CLASSICAL ASPECTS OF THE NUCLEAR SYMMETRY ENERGY

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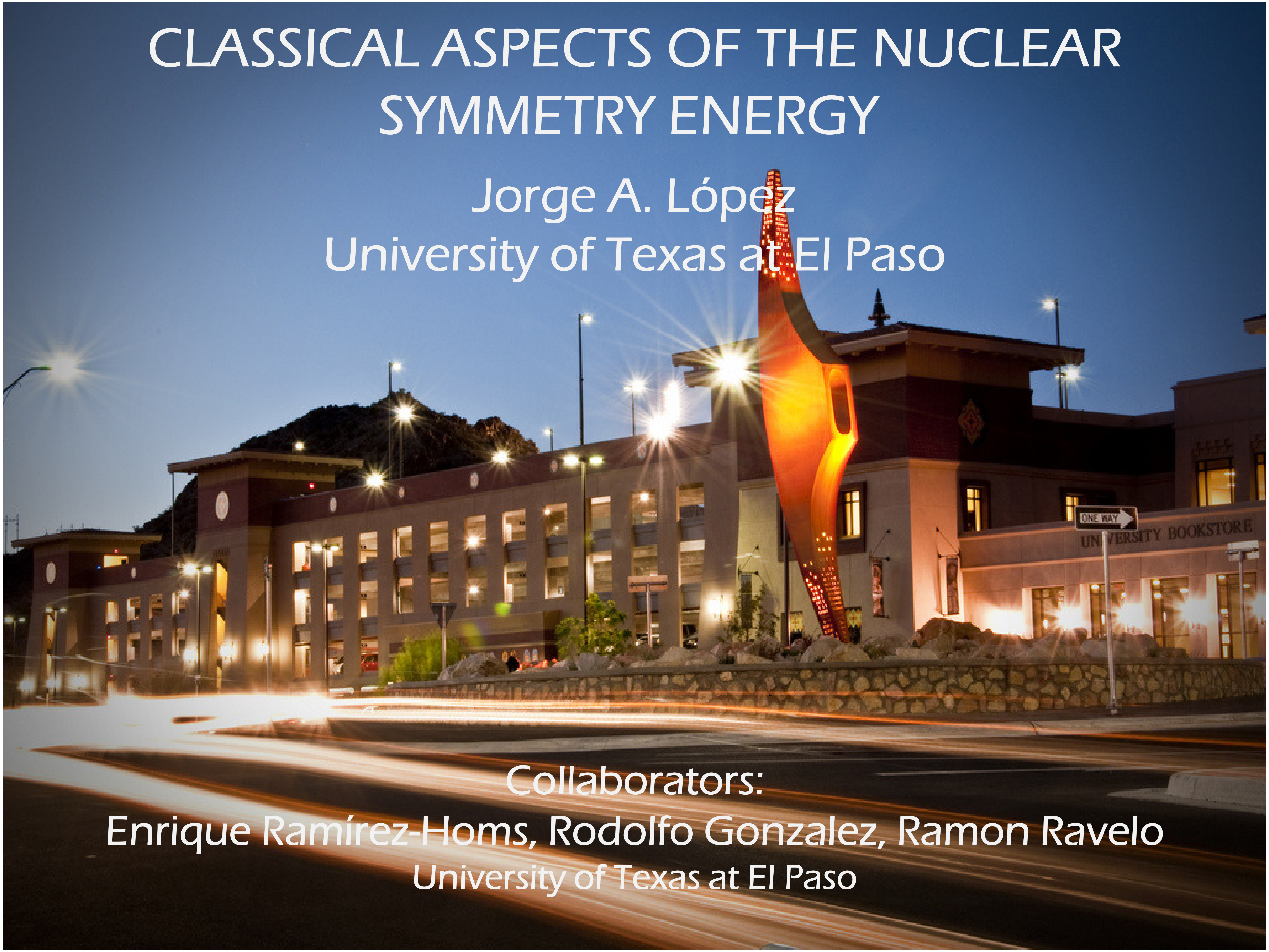

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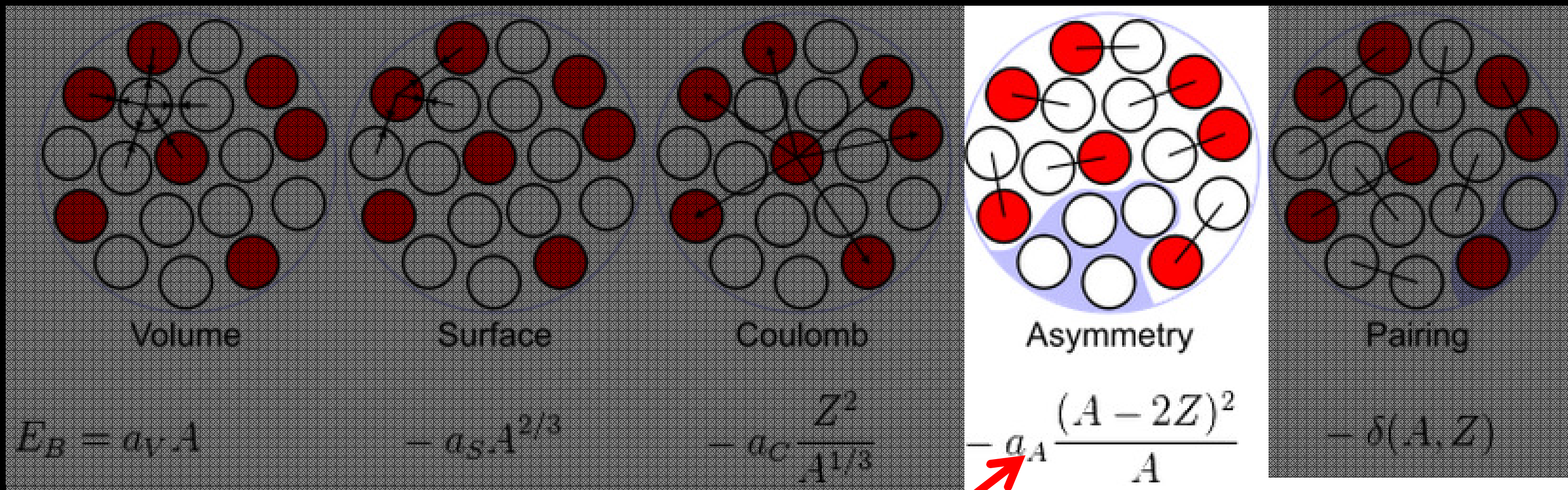
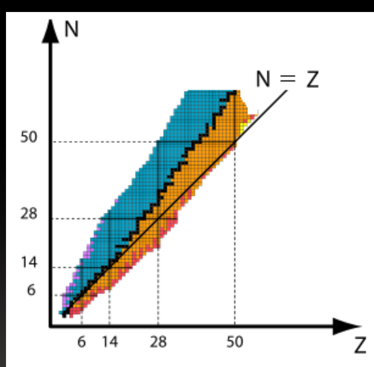
Outline

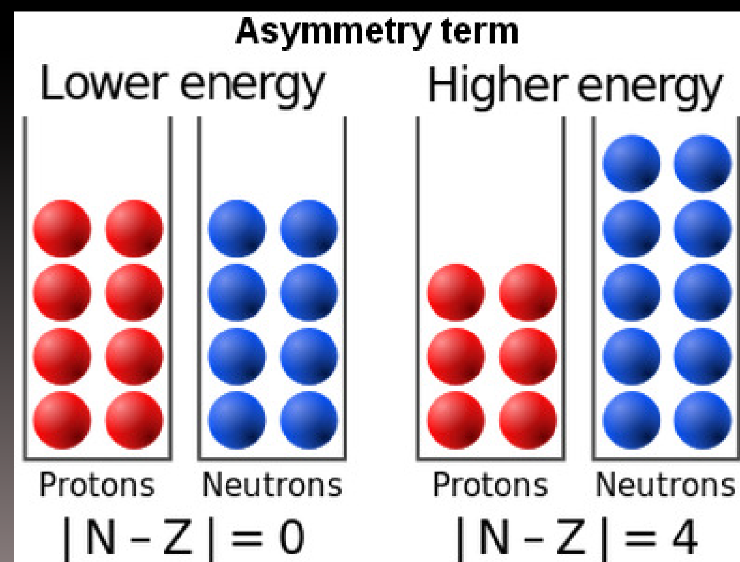
- Symmetry energy
- Considerations at higher energies
- Computational model
- Symmetry energy
- Conclusions

Adding more neutrons must affect asymmetry term



Asymmetry term penalizes nuclei with N ≠ Z





Quantum origin of the empirical asymmetry term

How to generalize to neutron-rich nuclei?

Extend liquid drop formula to non-symmetric isospin values

$$E(\rho,\alpha) = E(\rho,\alpha = 0) + E_{Sym}(\rho)\alpha^2 + O(\alpha^4)$$

Taylor expansion in terms of α
about isospin symmetric α=0
Odd-terms excluded due to
p-n exchange symmetry.

$$\alpha = \frac{(N-Z)}{A}$$

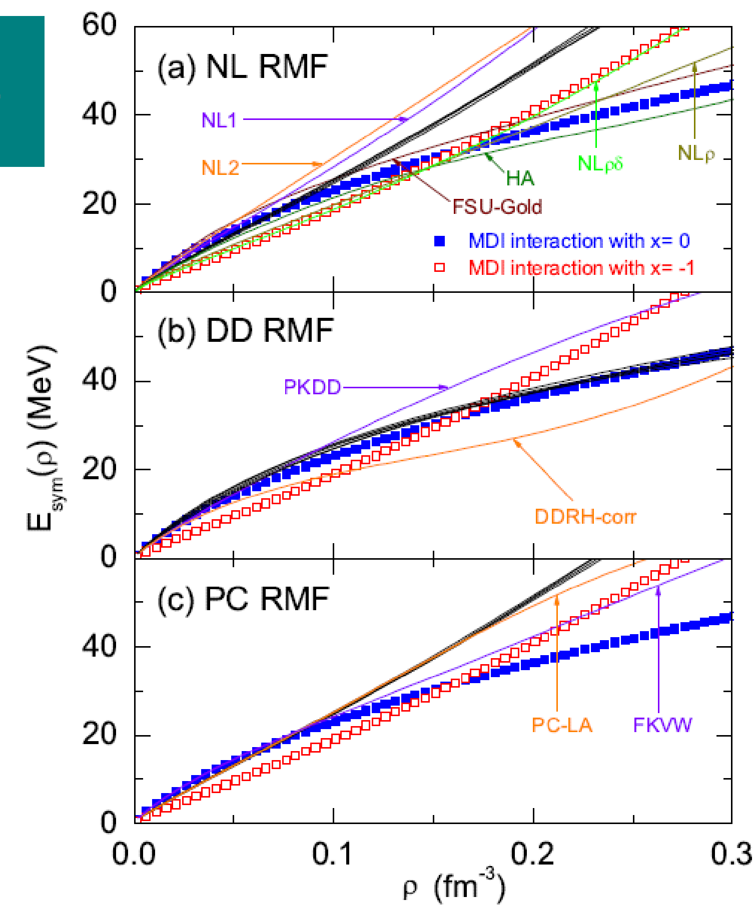
Obtain E_{sym} through:

