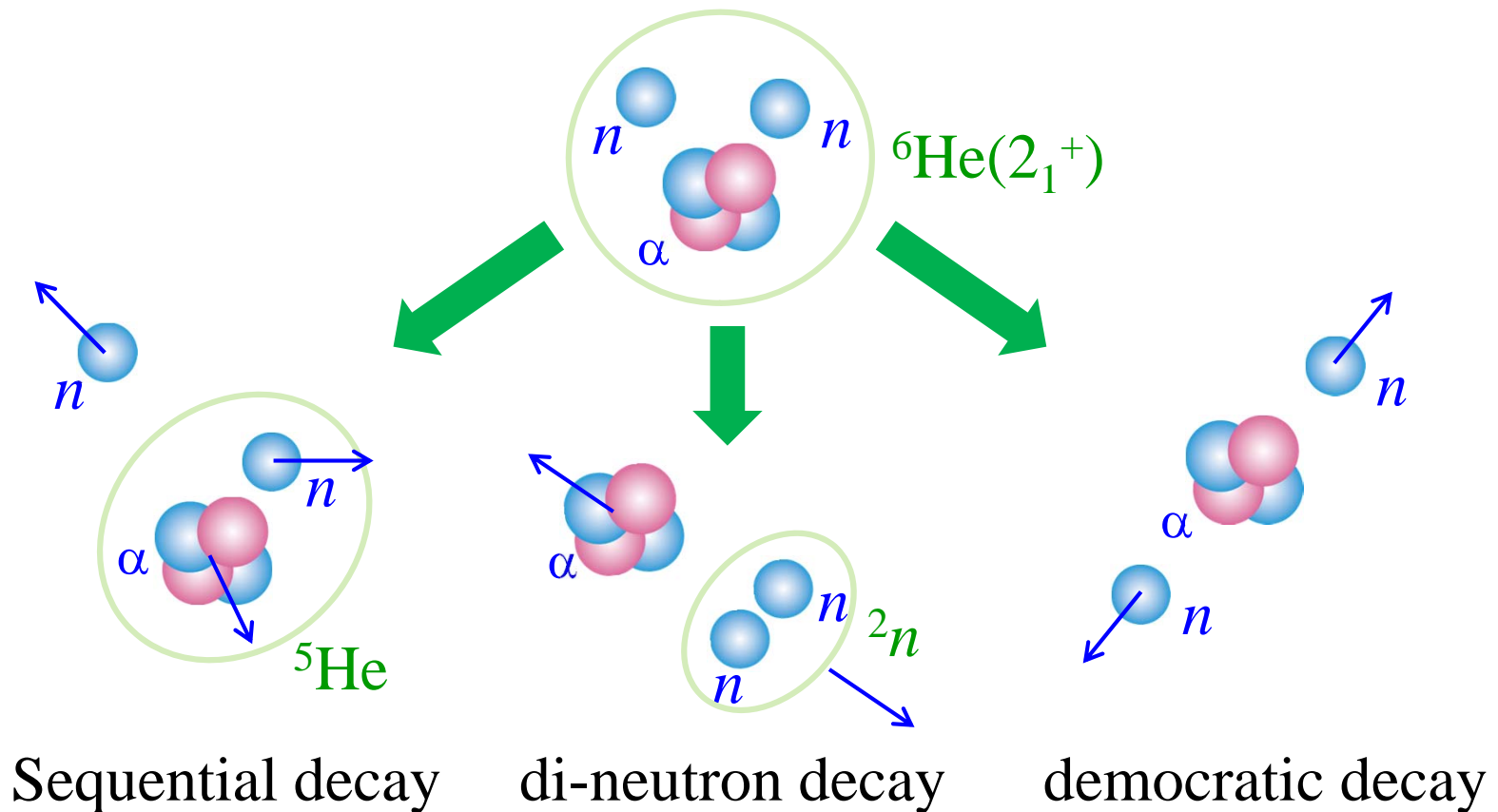
KO, Myo, Furumoto, Matsumoto, Yahiro, PRC88, 024616 (2013).

- ✓ The 2_1^+ state: Breit-Wigner form
- ✓ The 0_2^+ state: peculiar form due to the BGP effect (coexistence of the 0^+ resonant and nonresonant waves)
- ✓ The BGP has a strong scattering-angle dependence.
- ✓ We should be careful to identify the 0_2^+ state of ^{22}C in the observables.

2nd topic

What is the decay mode of the 2_1^+ state of ${}^6\text{He}$?

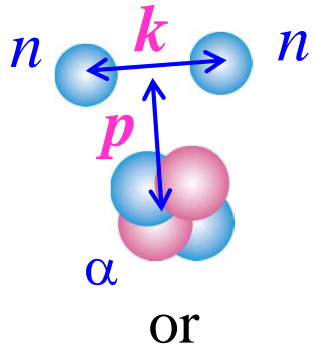
Y. Kikuchi, Matsumoto, Minomo, O, PRC88, 021602 (2013).



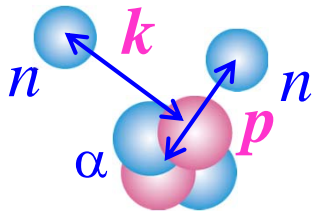
CDCC-CSLS

✓ The method of Complex-Scaled solutions of the Lippmann-Schwinger Eq.

Y. Kikuchi, Myo, Takashina, Kato, Ikeda, PTP122, 499 (2009).



or



$$T(\mathbf{p}, \mathbf{k}) = \left\langle \Phi^{(-)}(\mathbf{p}, \mathbf{k}) \underbrace{e^{i\mathbf{K} \cdot \mathbf{R}} |U| \Psi^{\text{CDCC}}}_{\sum_n |\Phi_n\rangle \langle \Phi_n| \approx 1} \right\rangle$$

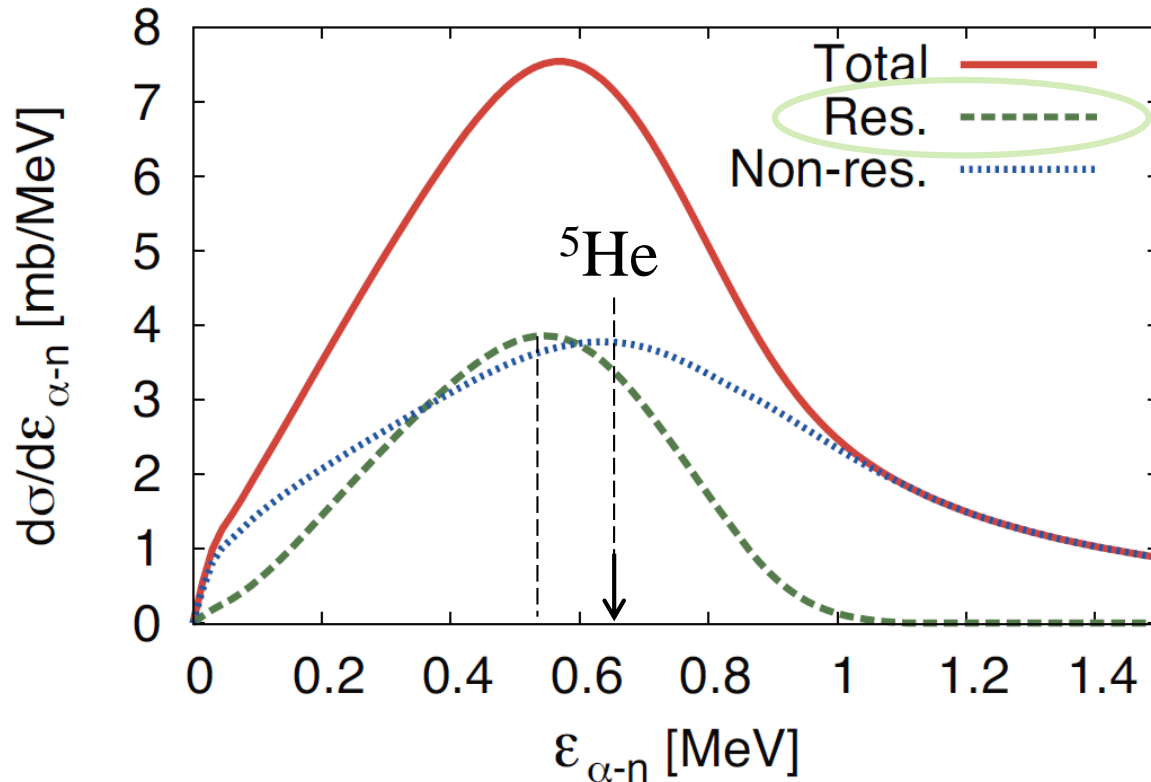
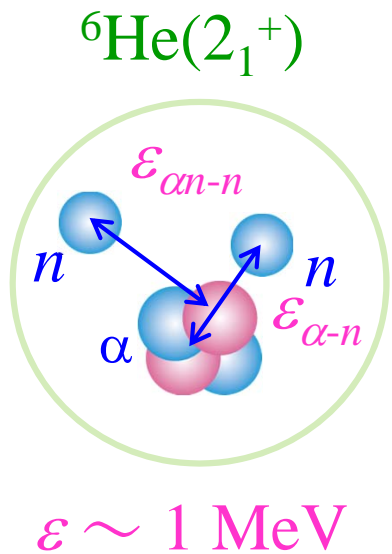
$$\approx \sum_n \left\langle \Phi^{(-)}(\mathbf{p}, \mathbf{k}) | \Phi_n \right\rangle T_n^{\text{CDCC}}$$

