Resonance and scattering aspects of weakly bound nuclei at low energy

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PKU-CUSTIPEN Nuclear Reaction Workshop
"Reactions and Spectroscopy of Unstable Nuclei"

MCAS for scattering, resonances, bound-states

- Developed a scattering method Multi-Channel Algebraic Scattering (MCAS)
- – Starts from Coupled-Channel interaction method.
- – Expands the CC potential using a Sturmian-expansion method
- – Computes algebraically the S-matrix
- – Extracts from the S-matrix all structure and resonance informations (bound-states, shape resonances, Fano-Feshbach, etc.)
- – Includes the effects of the Pauli exclusion principle between the incoming nucleon and the targets.
- Presently, analyzed spectra of medium-light nuclei C13 (n-C12), N13 (p-C12), C15 (n-C14), F15 (p-O14), He7, B7, Be7, Li7, Be9, B9, C17 (n-C16), Na-17 (p-Ne16) C-19 (n-C18), $^{9}{}_{\Lambda}$Be, $^{13}{}_{\Lambda}$C,
  current ... Ne23, Mn23, Na23, Al23, O17, O19 ...

Collaboration MCAS: L. C. (Padova), K. Amos (Melbourne), J.P.Svenne (Manitoba), S. Karataglidis (Sud Africa), D. van der Knijff (Melbourne), P. Fraser (Perth).

Produced publications: Nucl. Phys. A 728 (2003),

Phys.Rev.Lett.94:122503,2005. Phys.Rev.Lett.96:072502,2006.
Phys.Rev.Lett.101:242501,2008. Phys.Rev.Lett.99:089202,2007.
Phys.Rev.C72:014601,2005. Phys.Rev.C72:064604,2005.
Phys.Rev.C73:027601,2006. Phys.Rev.C74:064605,2006.
Phys.Rev.C83:047603,2011. Rev.Mex.Fis.57:20-29 2011
Nucl.Phys.A813:235-251,2008. Nucl.Phys.A808:192-219,2008.
Nucl.Phys.A790:251-256,2007. Eur.Phys.J.A35:69-80,2008.
Int.J.Mod.Ph.E19:1435-1450,2010. Nucl.Phys.A879:132-145,2012.
Nucl.Phys.A912:7-17,2013. JCAP.1206:030,2012.
Europhysics Lett. 99:12001, 2012. Phys.Rev.C,2014, in publication.

Sturmians (*aka* Weinberg states): a *different* way to QM.
Consider a two-body like Hamiltonian:

$$(E - H_o)\Psi_E = V\Psi_E \, , \tag{1}$$

where E is the spectral variable, and Ψ_E is the eigenstate.
Sturmians are the eigensolutions of:

$$(E - H_o)\Phi_i(E) = \frac{V}{\eta_i(E)}\Phi_i(E) \, , \tag{2}$$

where E is a parameter. The eigenvalue η_i is the potential scale.
SPECTRUM: all the potential rescalings that give solution to that equation, for given energy E, and with well-defined boundary conditions.

With sturmians, the S-matrix can be written as

$$S(E) = \frac{\Pi_i(1 - \eta_i(E^{(-)}))}{\Pi_i(1 - \eta_i(E^{(+)}))} \tag{3}$$

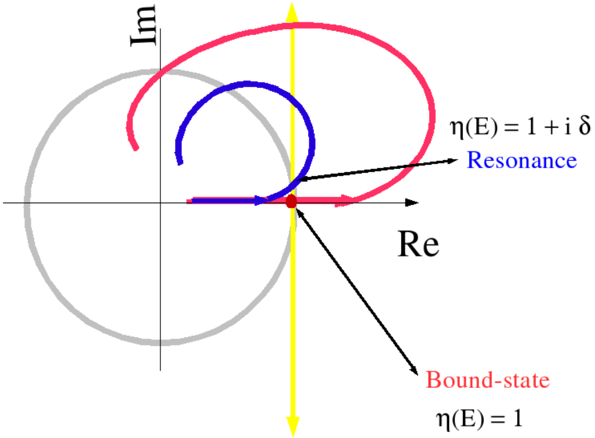
Alternatively, introducing the factor $\hat{\chi}_i(E,k)$ in momentum space

$$\hat{\chi}_i(E,k) = < k, c|V|\Phi_i(E) > , \tag{4}$$

the S-matrix can be rewritten also as

$$S(E) = 1 - i\pi k \sum_i \hat{\chi}_i(E^{(+)};k) \frac{1}{1 - \eta_i(E^{(+)})} \hat{\chi}_i(E^{(+)};k) \tag{5}$$

How resonances and bound states are found in Sturmian theory.