Nuclear symmetry energy

$$E_{\mathrm{sym}}(\rho) = \frac{1}{2}\frac{\partial^2 E(\rho,\alpha)}{\partial\alpha^2}|_{\alpha=0}$$

E can be calculated with many body techniques

Previous studies



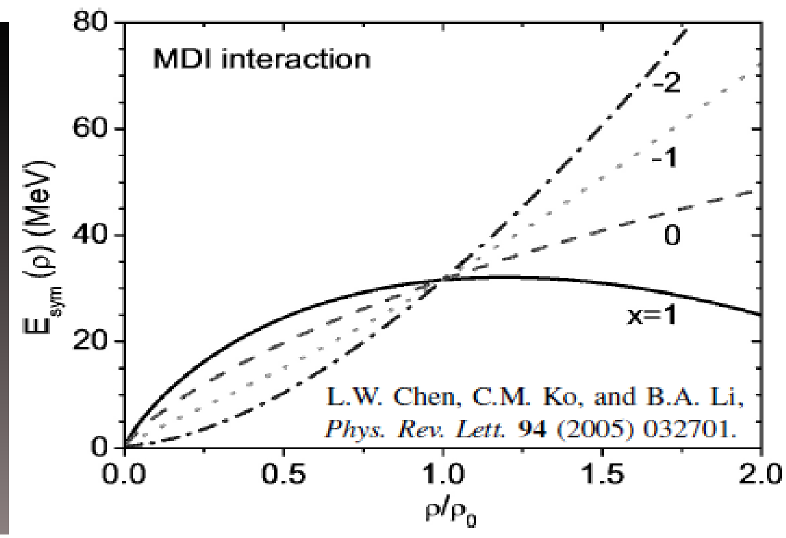
RELATIVISTIC MEAN-FIELD MODELS

The nonlinear RMF model

The density-dependent RMF model

The nonlinear point-coupling RMF model

The density-dependent point-coupling RMF model



All calculations at T=0

Tremendous work of Bao-An Li

Previous studies

Relativistic and Nonrelativistic Hartree Fock approximations:

- S. Ulrych and H. Muther, Phys. Rev. C56, 1788 (1997).
- E.N.E. van Dalen, C. Fuchs and A. Faessler, Nucl. Phys. A741, 227 (2004).
- Z.Y. Ma, J. Rong, B.Q. Chen, Z.Y. Zhu and H.Q. Song, Phys. Lett. B604, 170 (2004).
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- E.N.E. van Dalen, C. Fuchs, and A. Faessler, Phys. Rev. C72, 065803 (2005).
- J. Rong, Z.Y. Ma, and N. Van Giai, Phys. Rev. C73, 014614 (2006).
- I. Bombaci and U. Lombardo, Phys. Rev. C44, 1892 (1991).
- W. Zuo, L.G. Cao, B.A. Li, U. Lombardo, and C.W. Shen, Phys. Rev. C72, 014005 (2005).

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- C.B. Das, S. Das Gupta, C. Gale, and B.A. Li, Phys. Rev. C67, 034611 (2003).
- B.A. Li, C. B. Das, S. Das Gupta, and C. Gale, Phys. Rev. C69, 011603(R) (2004); Nucl. Phys. A735, 563 (2004).
- B.A. Li, Phys. Rev. C69, 064602 (2004).
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- J. Rizzo, M. Colonna, M. Di Toro, and V. Greco, Nucl. Phys. A732, 202 (2004).
- B. Behera, T.R. Routray, A. Pradhan, S.K. Patra, and P.K. Sahu, Nucl.Phys. A753, 367 2005
- J. Rizzo, M. Colonna, and M. Di Toro, Phys. Rev. C72, 064609 (2005).

Previous studies

In summary,

The knowledge we have about the symmetry energy is as good as the techniques used for solving the nuclear many-body problem, which are far from perfect.

How about $T > 0$?

Can we learn about the symmetry energy at higher energies ($T > 0$)?

At intermediate energies colliding nuclei fragment

Remember the liquid-gas phase transition

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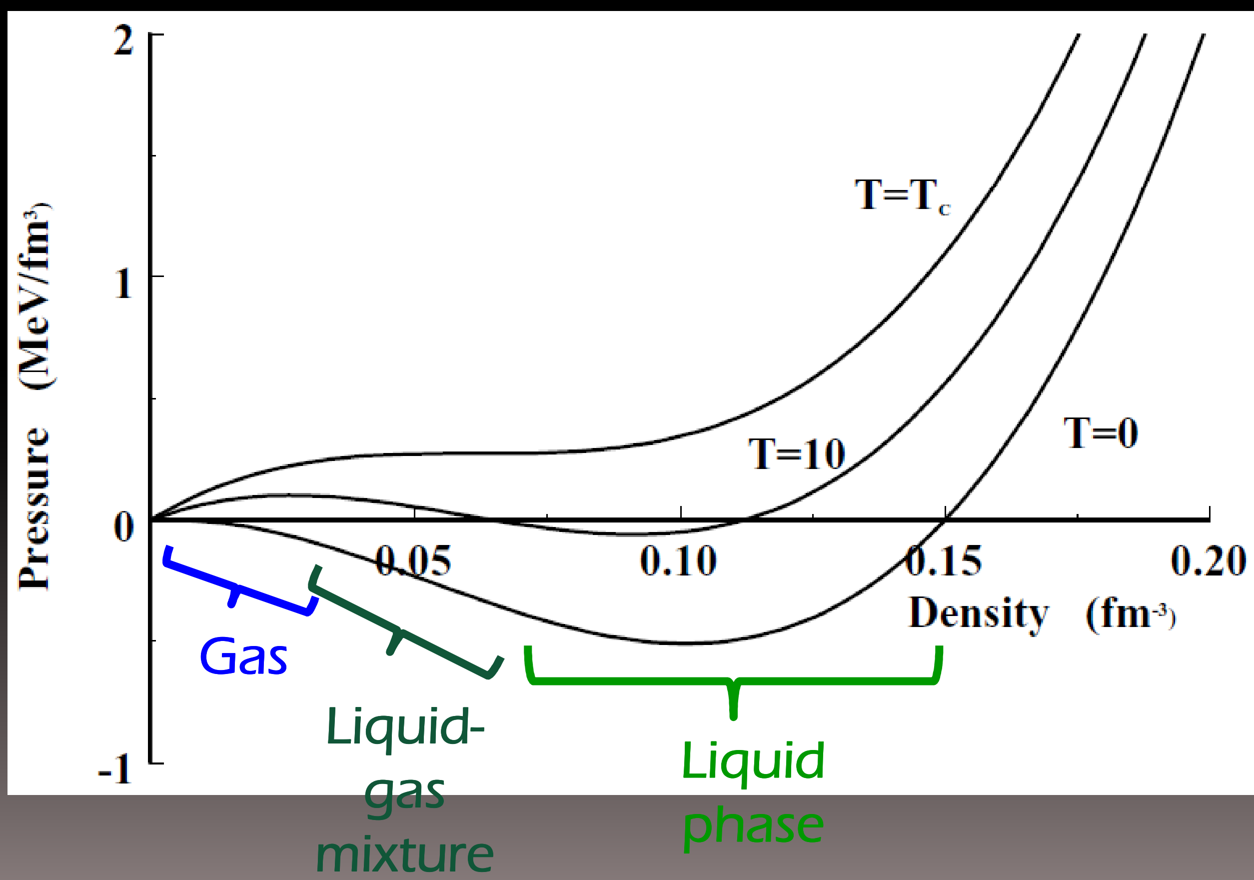
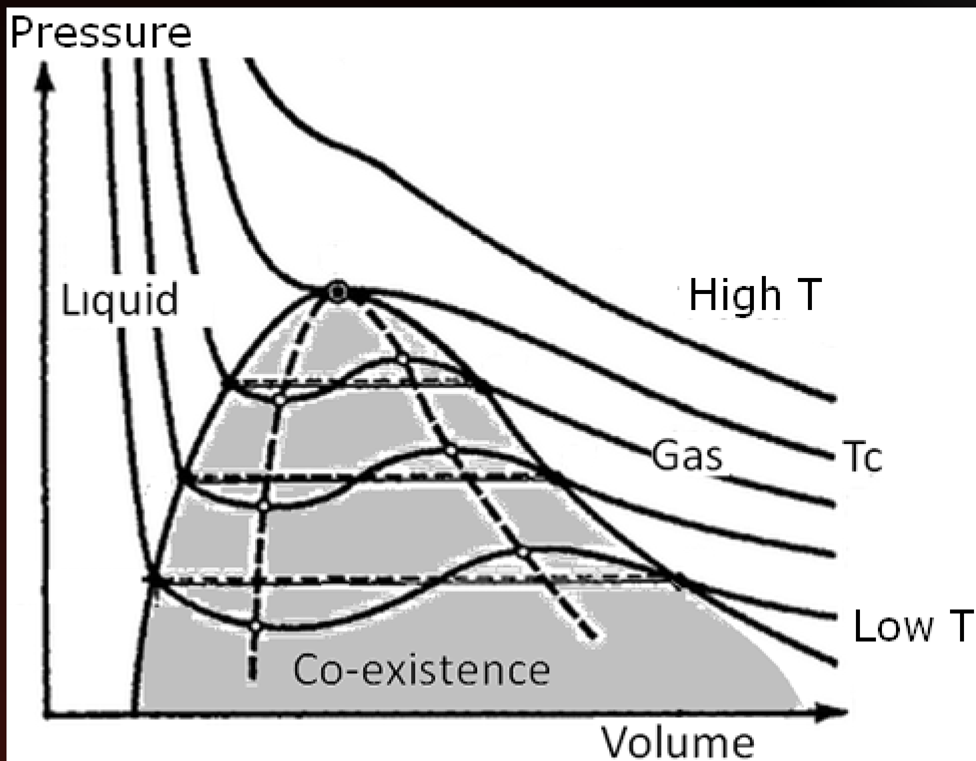
It is known that:

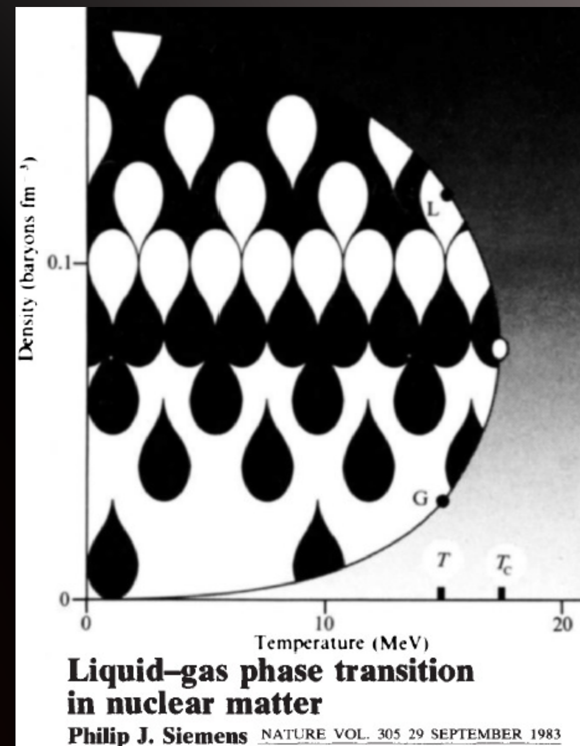
- Nuclear density is ~0.15 fm^{-3}
- Nuclear compressibility is ~200 MeV
- At low densities nuclear matter is a non-interacting Fermi gas

Can we extract an equation of state out of these data?