$$\equiv f_n(\mathbf{p}, \mathbf{k})$$

$$f_n(\mathbf{p}, \mathbf{k}) = \langle \varphi_{\text{free}}(\mathbf{p}, \mathbf{k}) | \Phi_n \rangle + \sum_i \langle \varphi_{\text{free}}(\mathbf{p}, \mathbf{k}) | V_{\alpha nn} C^{-1}(\theta) | \phi_i^\theta \rangle$$

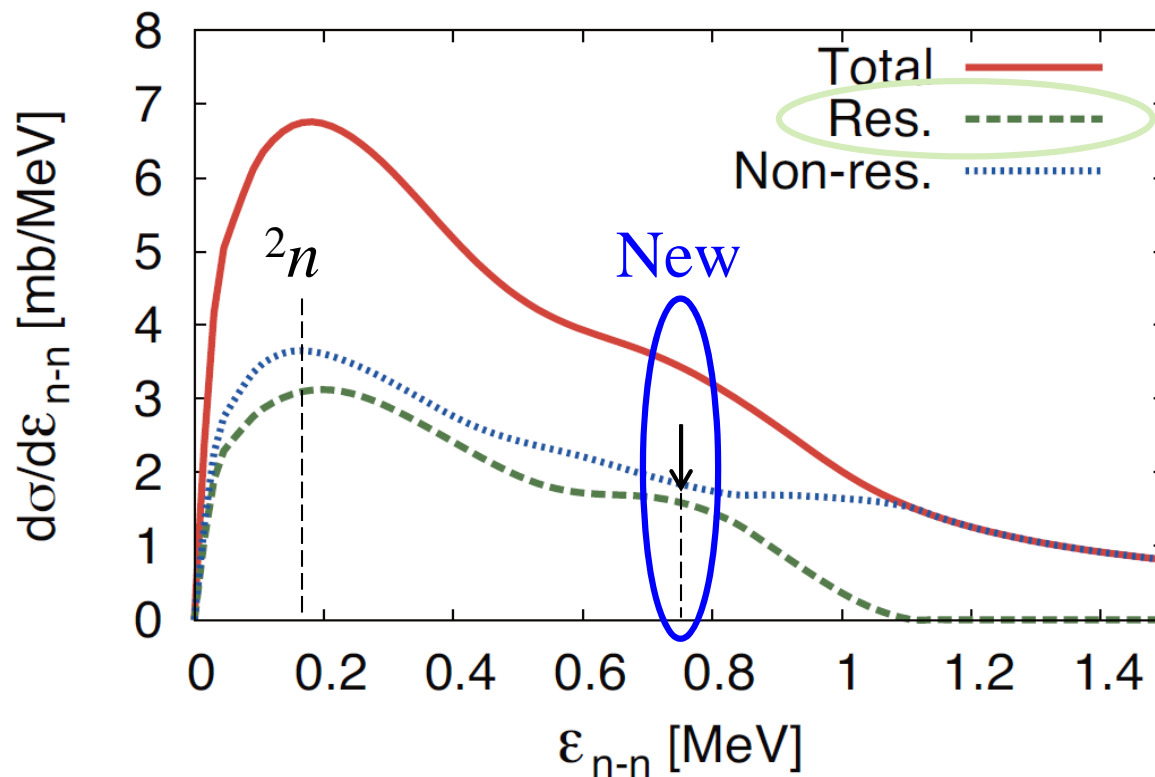
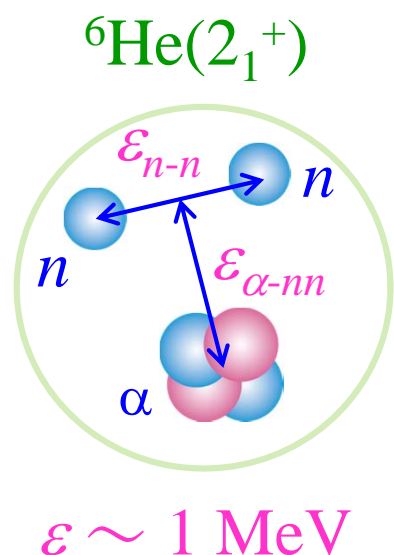
$$\times \frac{1}{\varepsilon - \varepsilon_i^\theta} \langle \tilde{\phi}_i^\theta | C(\theta) | \Phi_n \rangle$$

Sequential decay quenched



- ✓ When $\varepsilon \sim 1 \text{ MeV}$ and $\varepsilon_{\alpha-n} \sim 0.7 \text{ MeV}$, the other neutron ($\sim 0.3 \text{ MeV}$) hardly penetrates the centrifugal barrier (p -wave).
- ✓ The peak of the green line suggests the di-neutron decay or the democratic decay.