Model Potential

Current description: nucleon-nucleus scattering (light nuclei with 0^+ g.s.) including first core excitation of collective nature (quadrupole, octupole, etc).

$$V_{cc'}(r) = \sum_{n=C,LS,LL,SI} V_n < (\ell s)jI; J^\pi | \mathcal{O}_n f_n(r, R, \theta_{\mathbf{r},\mathbf{R}}) | (\ell' s)j' I'; J^\pi >$$

For all operators, the functional forms are expanded to second order in the core-deformation parameter $(R = R_0(1 + \beta_2 P_2(\theta))$

$$f_n(r, R, \theta) = f_n^{(0)}(r) - \beta_2 R_0 P_2(\theta) \frac{d}{dr} f_n^{(0)}(r)$$
$$+ \frac{\beta_2^2 R_0^2}{2\sqrt{\pi}} \left(P_0 - \frac{2\sqrt{5}}{7} P_2(\theta) + \frac{2}{7} P_4(\theta) \right) \frac{d^2}{dr^2} f_n^{(0)}(r)$$

The $n + {}^{12}$C system used in MCAS

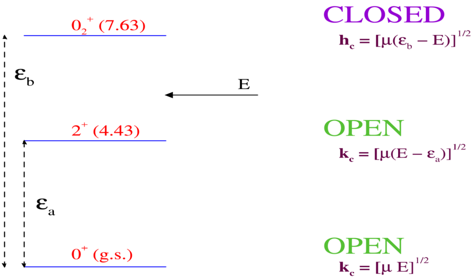


Figure: The core spectrum used as inut.

First application $n - C12/p - C12$ aborted: Why?

Bound states

^{13}C four observed $\rightarrow$ 12 computed

^{13}N one observed $\rightarrow$ 8 computed

The deep forbidden states contaminate the physical solution due to Coupled-Channel dynamics. Problems in CC formalisms (but not only)... (Embedded CC codes in nuclear reaction: GNASH, EMPIRE, TALYS)

Solution: introduce the OPP potential !

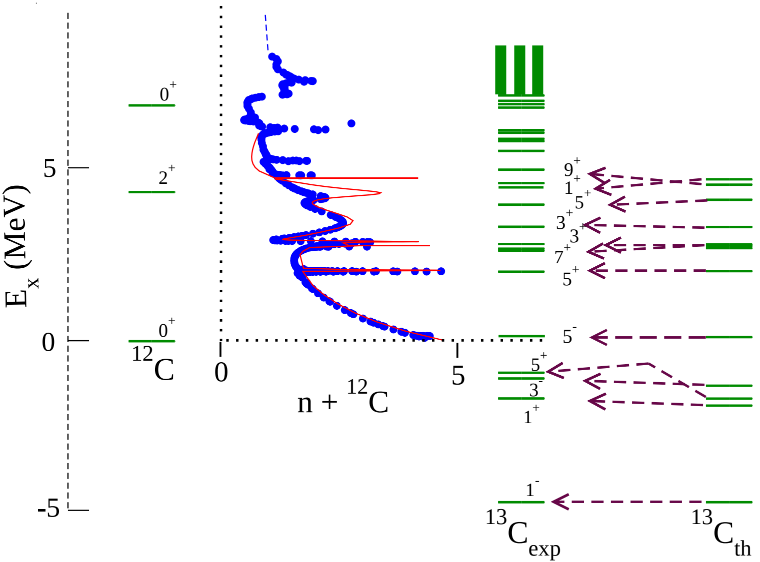
The full nuclear potential $\mathcal{V}_{cc}(r)$ is not the local potential $n - C12$:
The "complete" potential is (in partial-wave decomposition)

$$\mathcal{V}_{cc'}(r, r') = V_{cc'}(r)\delta(r - r')$$

$$+\delta_{cc'}\lambda_c A_c(r)A_c(r')(\delta_{c=s\frac{1}{2}^+}) + \delta_{cc'}\lambda_c A_c(r)A_c(r')(\delta_{c=p\frac{3}{2}^-})$$

