

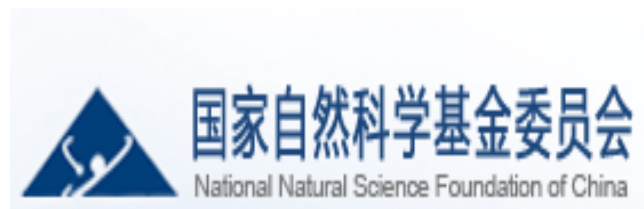
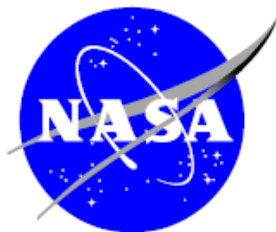
Effects of Short-Range Correlation on Symmetry Energy and Their Manifestation in Heavy-Ion Collisions

Bao-An Li



Collaborators:

W.J. Guo, X.H. Li and Z.Z. Shi ,Texas A&M University-Commerce, USA
Chang Xu (Nanjing University) and Ang Li (Xiamen University), China
Or Hen and Eli Piassetzky, Tel Aviv University, Israel
Larry Weinstein, Old Dominion University, USA



Outline

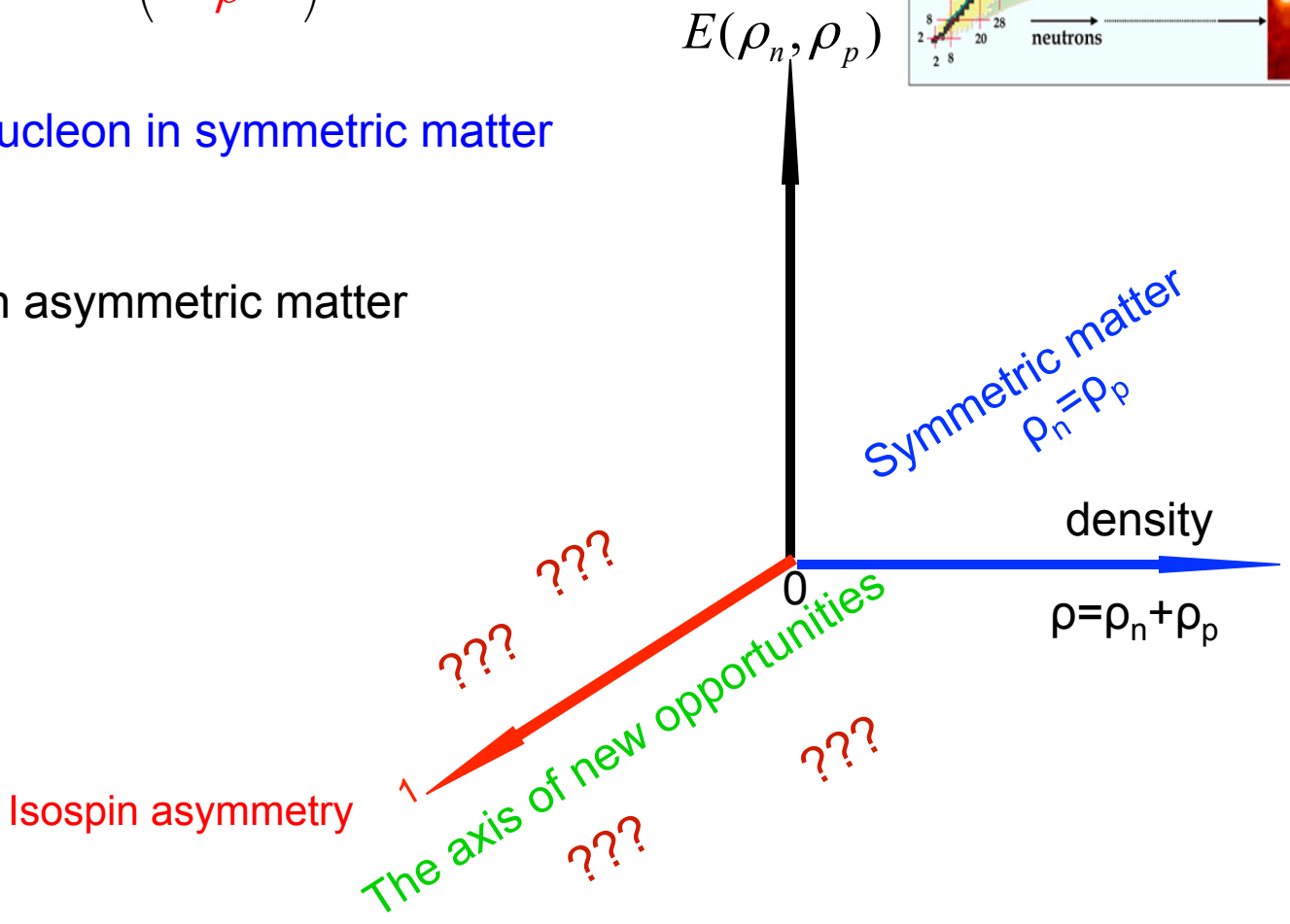
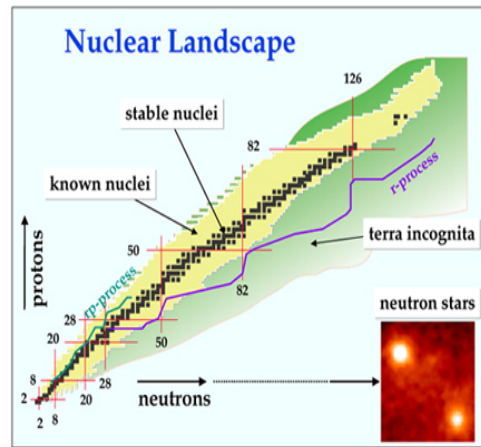
- Brief introduction on symmetry energy
- Isospin dependence of short-range correlation
- Effects of SRC on kinetic symmetry energy
- Evidence of negative kinetic symmetry energy from heavy-ion collisions