Coexistence of two decay modes

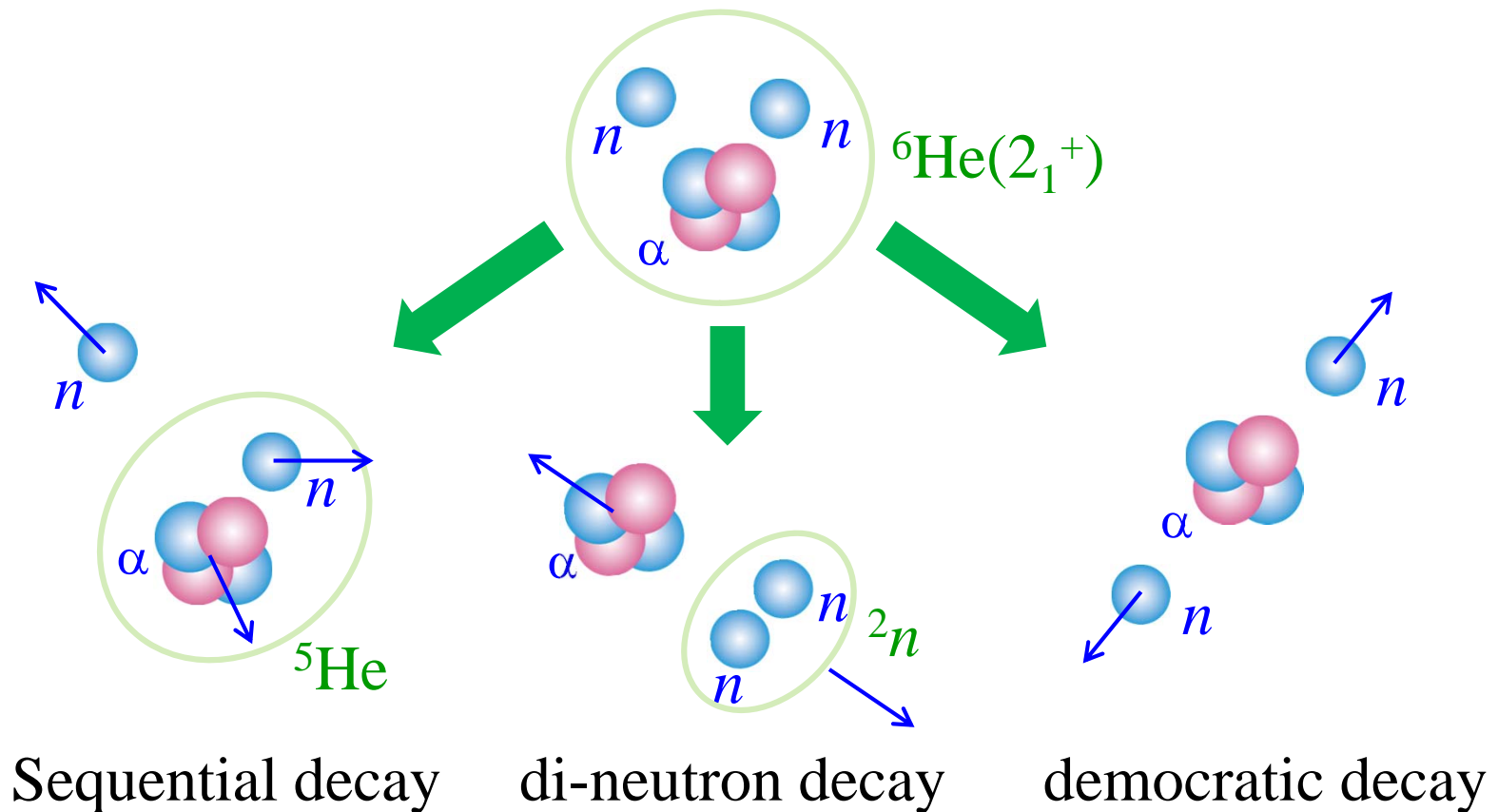


- ✓ The lower peak suggests the di-neutron decay due to the Fin. State Int. (FSI).
- ✓ The higher peak indicates the democratic decay.
- ➡ Decay of a di-neutron in the 2_1^+ state not due to the FSI.

Summary of the 2nd topic

What is the decay mode of the 2_1^+ state of ${}^6\text{He}$?

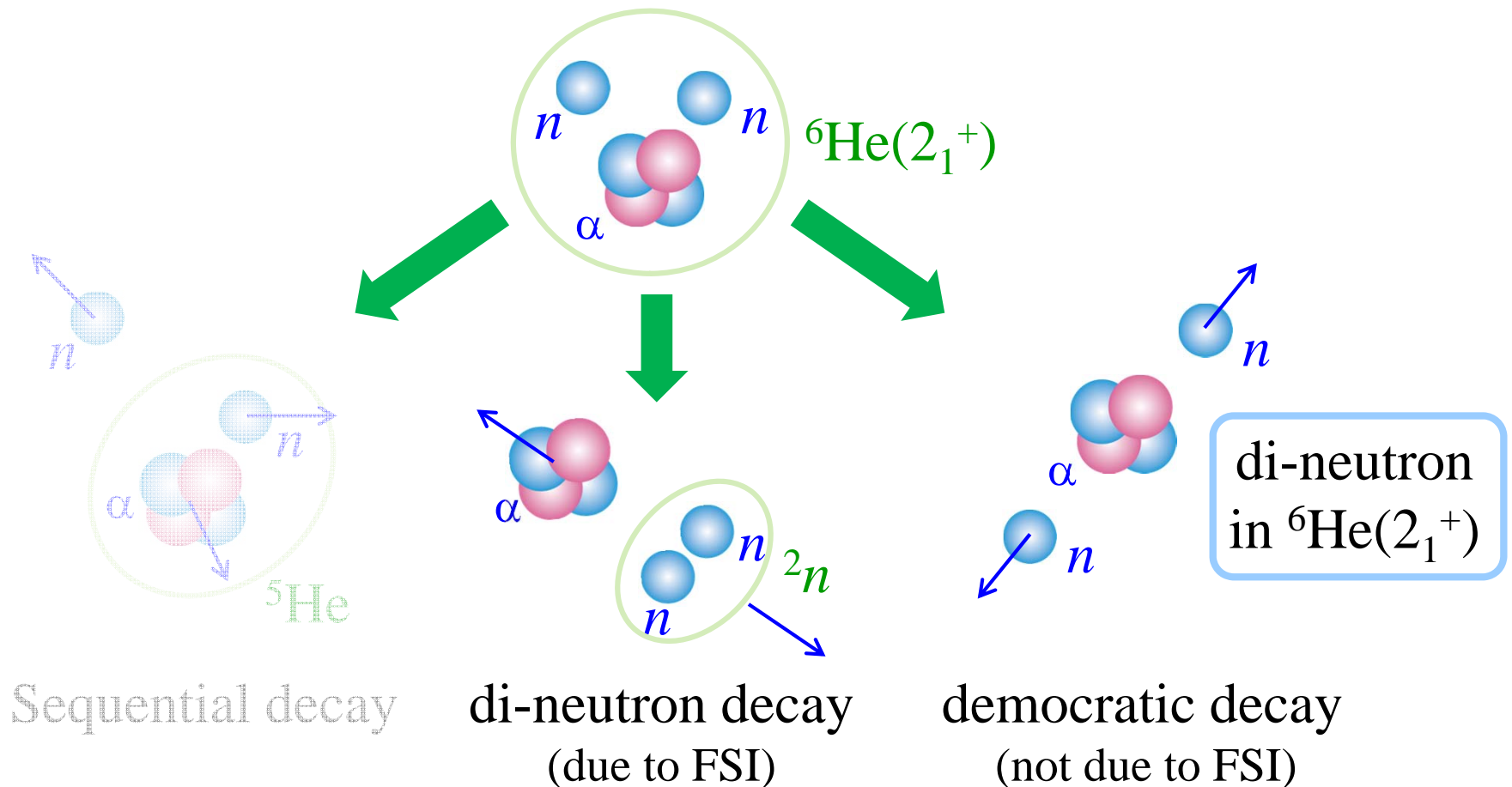
Y. Kikuchi, Matsumoto, Minomo, O, PRC88, 021602 (2013).



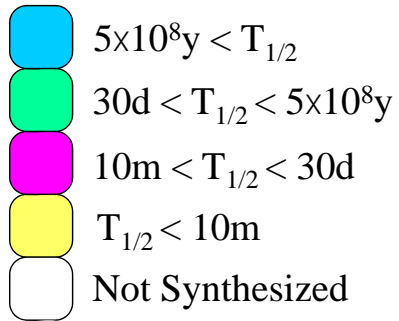
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What is the decay mode of the 2_1^+ state of ${}^6\text{He}$?

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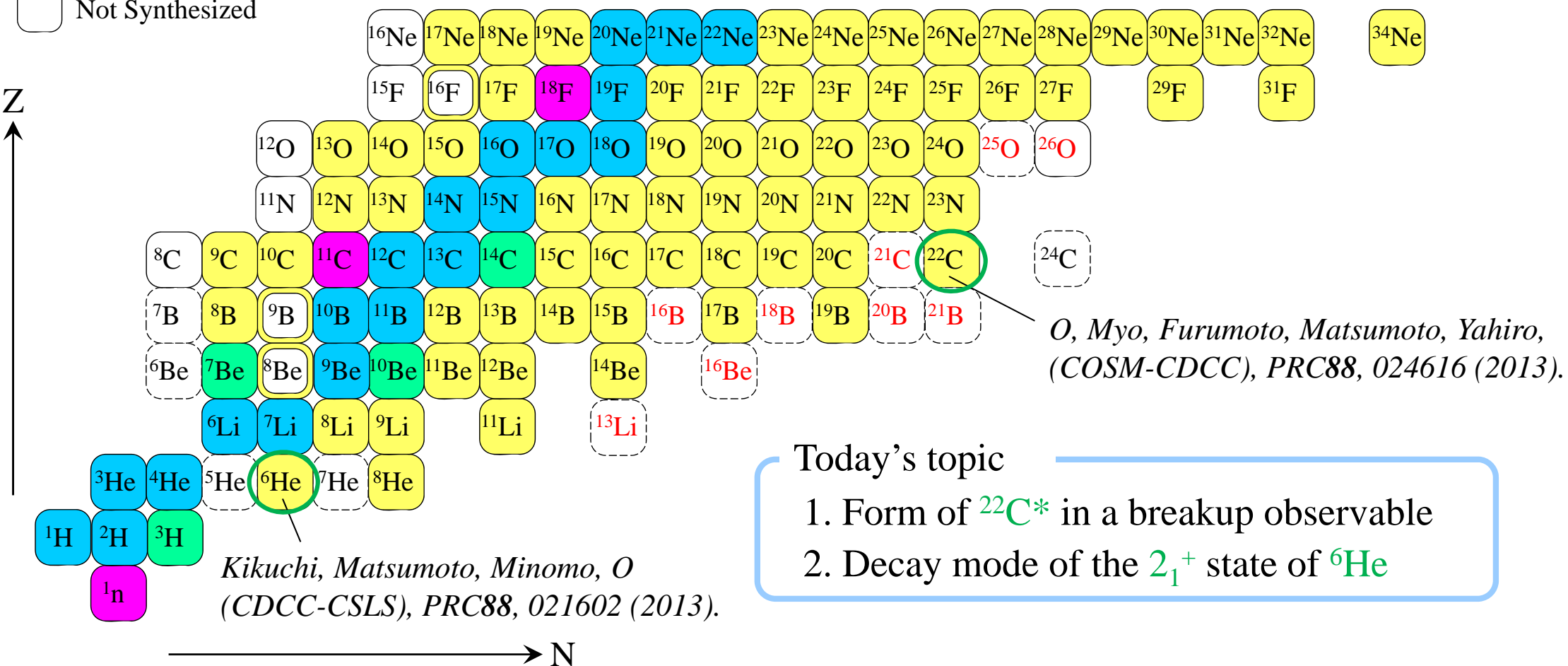