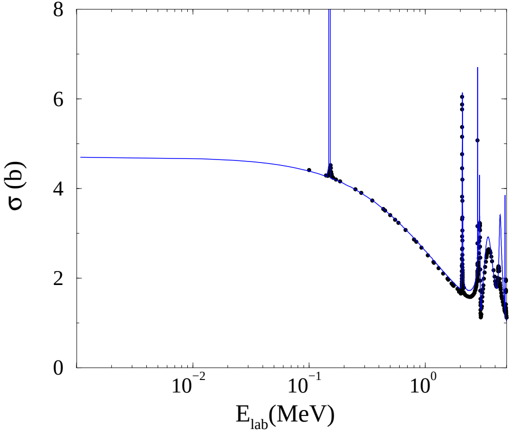
$A_c(r)$ are the Pauli-forbidden deep (CC-uncoupled) bound states.

A state in the OPP approach is:
forbidden in the limit $\lambda \to +\infty$
allowed when $\lambda \to 0$
hindered when $\lambda \to$ finite but $\neq 0$



E_x (MeV)
5
0
-5
0^+
2^+
0^+
^{12}C
0
5
n + ^{12}C
9^+
1^+
5^+
3^+
3^+
7^+
5^+
5^-
5^+
3^-
1^+
1^-
$^{13}C_{exp}$
$^{13}C_{th}$

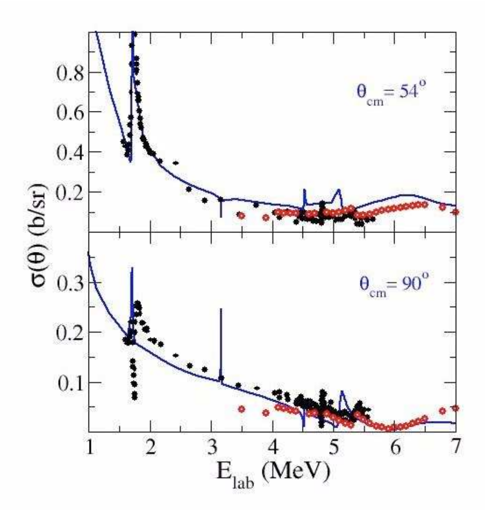
n-^{12}C: Low energy details



MCAS calculation

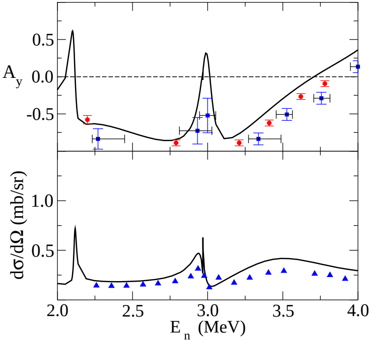
$\frac{5}{2}^{-}$ resonance centroid very sensitive to Pauli blocking

proton-C12 scattering/cross-sections

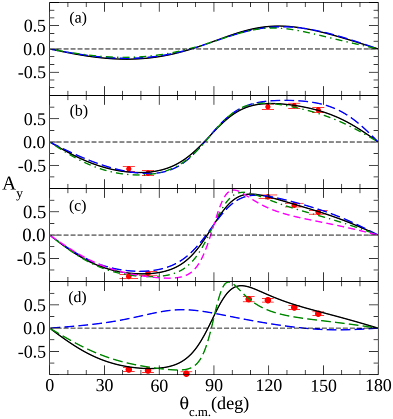


Analyzing powers of nucleons off ^{12}C

- First MCAS results:
  K.Amos *et al* NPA **728** ('03); PRC **72** ('05)
- Experimental data A_Y:
  TUNL data on $\vec{n}$-^{12}C A_y $2.2 < E < 8.0$ MeV
  ($\Delta E \simeq 0.2 \div 0.4$)
  C.D. Roper, W. Tornow, *et al.* PRC **72** ('05)
- Tests the spin-structure of our interaction. Spin-orbit needs additional terms at these energies?