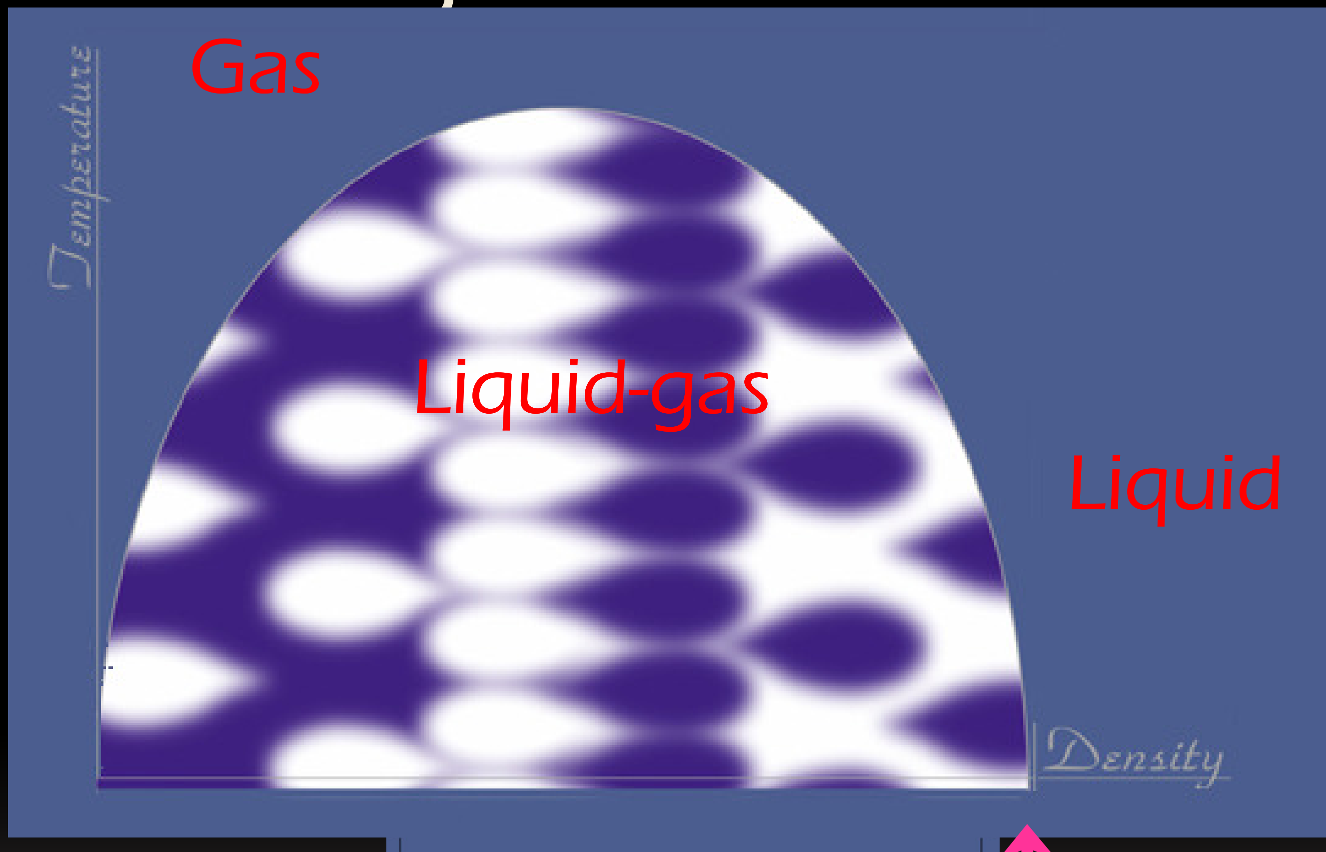
Behavior pressure-density resembles a liquid





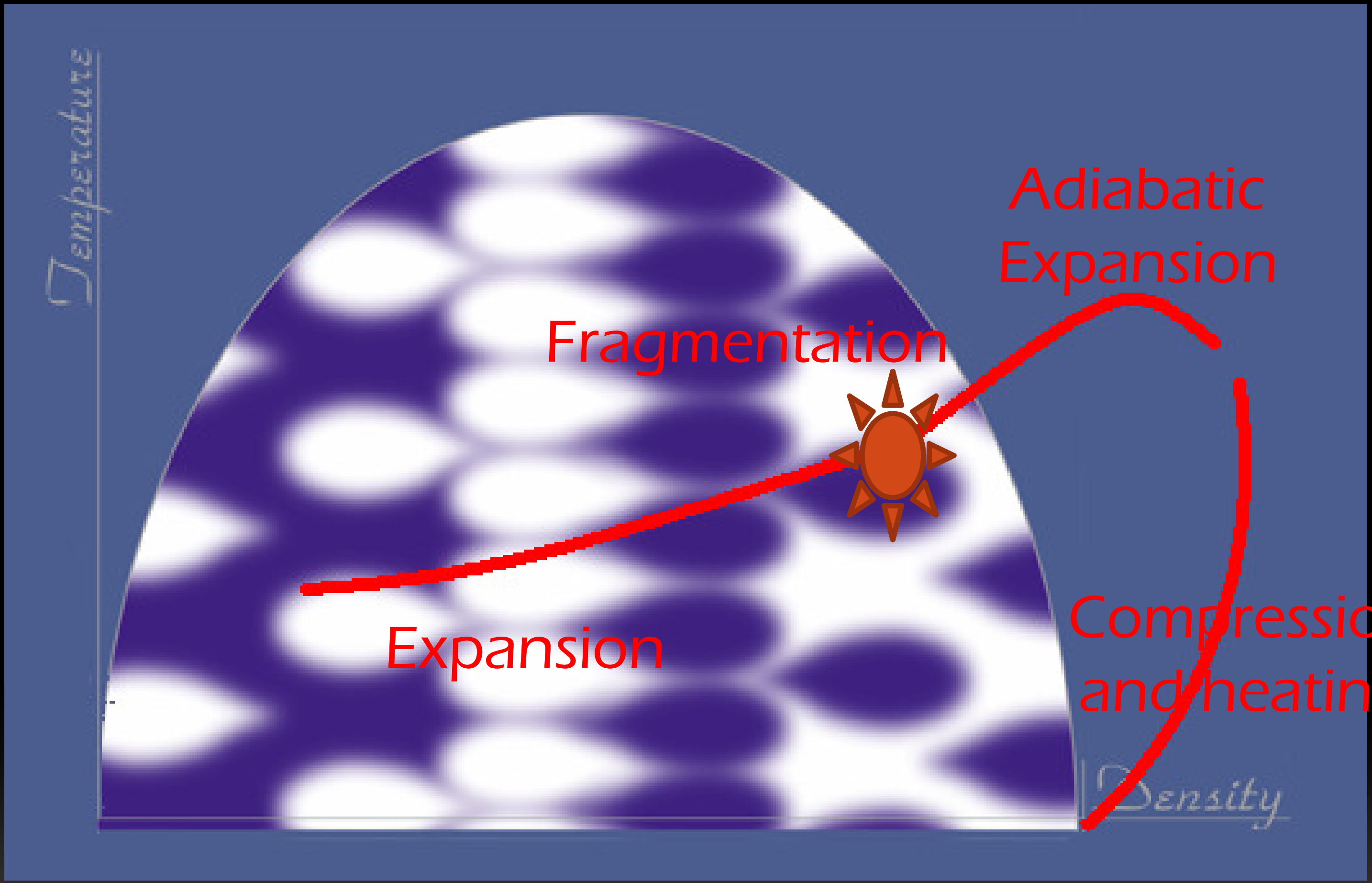
Liquid-gas phase transition is possible in nuclear matter

Phase diagram



Normal nucleus

Collision trajectory



Fragmentation reflects conditions in liquid-gas region

But at such conditions

1. Nucleons in a nucleus are not so restricted by Pauli principle

2. Nucleons and fragments obey classical dynamics

Let's check these quantum caveats

QUANTUM CAVEATS I

Take nucleons in fragments as particles in a box

Number of levels available at a given energy $\Phi(\epsilon) = \frac{\pi V}{6}\left(\frac{8M\epsilon}{h^2}\right)^{3/2}$

$\epsilon = 3T/2$
$M =$ nucleon mass neutron or proton
$\rho =$ number densities at saturation

Compare to number of particles N

$$N/\Phi(\epsilon) = \sqrt{\frac{\pi}{6}}\frac{\rho h^3}{(2\pi MT)^{3/2}} \ll 1$$

There are more states available than nucleons

Pauli blocking is not restrictive at

1 < T < 5 MeV

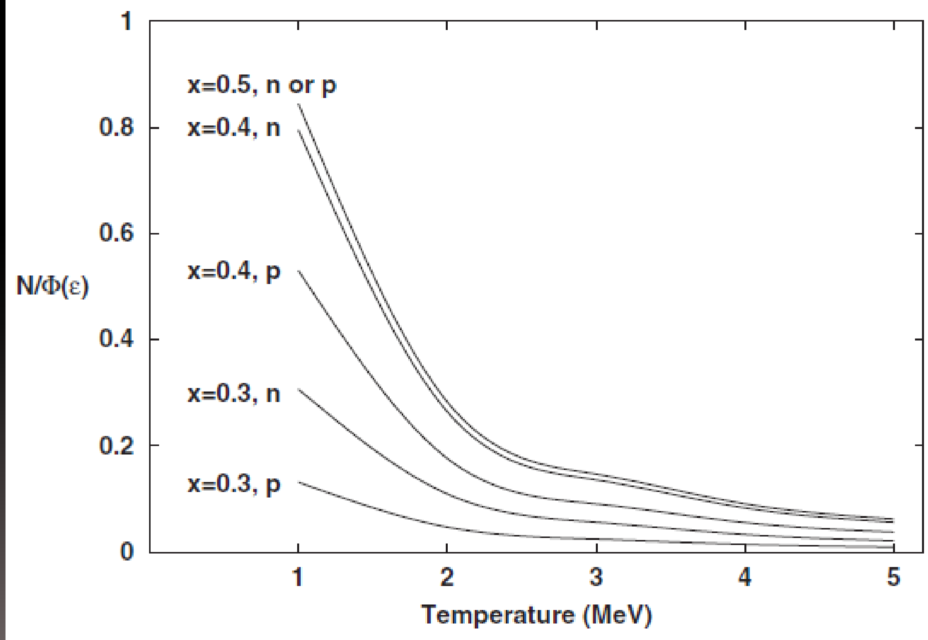


FIG. 9. Values of $N/\Phi(\epsilon)$ as a function of temperature for $x = 0.3$, 0.4, and 0.5 and for protons (p) and neutrons (n).