What is the Equation of State of neutron-rich nuclear matter?

$$E(\rho_n, \rho_p) = E_0(\rho_n = \rho_p) + E_{sym}(\rho) \left(\frac{\rho_n - \rho_p}{\rho} \right)^2 + o(\delta^4)$$

symmetry energy Isospin asymmetry δ

Energy per nucleon in symmetric matter
Energy per nucleon in asymmetric matter



Characterization of symmetry energy near normal density

$$E_{\text{sym}}(\rho) = E_{\text{sym}}(\rho_0) + \frac{L(\rho_0)}{3} \left(\frac{\rho - \rho_0}{\rho_0} \right) + O\left(\left(\frac{\rho - \rho_0}{\rho_0} \right)^2 \right)$$

$$L(\rho) = 3\rho \frac{dE_{\text{sym}}(\rho)}{d\rho}.$$

The physical importance of L

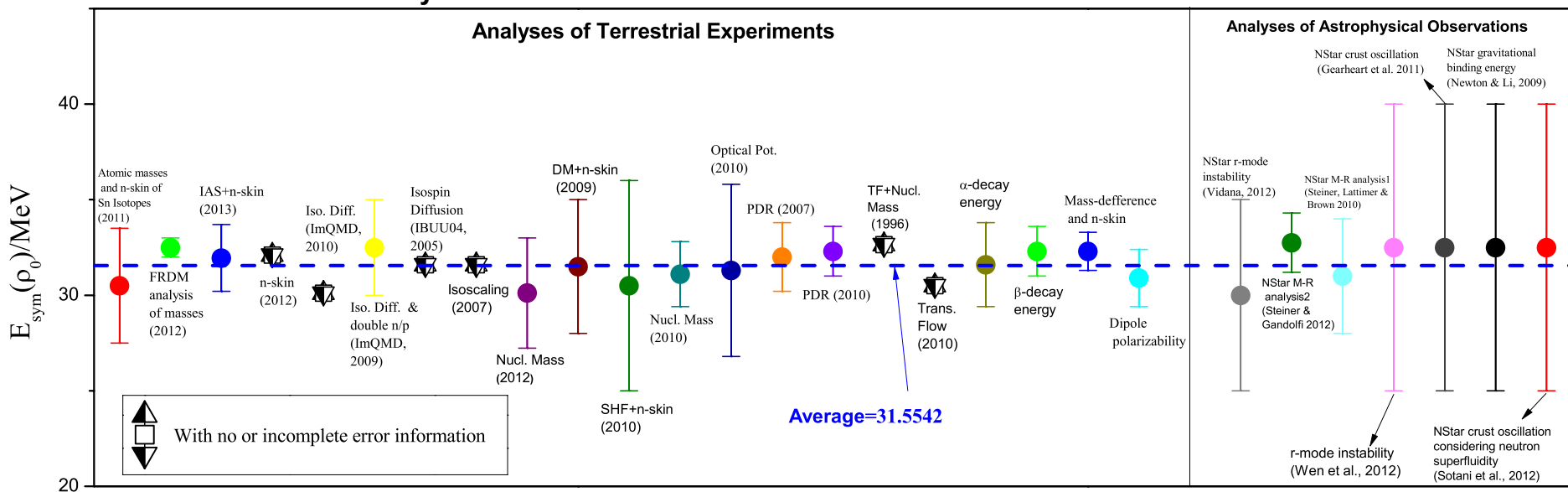
In npe matter in the simplest model of neutron stars at β -equilibrium