$(\theta_{cm} = 43.36^o)$
Red Data from TUNL (2005)
Blue Data A_y from W. Bucher et al (1959), $d\sigma$ from Fasoli et al. (1973)



1.9 MeV

2.2 MeV

2.8 MeV

3.2 MeV

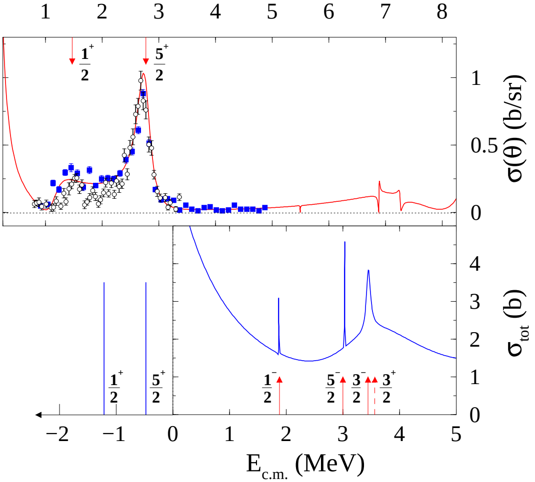
Analysis of ^{15}F (vs. ^{15}C mirror partner)

- Triggered by recent data by:
  V.Z. Goldberg *et al.* PRC **69** ('04)
  F.Q. Guo *et al.* PRC **72** ('05).

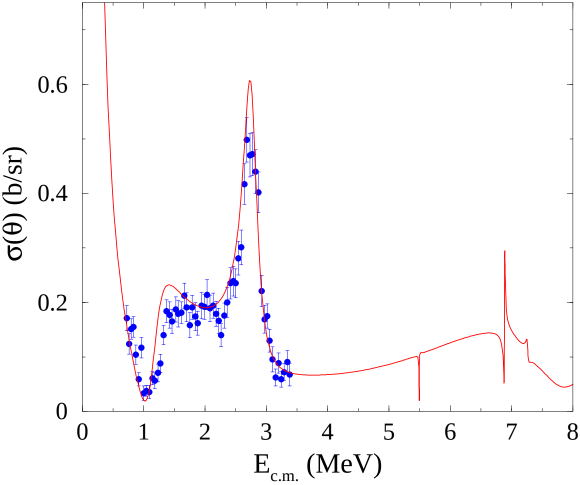
OUR STUDY

- L.Canton, G.Pisent, J.Svenne, K.Amos,*et al.*: PRL **96** ('06)
  used ^{15}C in fit-analysis to predict new states in ^{15}F

^{15}F (top, $p - ^{14}O$ $\theta_{cm} = 180^o$) & ^{15}C (bottom)



$^{15}F\ (p -^{14} O\ \ \theta_{cm} = 147^o)$



Voices from experiments ...

^{15}F resonant states

J^π	Theory $E, (\frac{1}{2}\Gamma)$	Experiment $E, (\frac{1}{2}\Gamma)$
$\frac{1}{2}^+$	1.31 (0.8)	1.47 (1.00)
$\frac{5}{2}^+$	2.78 (0.3)	2.77 (0.24)
$\frac{1}{2}^-$	5.49 (0.005)	
$\frac{5}{2}^-$	6.88 (0.01)	
$\frac{3}{2}^-$	7.25 (0.04)	
$\frac{1}{2}^+$	7.21 (1.2)	
$\frac{5}{2}^+$	7.75 (0.4)	
$\frac{3}{2}^+$	7.99 (3.6)	

Table: See publication PRL **96** 072502 (2006)

Voices from experiments ...

^{15}F resonant states

J^π	Theory $E, (\frac{1}{2}\Gamma)$	Experiment $E, (\frac{1}{2}\Gamma)$
$\frac{1}{2}^+$	1.31 (0.8)	1.47 (1.00)
$\frac{5}{2}^+$	2.78 (0.3)	2.77 (0.24)
$\frac{1}{2}^-$	5.49 (0.005)	4.9 (<0.2)
$\frac{5}{2}^-$	6.88 (0.01)	6.4 (<0.2)
$\frac{3}{2}^-$	7.25 (0.04)	
$\frac{1}{2}^+$	7.21 (1.2)	
$\frac{5}{2}^+$	7.75 (0.4)	7.8 (0.4) ?
$\frac{3}{2}^+$	7.99 (3.6)	?