QUANTUM CAVEATS II

Quantum mechanics yields to classical dynamics when the mean interparticle distance is much larger than the de Broglie wavelength

$$\rho\lambda_T^3 = \frac{\rho h^3}{(2\pi MT)^{3/2}} \ll 1$$

$M =$ nuclei masses
$\rho =$ overall system's density

In the liquid-gas phase the inter-particle distance is larger than de Broglie wavelength for all cluster sizes for 1 < T < 5 MeV

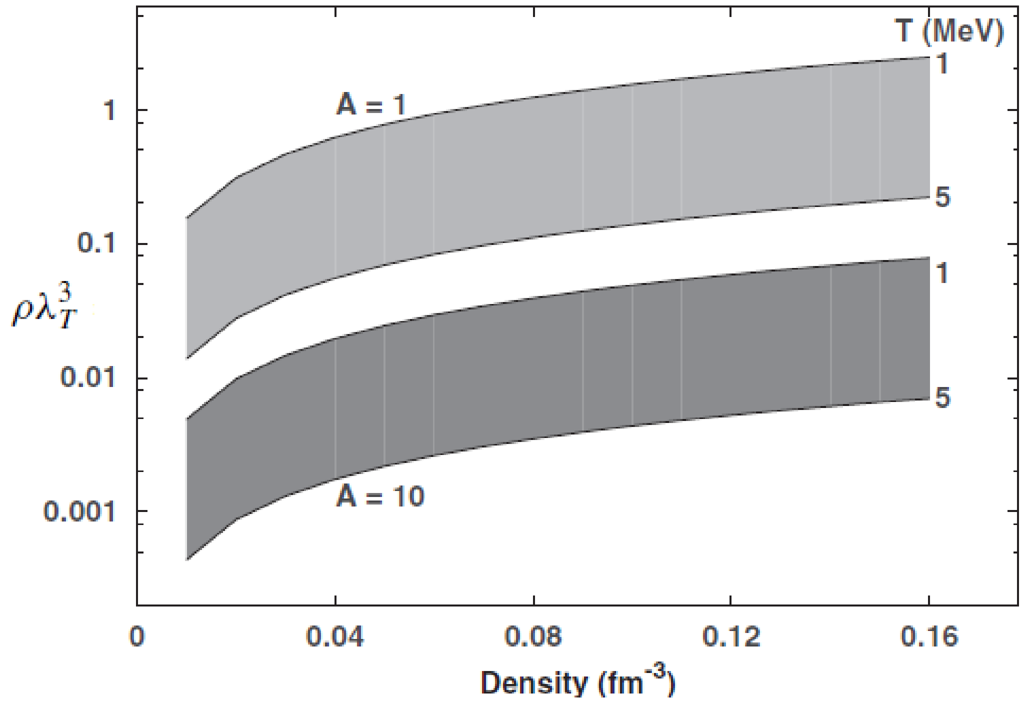


FIG. 10. Values of $\rho\lambda_T^3$ as a function of density and for a range of temperatures and for two values of the masses.

Then, since

1. Nucleons in a nucleus are not so restricted by Pauli principle and
2. Nucleons and fragments obey classical dynamics

One can use classical mechanics to study the symmetry energy at nuclear fragmentation energies !

Problem solved?

No, it is still a many-body problem

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COMPUTATIONAL MODEL

Classical molecular dynamics

- Inter-particle potential
- Uses protons & neutrons
- Proper dynamics and geometry
- Does not use “test particles”
- Does not use gaussian density distributions
- Produces fragments without external aid
- Deexcites fragments naturally
- Uses a unique set of parameters

But . . . is classic and not quantum

Classical Molecular Dynamics

- Potential
- Solve equation of motion
- Recognize clusters
- Track evolutions in space-time

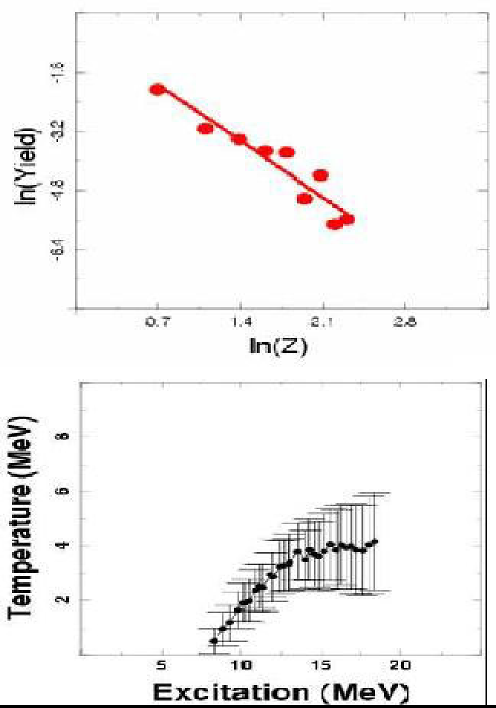
$$V_{np}(r) = V_r\left(\frac{e^{-\mu_r r}}{r} - \frac{e^{-\mu_r r_c}}{r_c}\right) - V_a\left(\frac{e^{-\mu_a r}}{r} - \frac{e^{-\mu_a r_a}}{r_a}\right),$$

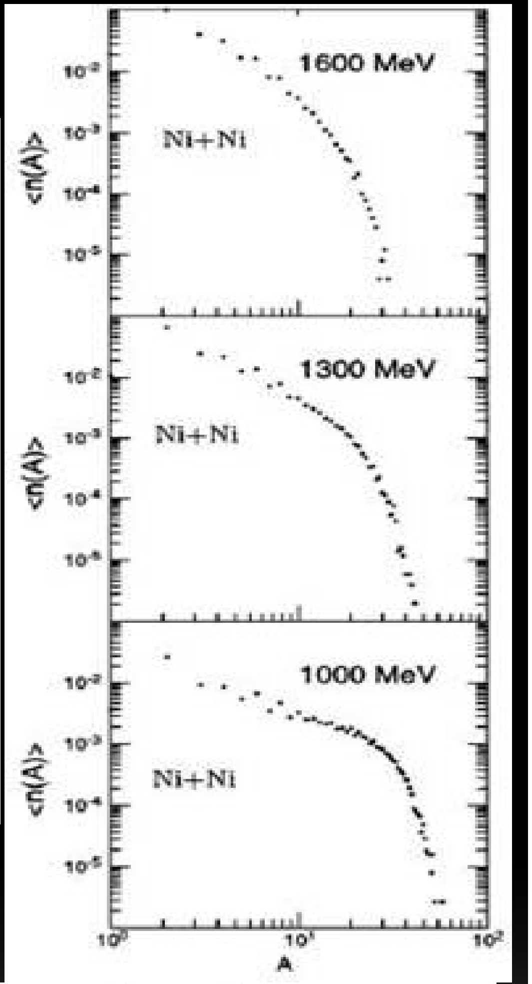
$$V_{nn}(r) = V_{pp}(r) = V_0\left(\frac{e^{-\mu_0 r}}{r} - \frac{e^{-\mu_0 r_c}}{r_c}\right)$$

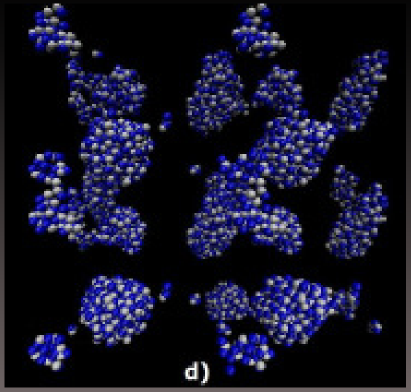
COMPUTATIONAL MODEL

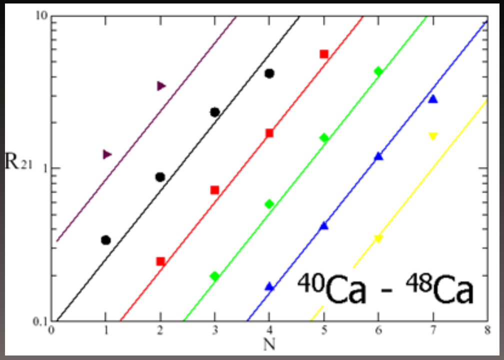
CMD can determine

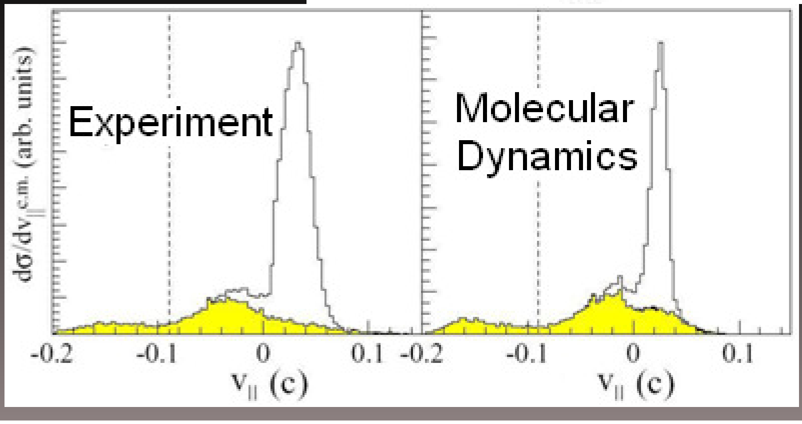
- *Mass distributions*
- *Critical phenomena*
- *Caloric curves*
- *Isoscaling*
- *Nuclear "Pasta"*