Exploration of unbound (but not free) systems



Our Aim

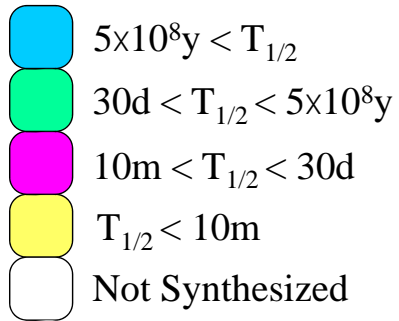
Dynamical description of Formation and Decay of unbound systems



Today's topic

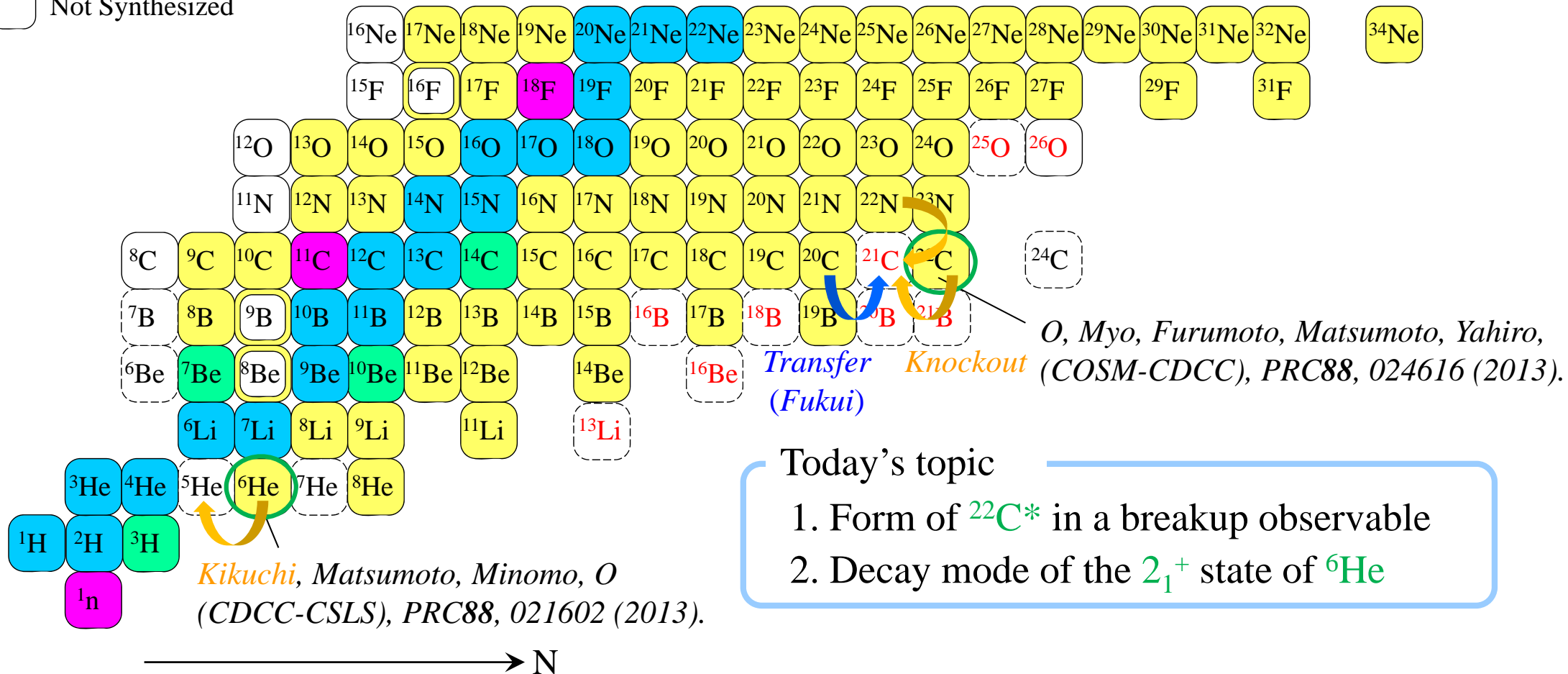
1. Form of ${}^{22}\text{C}^*$ in a breakup observable
2. Decay mode of the 2_1^+ state of ${}^6\text{He}$

Exploration of unbound (but not free) systems



Our Aim

Dynamical description of Formation and Decay of unbound systems



Today's topic

1. Form of $^{22}\text{C}^*$ in a breakup observable
2. Decay mode of the 2_1^+ state of ^6He

Numerical inputs

^{22}C wave function

- ✓ Minnesota force for n - n , Woods-Saxon potential for n - ^{20}C .
- ✓ $s_{1/2}$, $p_{3/2}$, $p_{1/2}$, $d_{5/2}$, $d_{3/2}$, $f_{7/2}$, $f_{5/2}$, $g_{9/2}$, $g_{7/2}$, $h_{11/2}$, and $h_{9/2}$ for the n s.p. orbit.
- ✓ Each orbit is described by 10 Gaussian basis functions.