Table: See publication @GSI Darmstadt Mukha et al. PRC **79** 061301 (2009)

A=7 nuclei as n + halo-core

$$
\begin{aligned}
{}^7Li \leftrightarrow p+ \quad & {}^6He(0_1^+[g.s.]; 2_1^+[1.78 MeV]; 2_2^+[5.6 MeV]) \\
{}^7He \leftrightarrow n+ \quad & {}^6He(0_1^+[g.s.]; 2_1^+[1.78 MeV]; 2_2^+[5.6 MeV]) \\
{}^7Be \leftrightarrow n+ \quad & {}^6Be(0_1^+[g.s.]; 2_1^+[1.70 MeV]; 2_2^+[5.6 MeV]) \\
{}^7B \leftrightarrow p+ \quad & {}^6Be(0_1^+[g.s.]; 2_1^+[1.70 MeV]; 2_2^+[5.6 MeV])
\end{aligned}
$$

See Phys. Rev. C, 74 (2006) for comparison with the cluster picture:

$$
\begin{aligned}
{}^7Li \leftrightarrow t+ \quad & \alpha \\
{}^7Be \leftrightarrow {}^3He+ \quad & \alpha
\end{aligned}
$$

& with shell model approach (Amos-Karataglidis):

$$
\begin{aligned}
{}^7Li \leftrightarrow 3p + 4n \quad & {}^7He \leftrightarrow 2p + 5n \\
{}^7Be \leftrightarrow 3n + 4p \quad & {}^7B \leftrightarrow 2n + 5p
\end{aligned}
$$

Exp spectra:

$^6He + p \rightarrow$

8 states below

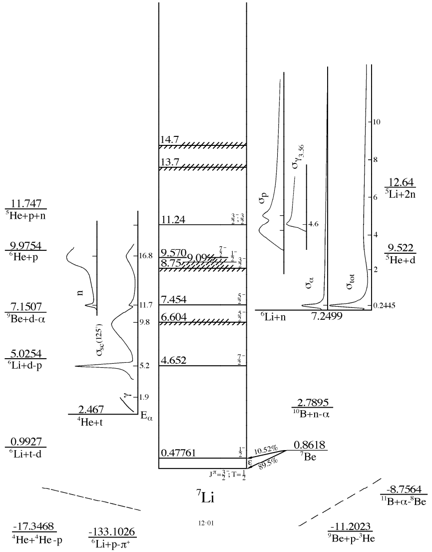


Figure 9: Energy levels of ^{7}Li. For notation see Fig. 5.

9

Table: Parameter values of the (negative parity) potential.

$V_0(\pi)$	$V_{\ell\ell}$	$V_{\ell s}$	V_{Is}
−36.817	−1.2346	14.9618	0.8511
R_0 fm;	a fm;	R_c fm;	β_2
2.8	0.88917	2.0	0.7298

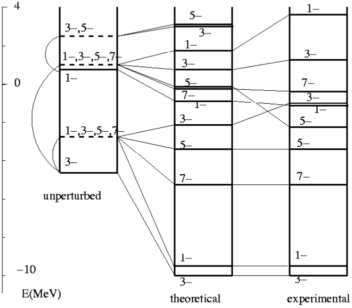


Table: Experimental data and theoretical results for ^{7}Li and ^{7}Be states (in MeV).

J^π	^{7}Li Exp.	^{7}Li Theory	^{7}Be Exp.	^{7}Be Theory
$\frac{3}{2}^-$	−9.975	−9.975	−10.676	−11.046
$\frac{1}{2}^-$	−9.497	−9.497	−10.246	−10.680
$\frac{7}{2}^-$	−5.323 [0.069]	−5.323	−6.106 [0.175]	−6.409
$\frac{5}{2}^-$	−3.371 [0.918]	−3.371	−3.946 [1.2]	−4.497
$\frac{5}{2}^-$	−2.251 [0.08]	−0.321	−3.466 [0.4]	−1.597
$\frac{3}{2}^-$	−1.225 [4.712]	−2.244	−−	−−
$\frac{1}{2}^-$	−0.885 [2.752]	−0.885		−2.116
$\frac{7}{2}^-$	−0.405 [0.437]	−0.405	−1.406 [?]	−1.704
$\frac{3}{2}^-$	−−	−−	−0.776 [1.8]	−3.346
$\frac{3}{2}^-$	1.265 (0.26)	0.704 (0.056)	0.334 (0.32)	−0.539
$\frac{1}{2}^-$		1.796 (1.57)		0.727 (0.699)
$\frac{3}{2}^-$	3.7 (0.8) ?[a]	2.981 (0.99)		1.995 (0.231)
$\frac{5}{2}^-$	4.7 (0.7) ?[a]	3.046 (0.75)		2.009 (0.203)
$\frac{5}{2}^-$		5.964 (0.23)		4.904 (0.150)
$\frac{7}{2}^-$		6.76 (2.24)	6.5 (6.5) ?[b]	5.78 (1.65)

Table: Experimental data and theoretical results for ^{7}Li (Energies are in MeV, widths are in keV). All energies are defined with thresholds, $p+^6$He $= 9.975$ MeV with respect to ^{7}Li ground state. See text for explanation of the labels "orig", "II", and "III".