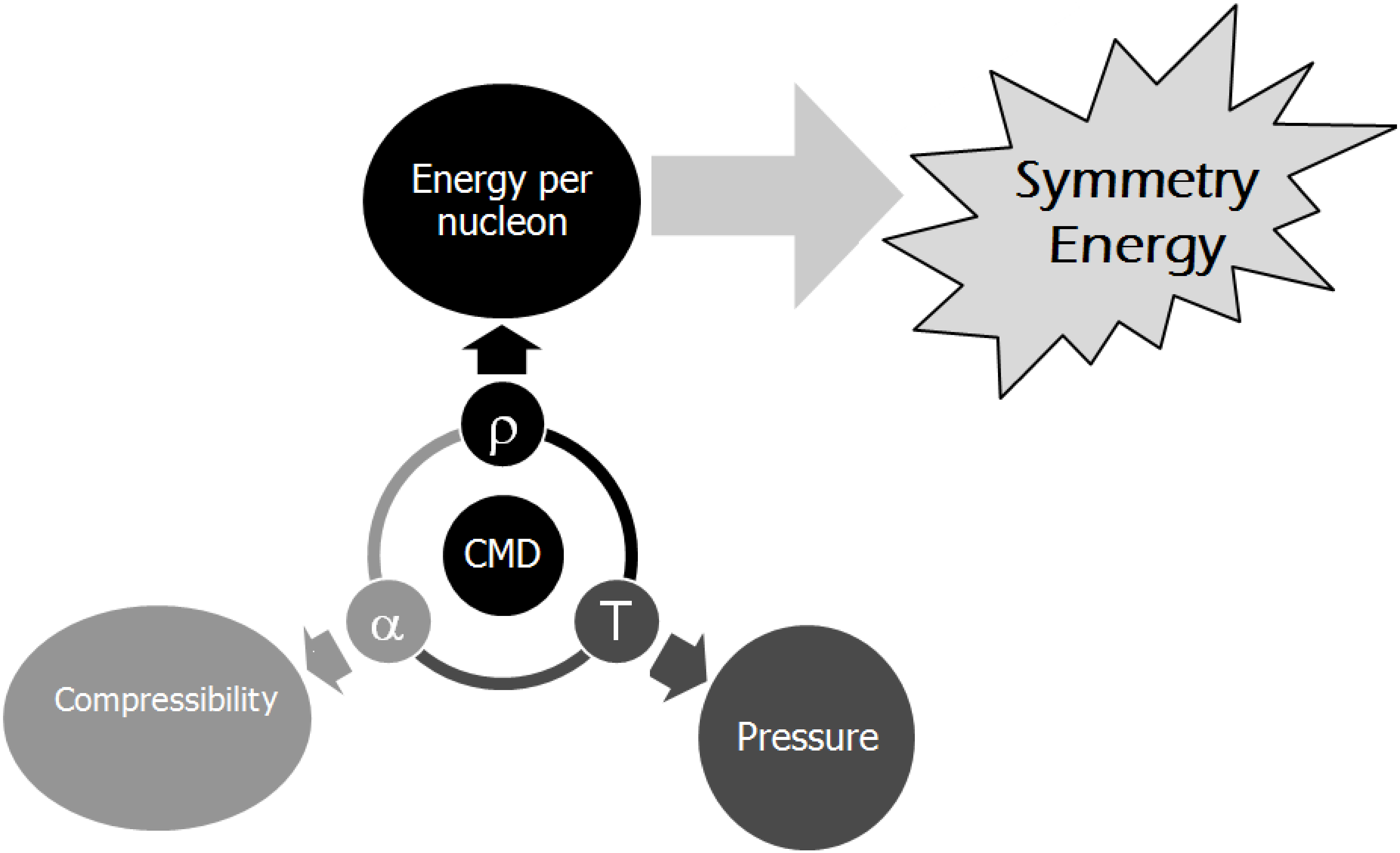
Procedure to study infinite nuclear matter

- Create an infinite system
  - Select density ρ
  - Select Temperature
- Equilibrate
- Measure
  - Binding energy E(ρ,T)
  - Pressure p(ρ,T)
  - Compressibility K(ρ,T)
- Obtain equation of state
- Study other properties of system

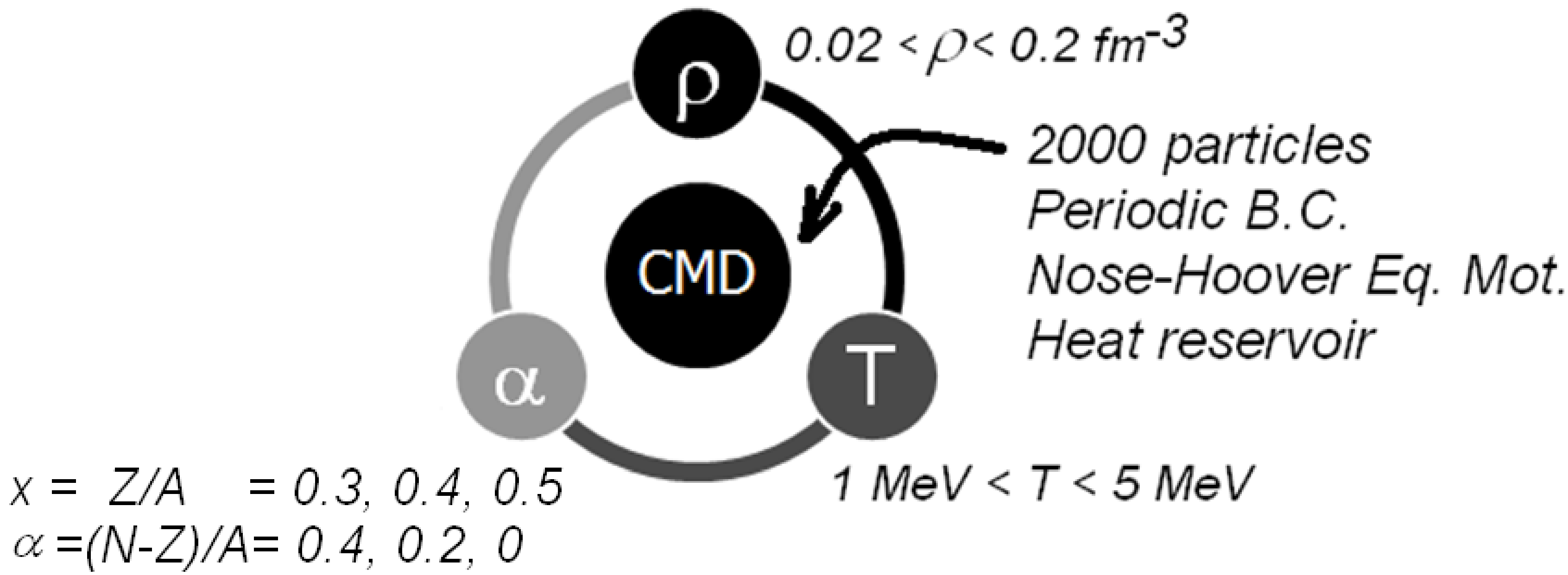
INFINITE MATTER

- Method 1: Lattices (SC, FCC and BCC)
- Method 2: Molecular dynamics
- Symmetric matter (1,000 p & 1,000 n)
- $0 < \rho < \rho_0$, $0 < T < 1.0$ MeV
- Periodic boundary conditions
- Andersen thermostat
- Minimum spanning tree to identify clusters
- Potentials:
  - Pandharipande medium
  - Pandharipande stiff
  - Horowitz

Procedure


Energy per nucleon
Symmetry Energy
ρ
CMD
α
T
Compressibility
Pressure

Some details



- Statistical averages were obtained out of ensembles of 200 systems at every combination of (ρ, T, x).
- The average of all standard deviations of E/A was 0.036 MeV.