D. R. Thompson *et al.*, NP **A286**, 53 (1977).

0^+ ground state with $S_{2n} = 289$ keV, 604 0^+ and 1,385 2^+ PS

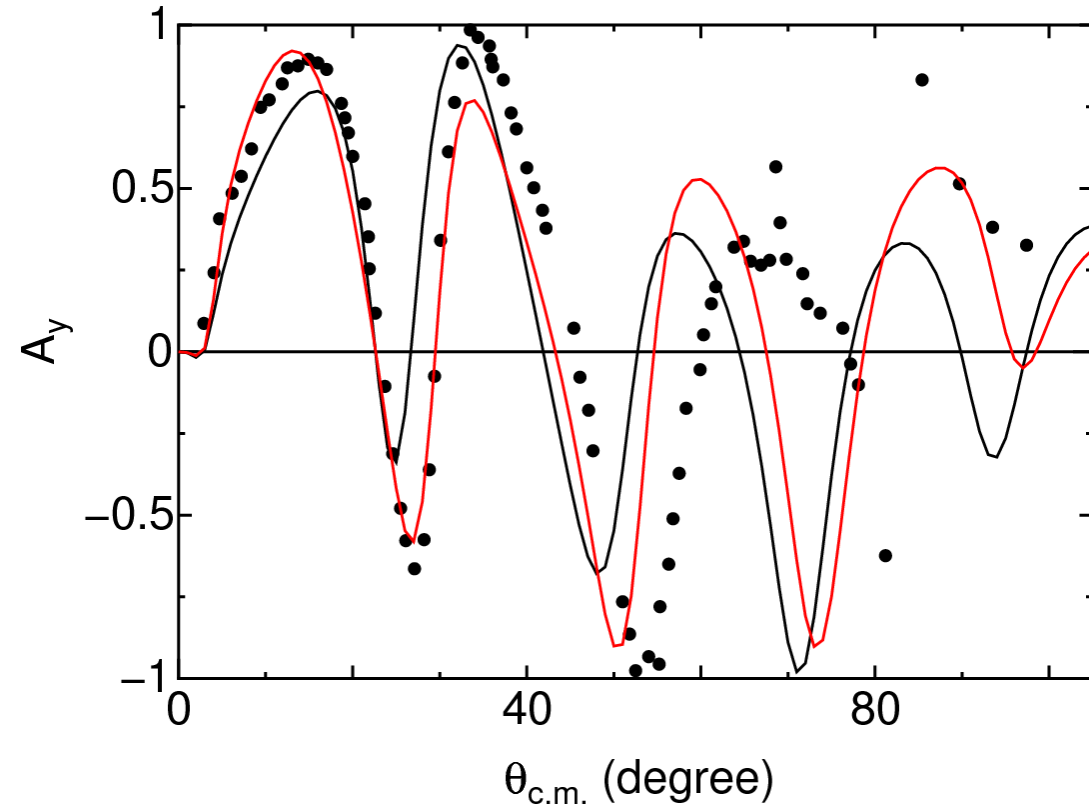
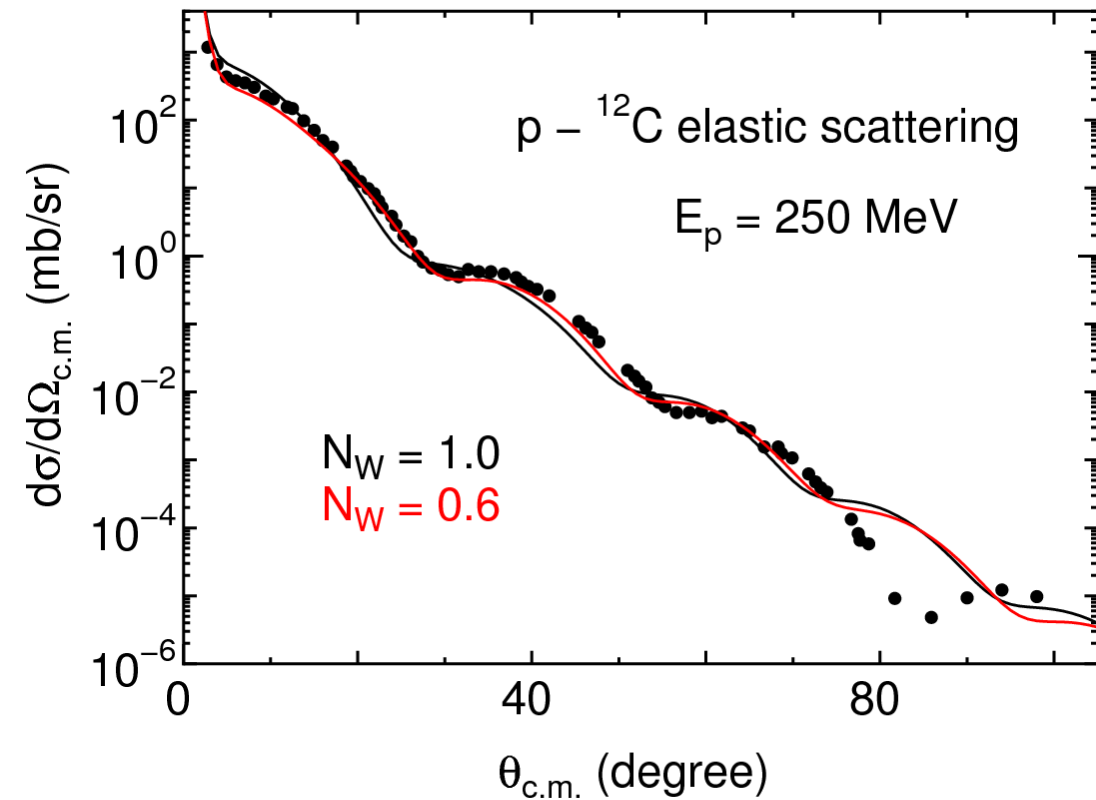
^{22}C - ^{12}C breakup reaction

- ✓ 77 (0^+) + 164 (2^+) PS below 10 MeV are included as breakup states of ^{22}C .
- ✓ Distorting potentials are calculated by a microscopic folding model with CEG07 nucleon-nucleon g matrix.
- ✓ We adopt the so-called no-recoil approximation for the ^{20}C core nucleus.

T. Furumoto *et al.*, PRC **78**, 044610 (2008).

p-¹²C scattering at 250 MeV

Nuclear density: L. C. Chamon *et al.*, PRC 66, 014610 (2002) [Sao Paulo group]



Complex Scaling Method (CSM)

S. Aoyama, T. Myo, K. Kato, and K. Ikeda,
Prog. Theor. Phys. 116, 1 (2006)

Courtesy of Matsumoto

Complex-scaling operator: U^θ

$$U^\theta f(r) = e^{i3/2\theta} f(re^{i\theta})$$

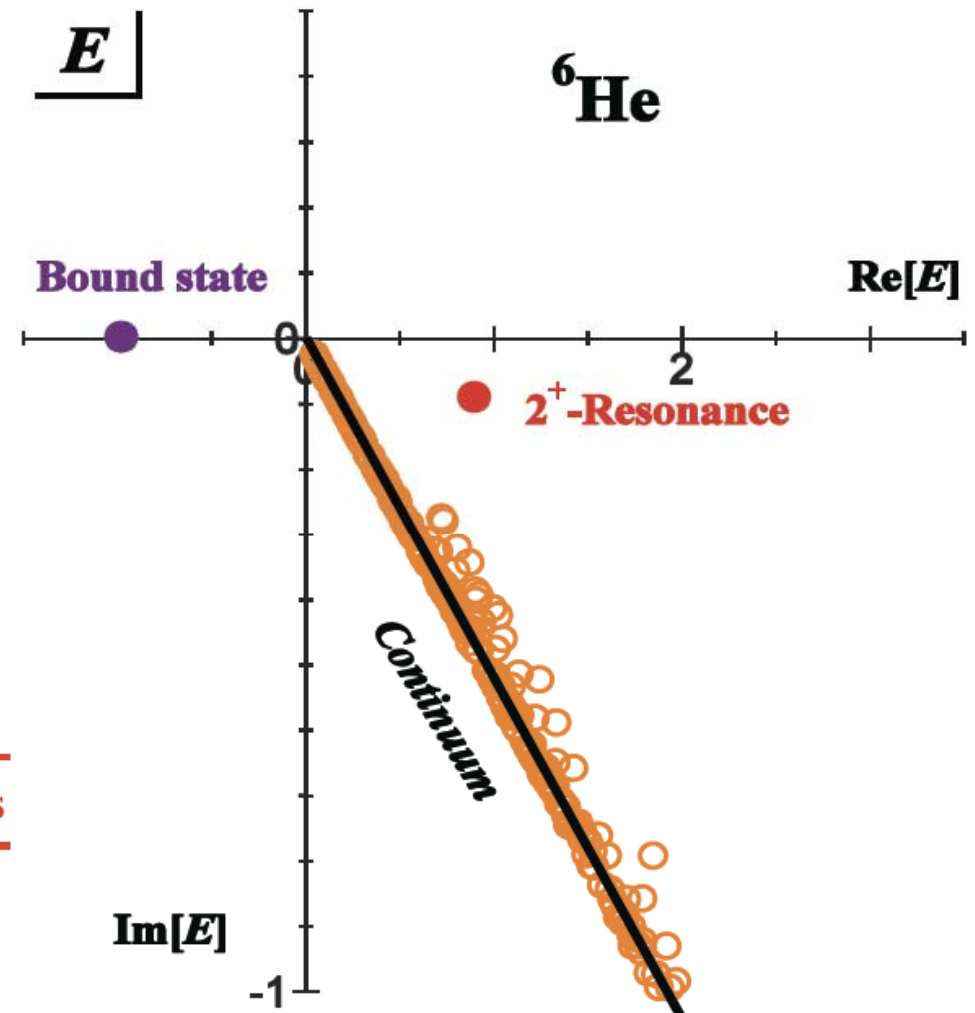
Coordinate: $r \rightarrow re^{i\theta}$

Momentum: $k \rightarrow ke^{-i\theta}$

Asymptotic form

$$e^{ikr} \longrightarrow e^{ikr \cos \theta} e^{-kr \sin \theta}$$

Useful for searching many-body resonances



Green's function with Complex-Scaling Method (CDCS Green's function)

$$\mathcal{G}^{(-)}(E, \xi, \xi') = \frac{1}{E - H - i\epsilon} \approx \sum_{\nu} U^{-\theta} \frac{|\Phi_{\nu}^{\theta}\rangle \langle \tilde{\Phi}_{\nu}^{\theta}|}{E - E_{\nu}^{\theta}} U^{\theta}$$