$$\begin{aligned} P(\rho, \delta) &= P_0(\rho) + P_{\text{asy}}(\rho, \delta) = \rho^2 \left(\frac{\partial E}{\partial \rho} \right)_{\delta} + \frac{1}{4} \rho_e \mu_e \\ &= \rho^2 \left[E'(\rho, \delta = 0) + E'_{\text{sym}}(\rho) \delta^2 \right] + \frac{1}{2} \delta(1 - \delta) \rho E_{\text{sym}}(\rho), \end{aligned}$$

In pure neutron matter at saturation density of nuclear matter

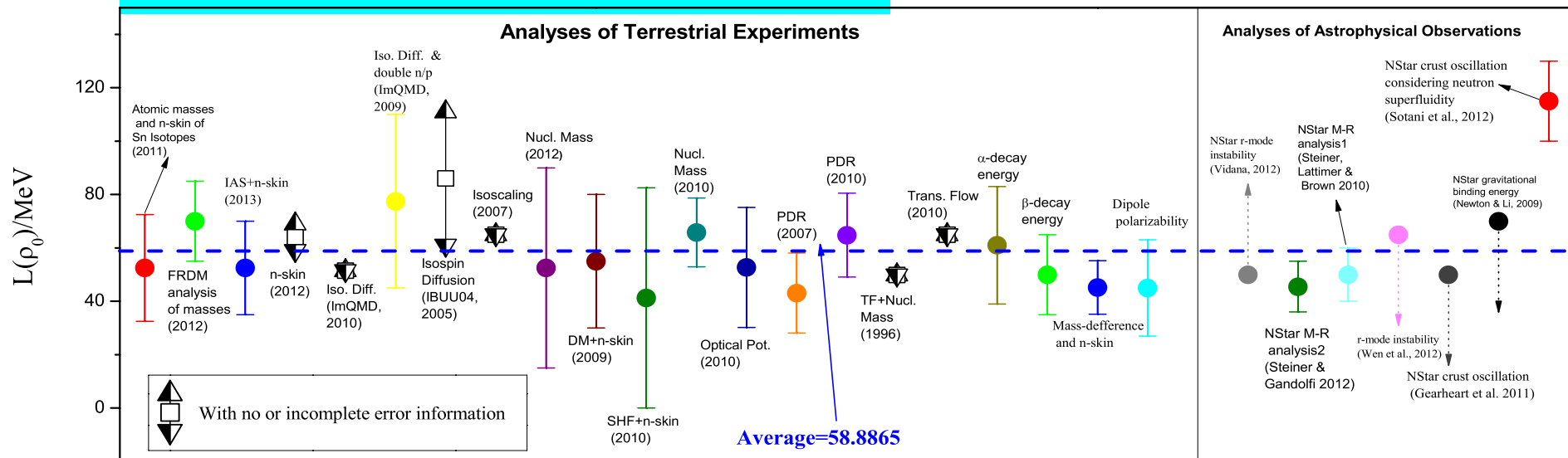
$$P_{\text{PNM}}(\rho_0) = \rho_0^2 E'_{\text{sym}}(\rho_0) = \frac{1}{3} \rho_0 L,$$