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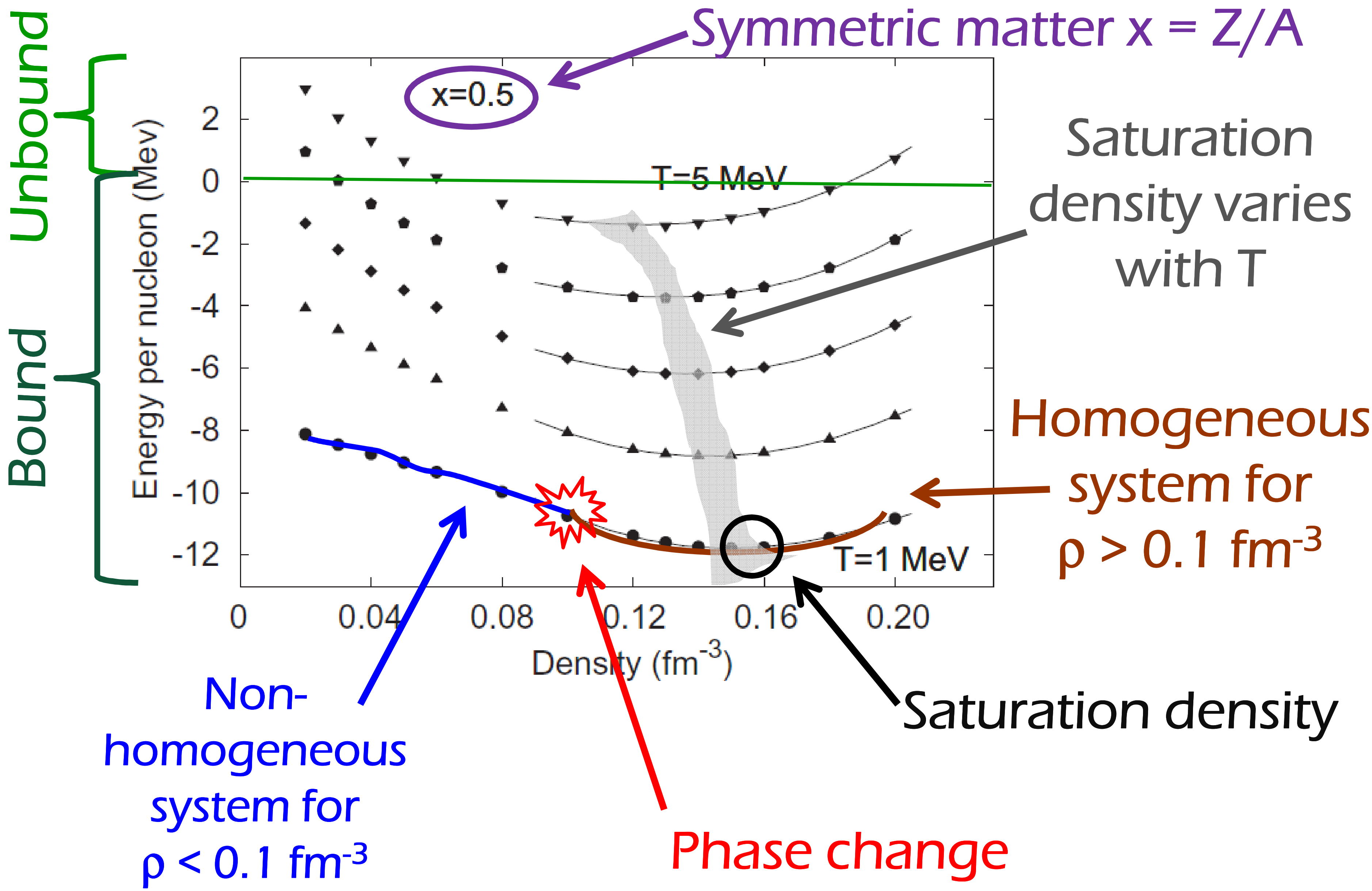
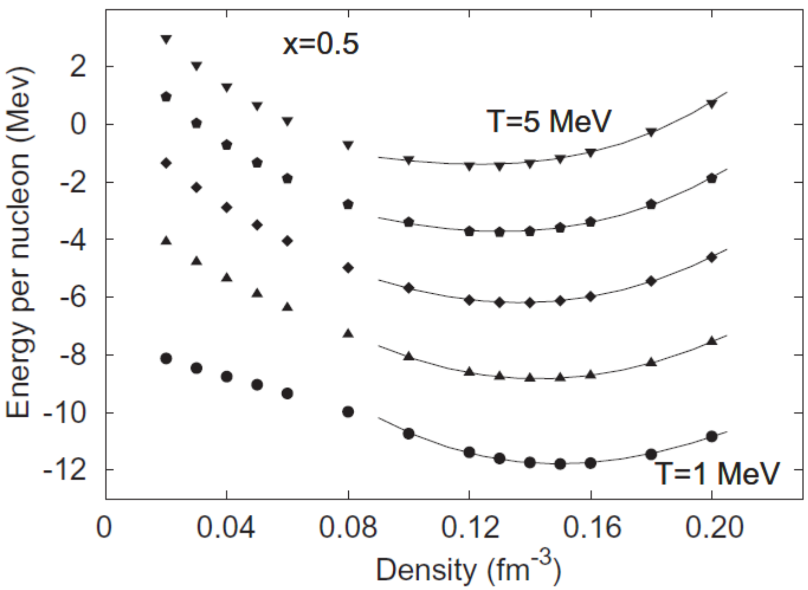
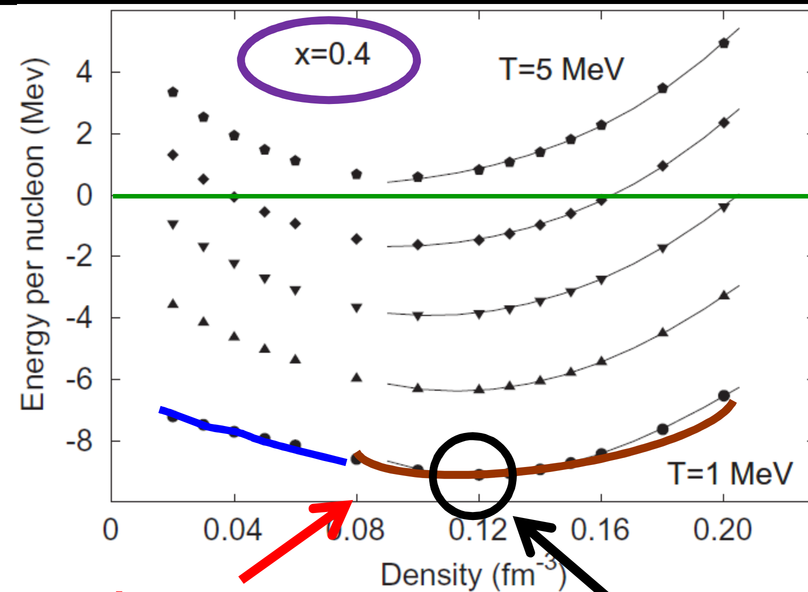
Energy per nucleon



NEUTRON RICH MATTER — Energy per nucleon: variation with isospin



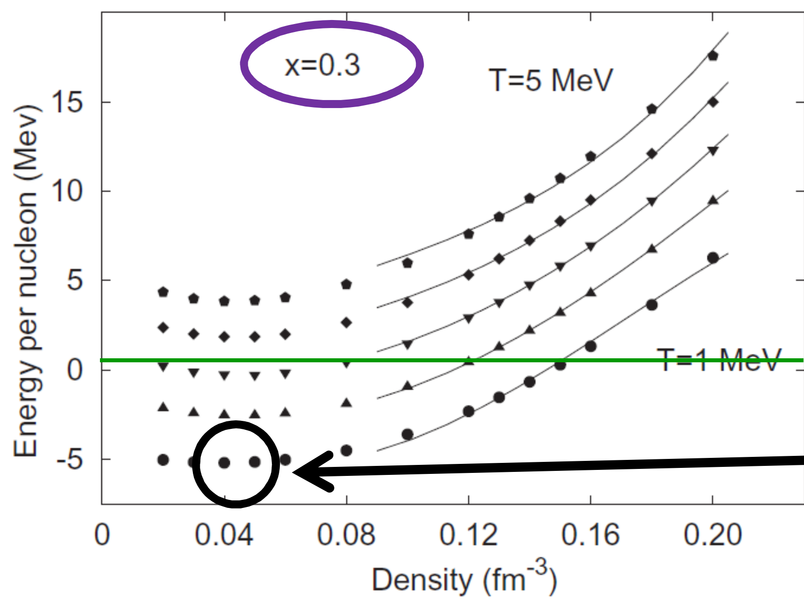


Phase change

Lower saturation density

Unbound since very low T

Bound only at very low T



Phase change?

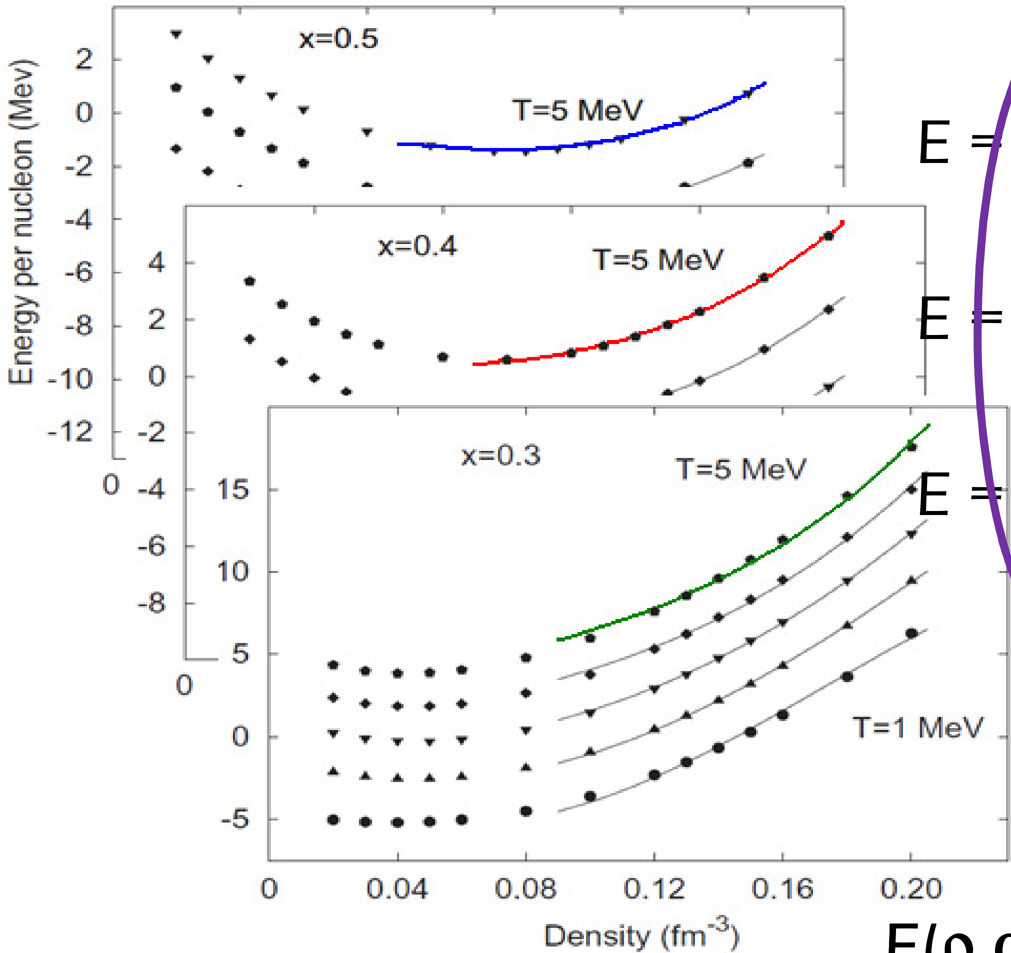
Very low saturation density

SYMMETRY ENERGY

$$E_{Sym} = \frac{1}{2}\frac{\partial^2 E(\rho,\alpha)}{\partial \alpha^2}\bigg|_{\alpha=0}$$

How to extract the symmetry energy from CMD simulations

Procedure: pick a temperature



Obtain fits

$\alpha = 0$

$E = -1.15\,\rho - 254.00\,\rho^2 + 1396.38\,\rho^3$

$\alpha = 0.2$

$E = 10.05\,\rho - 171.11\,\rho^2 + 1226.08\,\rho^3$

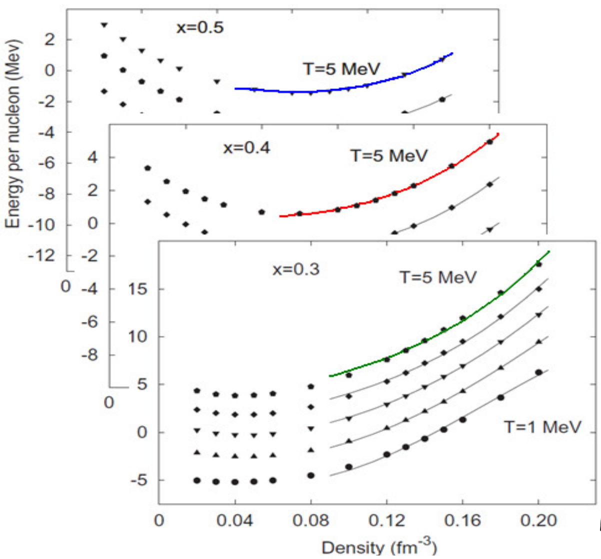
$\alpha = 0.4$

$E = 94.64\,\rho - 581.32\,\rho^2 + 2775.69\,\rho^3$

$E(\rho,\alpha) = E_1(\alpha)\,\rho + E_2(\alpha)\,\rho^2 + E_3(\alpha)\,\rho^3$

Extract coefficients

SYMMETRY ENERGY



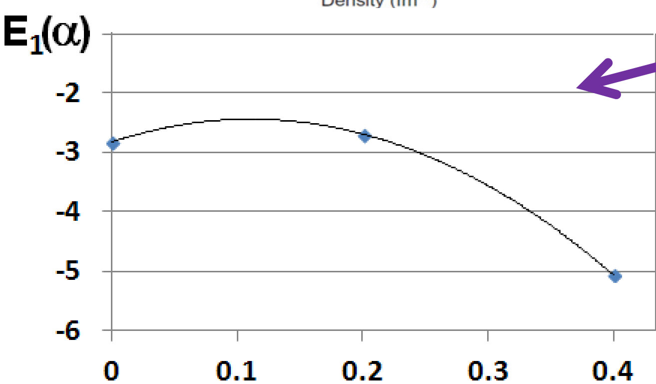
$$E = -1.15\,\rho - 254.00\,\rho^2 + 1396.38\,\rho^3 \qquad \alpha=0$$