New Smoothing Procedure with *CSM*

T.M., K. Kato, and M. Yahiro, PRC82, 051602 (2010).

$$\frac{d\sigma}{dE} = \int T^\dagger(E')T(E')\delta(E - E')dE' = \frac{1}{\pi}\text{Im}\mathcal{R}(E)$$

$$T(E) = \psi^{(-)}(E, \xi) \chi_C^{(-)}(\mathbf{R}) |V| \Psi^{(+)}(\xi, \mathbf{R})$$

Response function

Final state of the projectile

$$\mathcal{R}(E) = \int d\xi d\xi' \langle \Psi^{(+)}(\xi, \mathbf{R}) | V^* | \chi_C^{(-)}(\mathbf{R}) \rangle_{\mathbf{R}} \mathcal{G}^{(-)}(E, \xi, \xi') \langle \chi_C^{(-)}(\mathbf{R}) | V | \Psi^{(+)}(\xi, \mathbf{R}) \rangle_{\mathbf{R}}$$

Green's function with Complex-Scaling Method (CDCS Green's function)

$$\mathcal{G}^{(-)}(E, \xi, \xi') = U^{-\theta} \frac{1}{E - H^\theta - i\epsilon} U^\theta \approx \sum_{\nu} U^{-\theta} \frac{|\Phi_{\nu}^{\theta}\rangle \langle \tilde{\Phi}_{\nu}^{\theta}|}{E - E_{\nu}^{\theta}} U^{\theta}$$

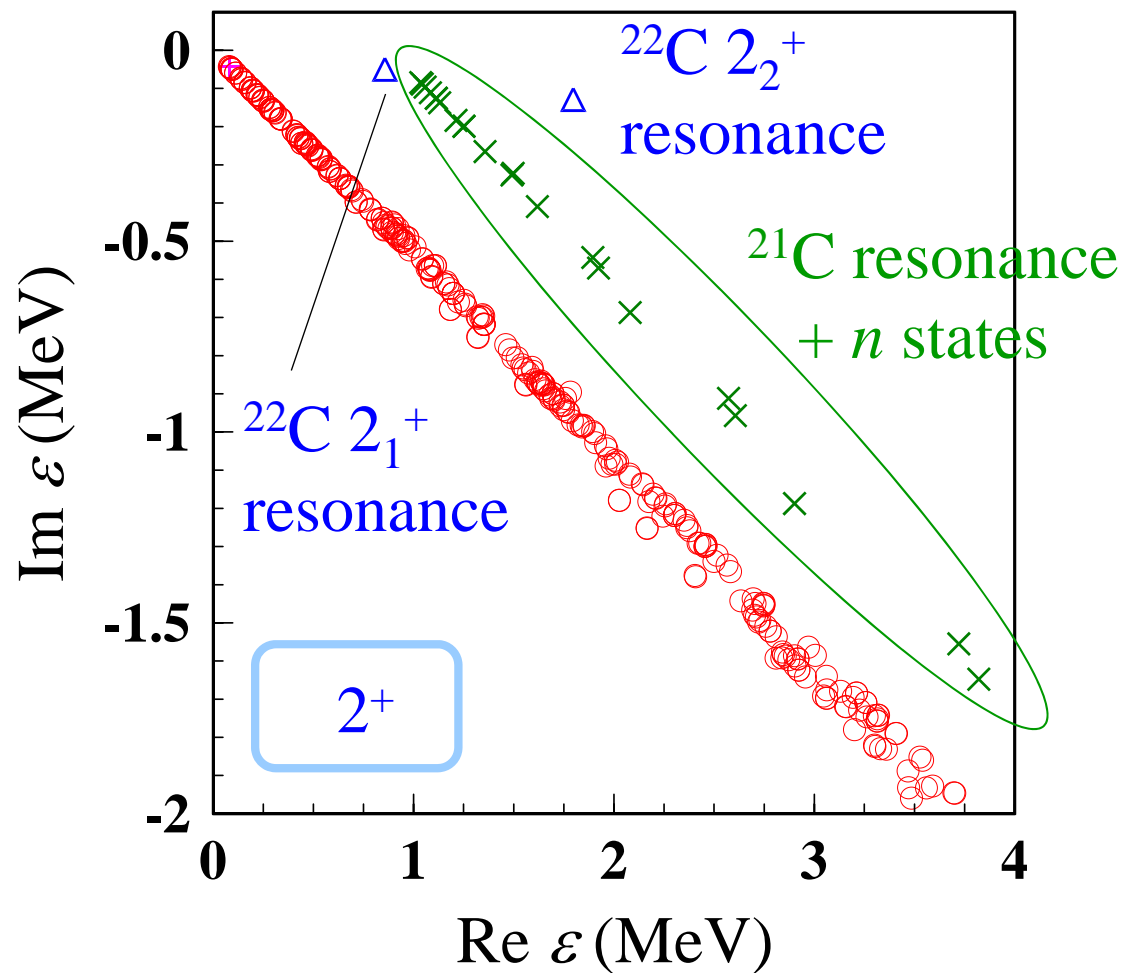
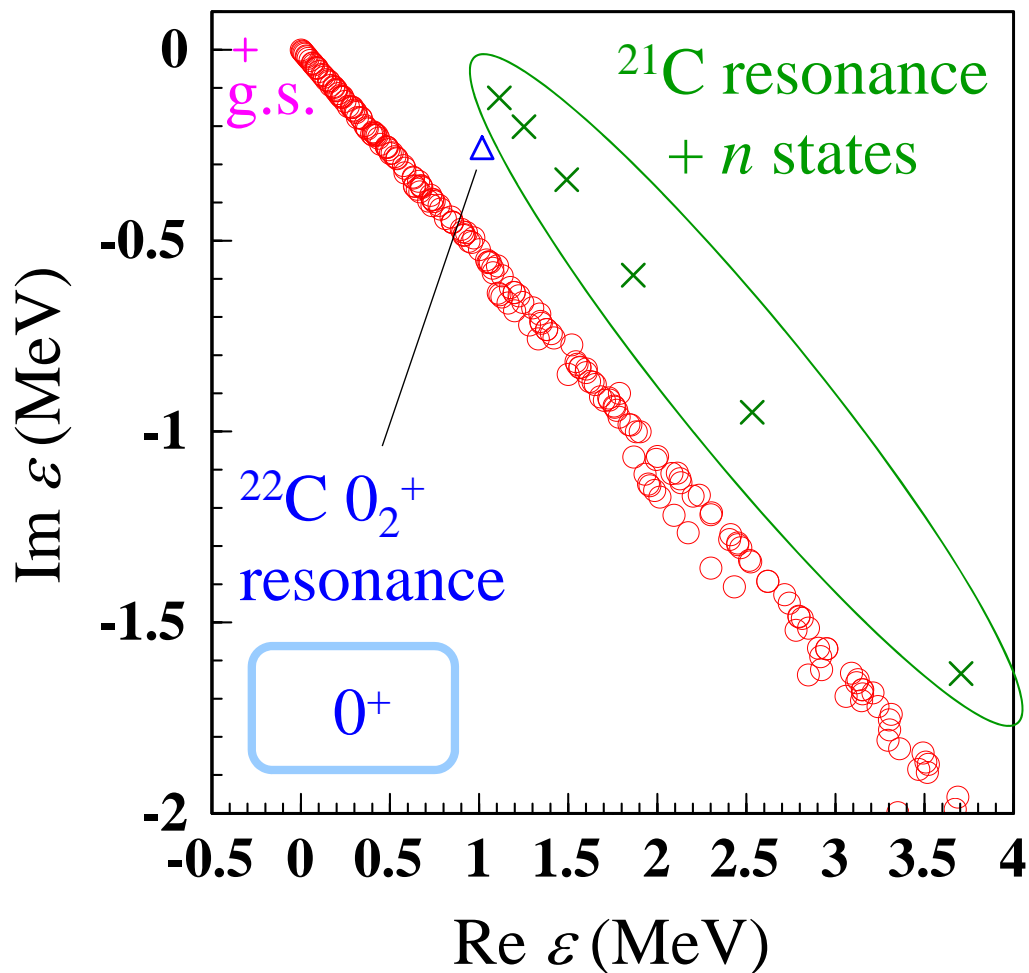
➔

$$\mathcal{G}^{(-)}(E, \xi, \xi') \approx \sum_{\nu} \sum_{i,j} |\Phi_i\rangle \frac{\langle \Phi_i | U^{-\theta} | \Phi_{\nu}^{\theta} \rangle \langle \tilde{\Phi}_{\nu}^{\theta} | U^{\theta} | \Phi_j \rangle}{E - E_{\nu}^{\theta}} \langle \Phi_j |$$

$$\mathcal{R}(E) = \sum_{\nu} \sum_{i,j} \langle \Psi^{(+)} | V^* | \chi_C^{(-)} \Phi_i \rangle \frac{\langle \Phi_i | U^{-\theta} | \Phi_{\nu}^{\theta} \rangle \langle \tilde{\Phi}_{\nu}^{\theta} | U^{\theta} | \Phi_j \rangle}{E - E_{\nu}^{\theta}} \langle \Phi_j \chi_C^{(-)} | V | \Psi^{(+)} \rangle$$