J^π	^{7}Li		^{7}Li	
	Exp.	Theory (orig)	Theory (II)	Theory (III)
$\frac{3}{2}^-$	-9.975	-9.975	-9.9200	-9.9200
$\frac{1}{2}^-$	-9.497	-9.497	-9.3165	-9.3165
$\frac{7}{2}^-$	-5.323 [69]	-5.323	-5.2431	-5.2431
$\frac{5}{2}^-$	-3.371 [918]	-3.371	-3.3661	-3.3661
$\frac{5}{2}^-$	-2.251 [80]	-0.321	0.1248 (7xE-6)	0.1250 (65)
$\frac{3}{2}^-$	-1.225 [4712]	-2.244	-2.228	-2.228
$\frac{1}{2}^-$	-0.885 [2752]	-0.885	-0.5283	-0.5283
$\frac{7}{2}^-$	-0.405 [437]	-0.405	0.1869 (3xE-4)	0.1871 (85)
$\frac{3}{2}^-$	1.265 (260)	0.704 (56)	1.3034 (140)	1.3044 (223)
$\frac{1}{2}^-$		1.796 (1570)	2.1151 (1628)	2.1301 (1714)
$\frac{3}{2}^-$	3.7 (800) ?[a]	2.981 (990)	3.1053 (839)	3.1158 (891)
$\frac{5}{2}^-$	4.7 (700) ?[a]	3.046 (750)	3.3260 (876)	3.3352 (942)
$\frac{5}{2}^-$		5.964 (230)	6.0137 (254)	6.0361 (318)
$\frac{7}{2}^-$		6.76 (2240)	6.9619 (2420)	6.9642 (2072)

[a] For these states spin and parity are unknown.

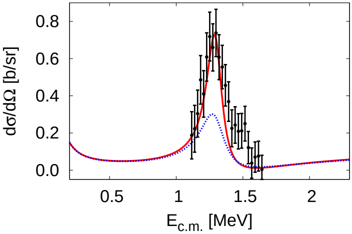


Figure: The angular cross section for p + ^{6}He elastic scattering at 180^o and 140^o in c.m., respectively. Measurements taken at RIBRAS (Brasil) [R. Lichtenthäler Priv. Comm.] The solid red line refers to "Theory(II)" in Table. It is compared with another calculation (denoted "Theory(III)" in previous Table and represented with the dashed blue line) where we consider the decay width of the 2_2^+ level of ^{6}He in the coupled-channel formalism.

Perspectives ...

- Extension of the MCAS method for radiative-capture processes of astrophysical interests, radioactive beams, Nuclear-Data-group applications
- Connecting the model Hamiltonian to its microscopic origin.
- Extension of the model to medium-heavy nuclei. (Oxygen, Sodium)
- Extension from nucleon-core to α-core dynamics.

The elastic integral cross section for n + ^{22}Ne at low energy.

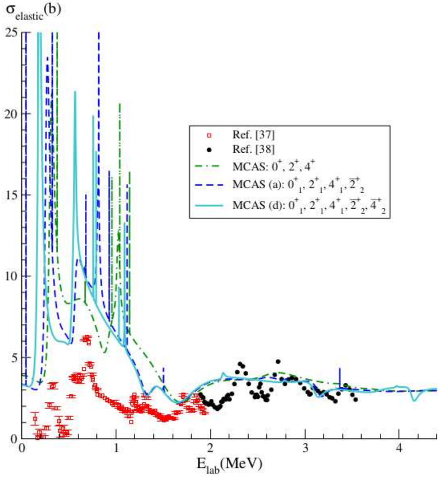


Figure: Experimental data of S. Sikkema et al (1958) and of S. R. Salisbury et al (1966). See publication P.R.Fraser, L.Canton, K.Amos, S.Karataglidis, J.P.Svenne, D. van der Knijff, Phys Rev. C 2014