$$E = 10.05\,\rho - 171.11\,\rho^2 + 1226.08\,\rho^3 \qquad \alpha=0.2$$

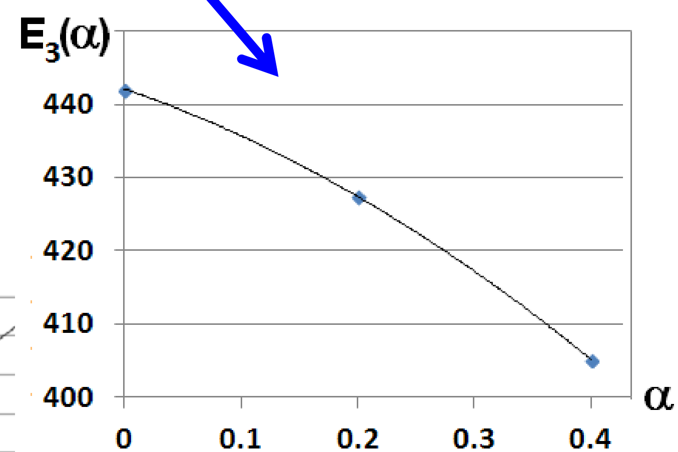
$$E = 94.64\,\rho - 581.32\,\rho^2 + 2775.69\,\rho^3 \qquad \alpha=0.4$$

$$E(\rho,\alpha) = E_1(\alpha)\,\rho + E_2(\alpha)\,\rho^2 + E_3(\alpha)\,\rho^3$$

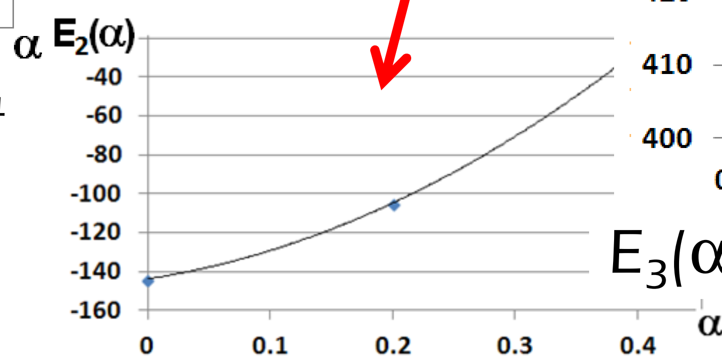
Fit coefficients



$$E_1(\alpha)=E_{00}+E_{02}\alpha^2+E_{04}\alpha^4$$

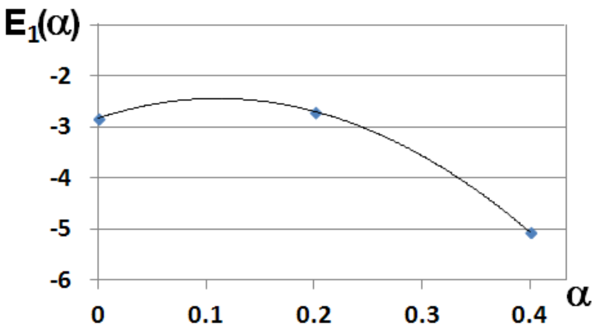


$$E_3(\alpha)=E_{20}+E_{22}\alpha^2+E_{24}\alpha^4$$

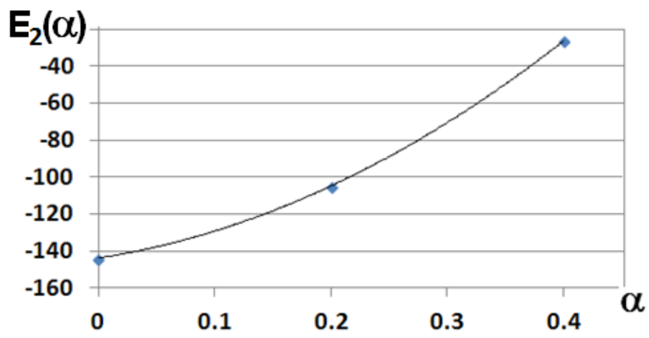


$$E_2(\alpha)=E_{10}+E_{12}\alpha^2+E_{14}\alpha^4$$

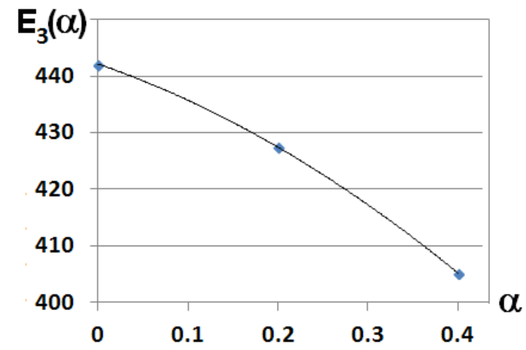
SYMMETRY ENERGY



$E_1(\alpha)=E_{00}+E_{02}\alpha^2 +E_{04}\alpha^4$



$E_2(\alpha)=E_{10}+E_{12}\alpha^2+E_{14}\alpha^4$



$E_3(\alpha)=E_{20}+E_{22}\alpha^2 +E_{24}\alpha^4$

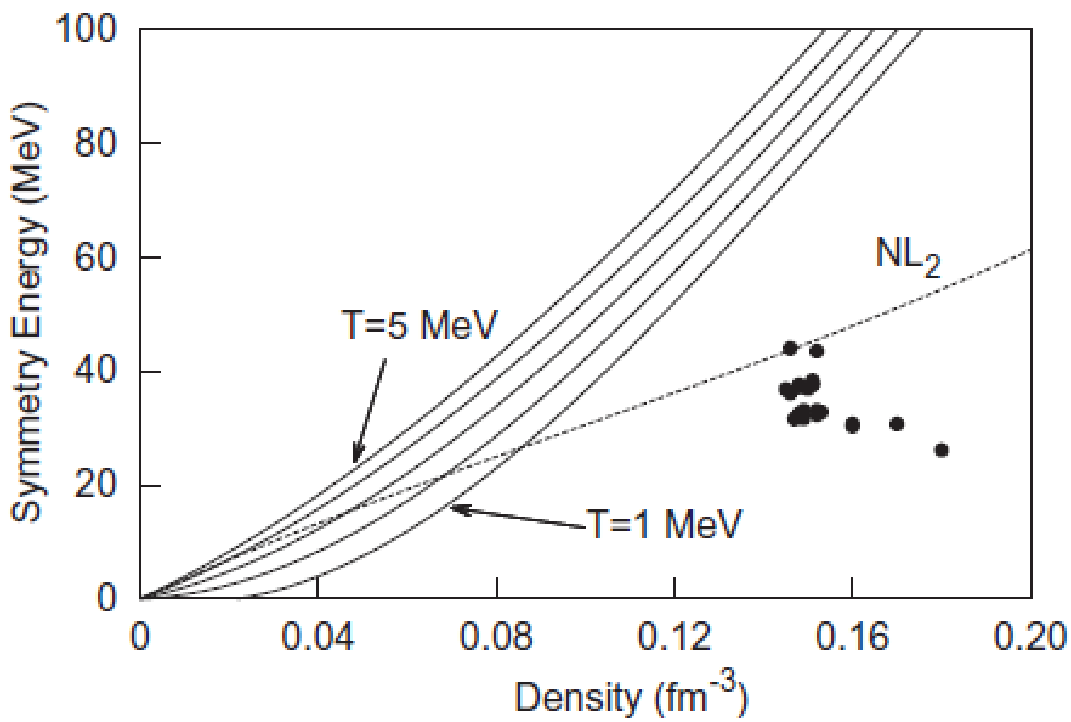
In general:

$$E(\rho,\alpha)=[E_{00}+E_{02}\alpha^2+E_{04}\alpha^4]\rho+[E_{10}+E_{12}\alpha^2+E_{14}\alpha^4]\rho^2+[E_{20}+E_{22}\alpha^2+E_{24}\alpha^4]\rho^3$$

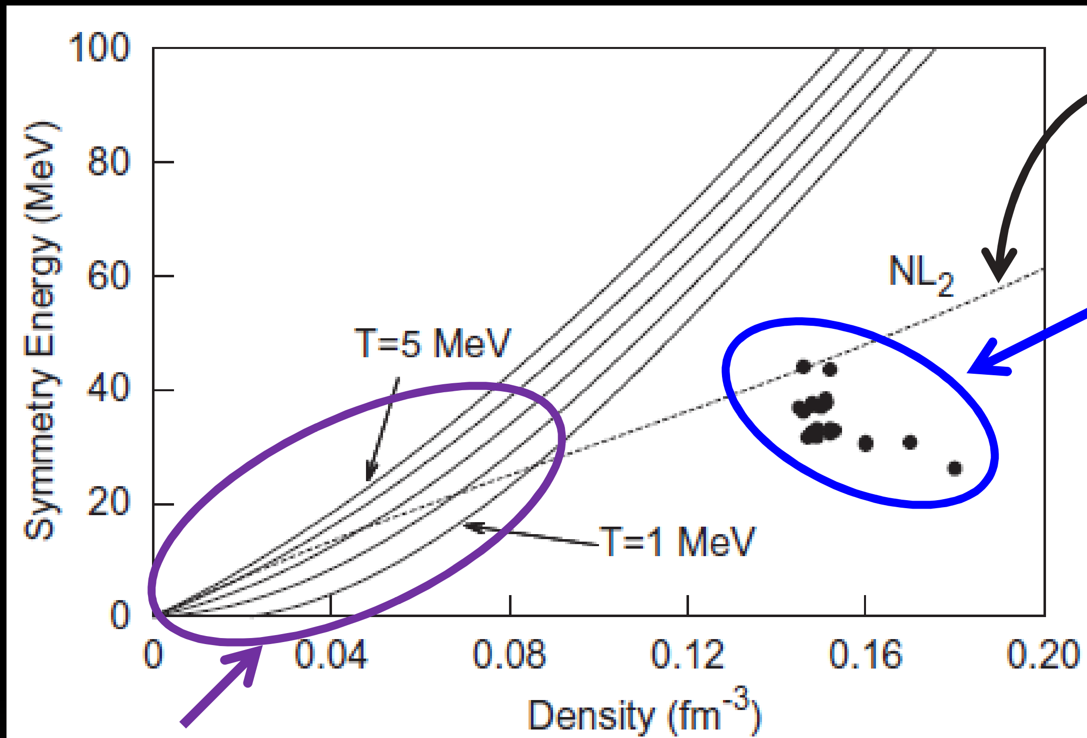
$$E_{Sym}=\frac{1}{2}\frac{\partial^2 E(\rho,\alpha)}{\partial\alpha^2}\bigg|_{\alpha=0}$$