T-matrix calculated by CDCC

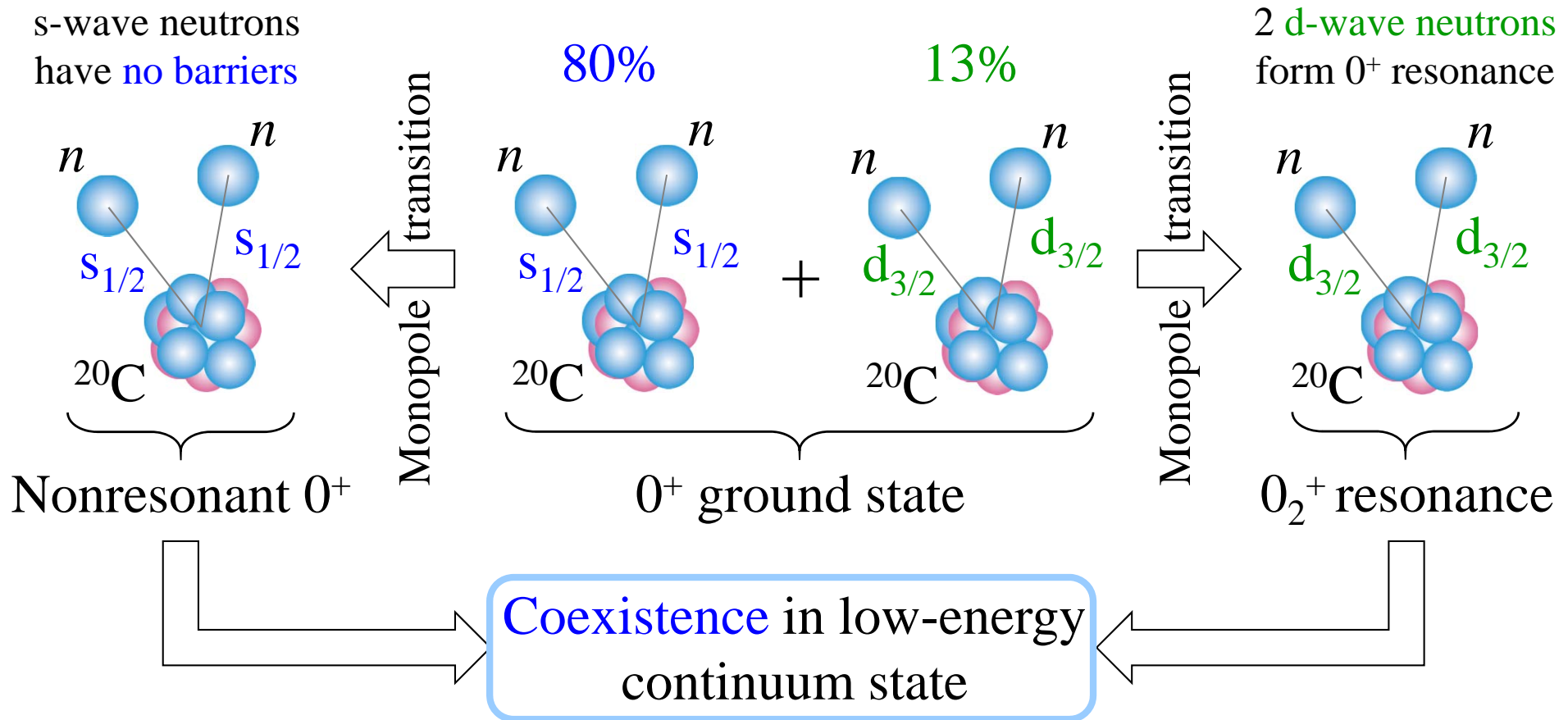
PS in the complex energy plane



✓ The complex-scaling method **classifies** the continuum states of ^{22}C .

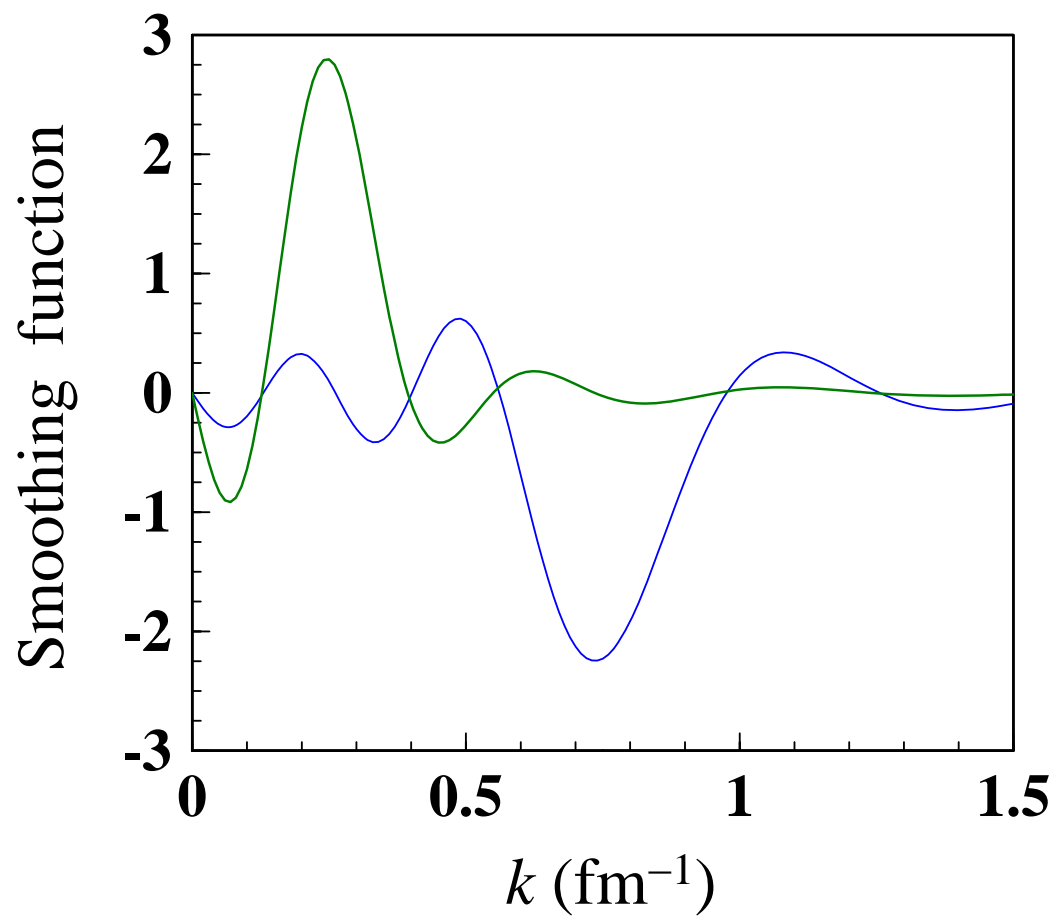
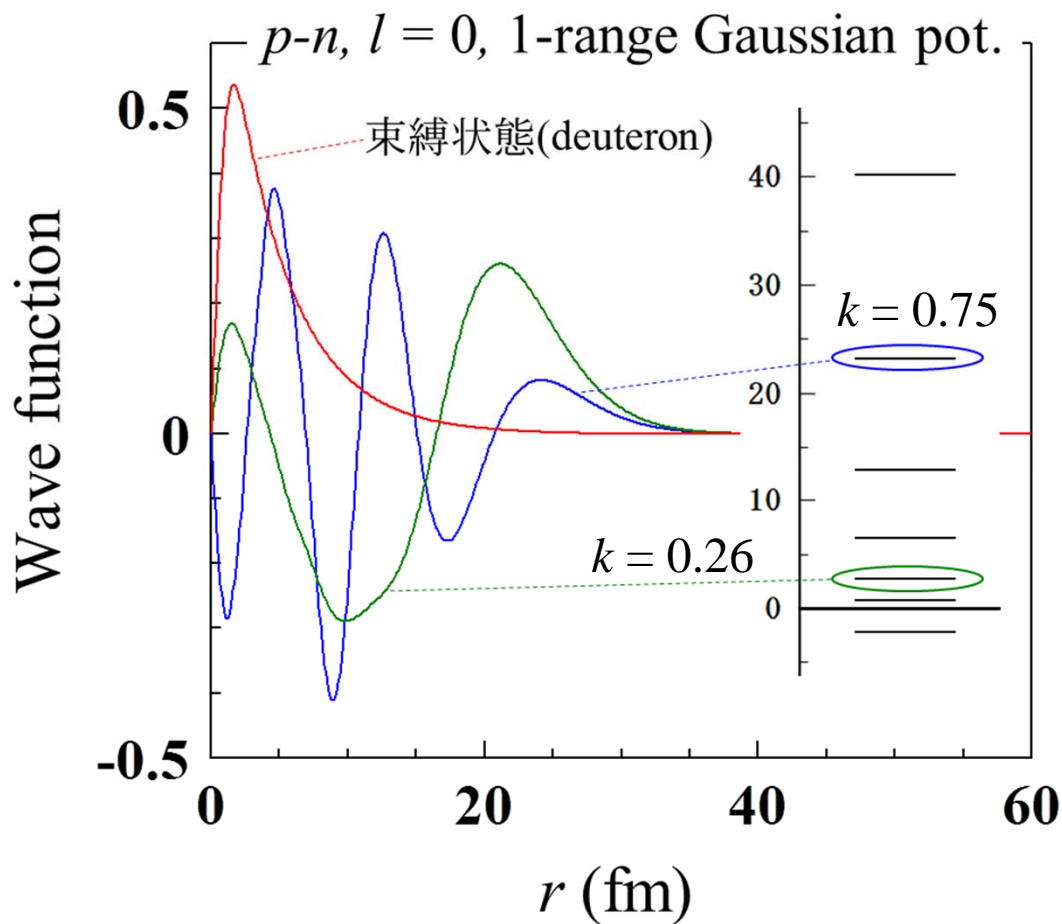
J. Aguilar and J. M. Combes, *Comm. Math. Phys.* **22**, 269 (1971);
E. Balslev and J. M. Combes, *ibid.* **22**, 280 (1971).