$$=E_{02}\rho+E_{12}\rho^2+E_{22}\rho^4$$

For each temperature



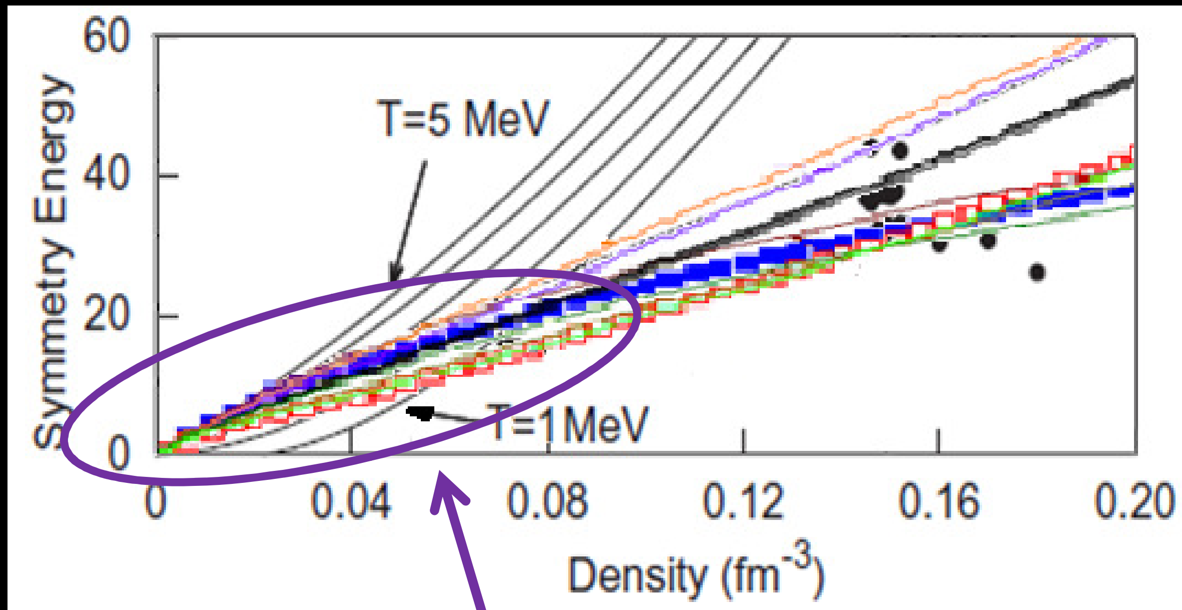
SYMMETRY ENERGY



S.-J. Lee, J. Fink, A.B. Balantekin et al., PRL 57, 2916 (1986).

Rel. and non-rel. HF approximations.
Rel. and non-rel Mean Field theories

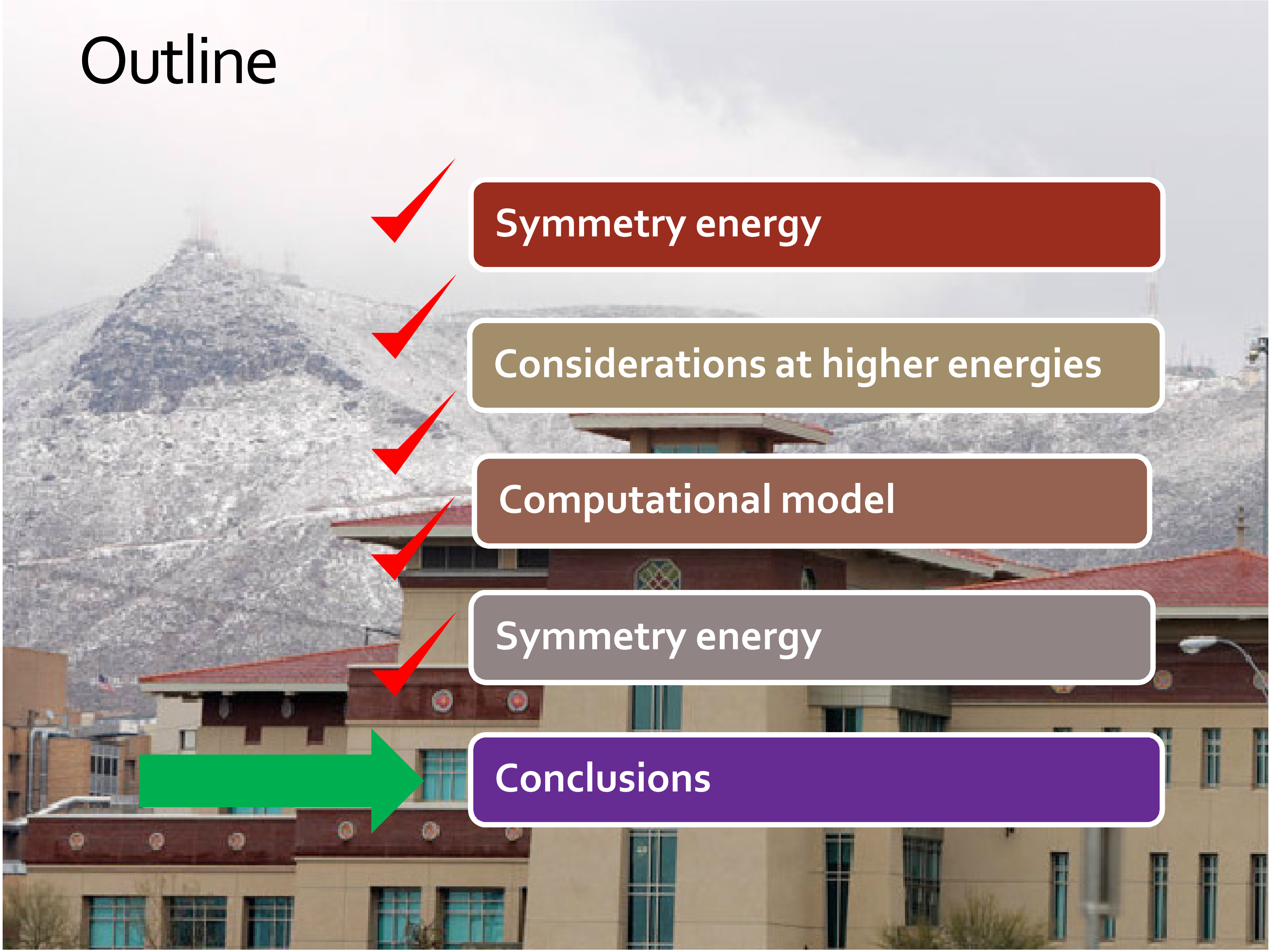
- Mostly classical behavior at low densities



CMD – RMFT

Agreement at low densities?

$\rho \leq 0.08\ fm^{-3}$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- Nuclear symmetry energy can be studied with CMD
- Symmetry energy is mostly classical at low ρ

PHYSICAL REVIEW C
nuclear physics

Isospin-asymmetric nuclear matter

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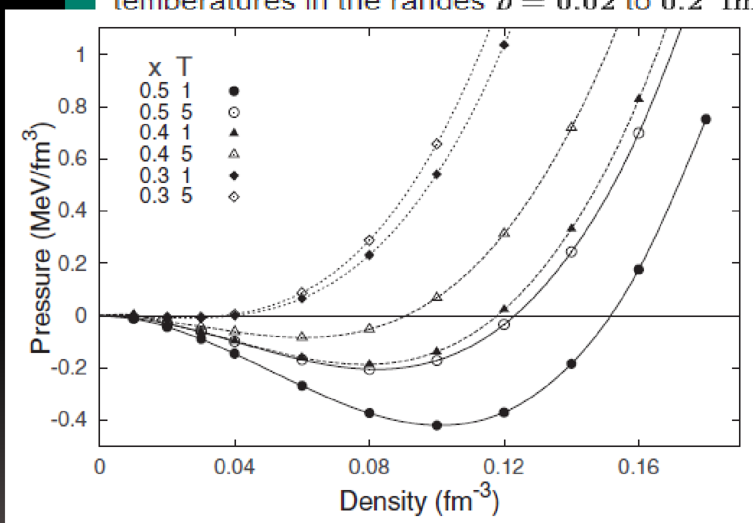
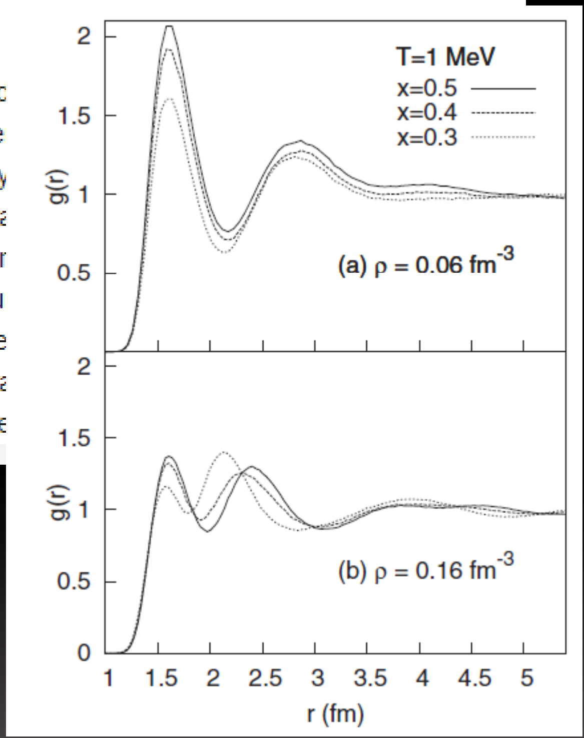
ABSTRACT

This study uses classical molecular dynamics to simulate infinite nuclear matter anc
asymmetry on bulk properties such as energy per nucleon, pressure, saturation de
energy. The simulations are performed on systems embedded in periodic boundary
temperatures in the ranges $\rho = 0.02$ to $0.2\ \mathrm{fm}^{-3}$ and $T = 1, 2, 3, 4,$ and 5 MeV, a
ate that symmetric and asymmetric r
n a liquid phase to a liquid-gas mixtu
uilibrium densities, a softening of the
n. A procedure leading to the evalua
plemented and compared to mean fie





Symmetry energy and phase diagram of warm asymmetric matter

¡Thanks !

Current work:

- Isospin diffusion

Future work:

- Connection with reactions

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Thanks to Lawrence Berkeley Lab for hospitality