Why so large BGP effect?



- ✓ In a core + n system, this will hardly be realized.
- ✓ This resonant-nonresonant 0^+ coexistence is expected for (s-wave) two-neutron halo nuclei generally.

平滑化関数(PS法)

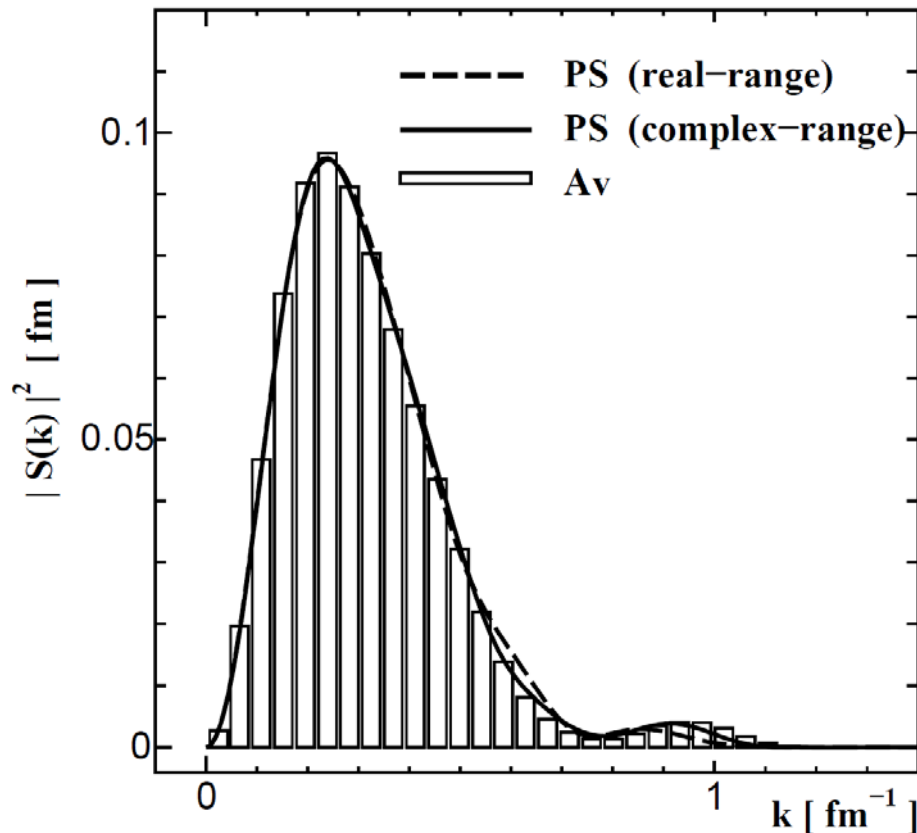


固有値に対応する k にピークを持つが、かなりの拡がりを持つ。

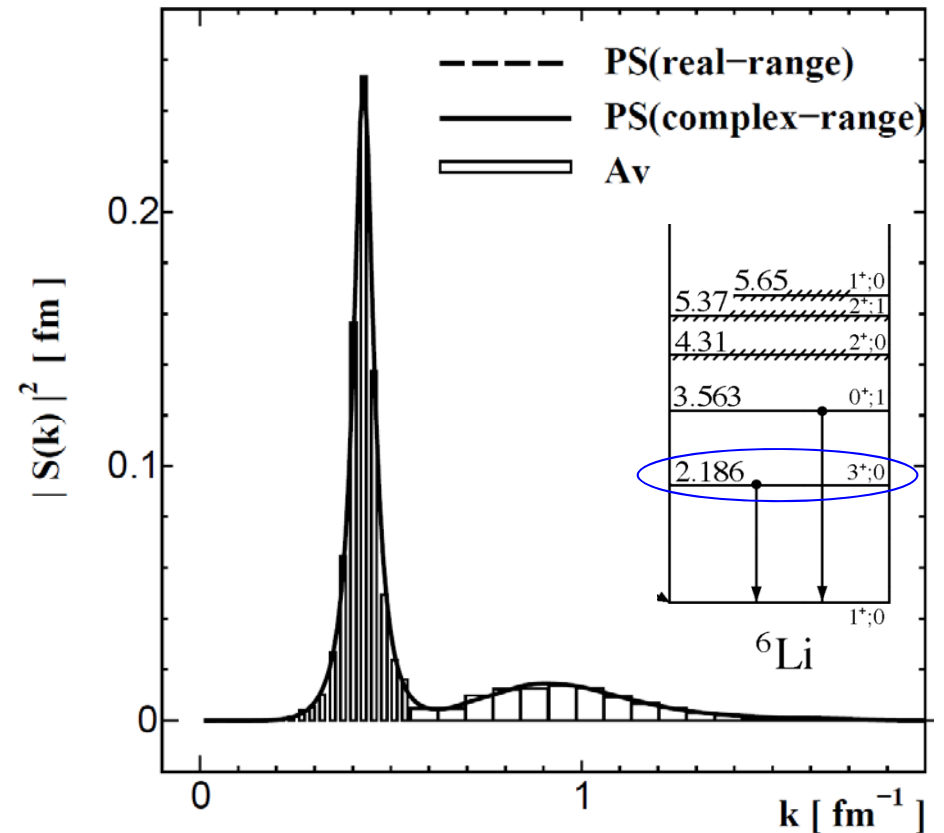
平滑化の実例(A_v法 vs PS法)

T. Matsumoto, Kamizato, O, Iseri, Hiyama, Kamimura, Yahiro, Phys. Rev. C **68**, 064607 (2003).

$d+^{58}\text{Ni}$ at 80 MeV



$^6\text{Li}+^{40}\text{Ca}$ at 156 MeV



平滑化した遷移強度は両者で極めて良く一致。