Constraints on $E_{\text{sym}}(\rho_0)$ and L based on 29 analyses of some data, Aug. 2013



	$E_{\text{sym}}(\rho_0)$	Slope L
2013 average of the means	31.55415	58.88646
2013 "standard deviation"	2.66	16.52645

Bao-An Li and Xiao Han,
Phys. Lett. B727, 276 (2013).



Where does the $S_0 = E_{\text{sym}}(\rho_0) \approx 31 \text{ MeV}$ come from?

In the literatures in both astrophysics and nuclear physics

$$S = E_{\text{sym}} = E_{\text{sym}}^{\text{Kin}} + E_{\text{sym}}^{\text{pot}}$$

$$E_{\text{sym}}^{\text{kin}}(\rho) \approx \frac{1}{3} E_F(\rho_0) (\rho / \rho_0)^{2/3} \approx 12.5 \text{ MeV at } \rho_0$$

At abnormal densities, it is customary to parameterize

$$S(\rho) = \frac{C_{s,k}}{2} \left(\frac{\rho}{\rho_0}\right)^{2/3} + \frac{C_{s,p}}{2} \left(\frac{\rho}{\rho_0}\right)^{\gamma_i}$$

Kinetic

Potential

The only term goes into the reaction dynamics

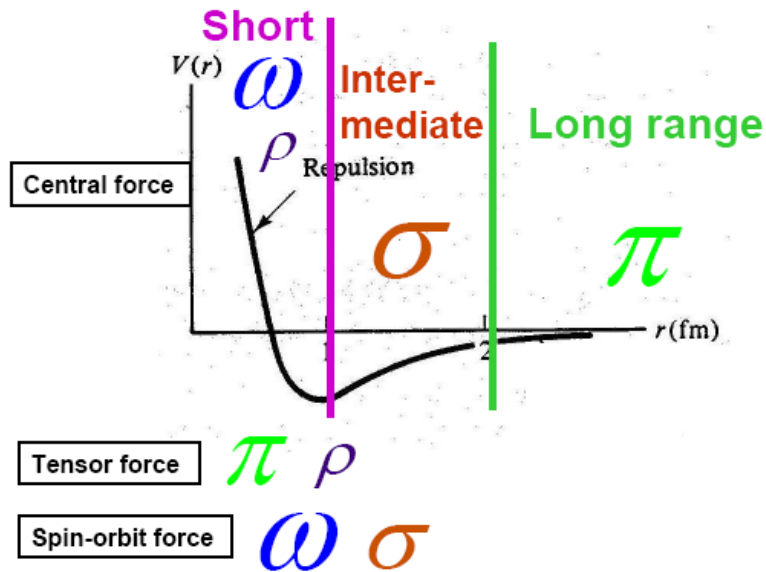
Probing all of the unknown physics

$$C_{s,k} = 25 \text{ MeV}$$

$$C_{s,p} = 2S_0 - C_{s,k}$$

The short and long range tensor force

Lecture notes of R. Machleidt
 CNS summer school, Univ. of Tokyo
 Aug. 18-23, 2005



π (138)

$$V_{\pi} = \frac{f_{\pi}^2 M^2}{3m_{\pi}^2} \frac{\vec{q}^2}{\vec{q}^2 + m_{\pi}^2} [-\vec{\sigma}_1 \cdot \vec{\sigma}_2 - S_{12}(\hat{q})] \vec{r}_1 \cdot \vec{r}_2$$

Long-ranged tensor force

σ (600)

$$V_{\sigma} \approx \frac{g_{\sigma}^2}{\vec{q}^2 + m_{\sigma}^2} \left[-1 - \frac{\vec{L} \cdot \vec{S}}{2M^2} \right]$$

intermediate-ranged, attractive central force plus LS force

ω (782)

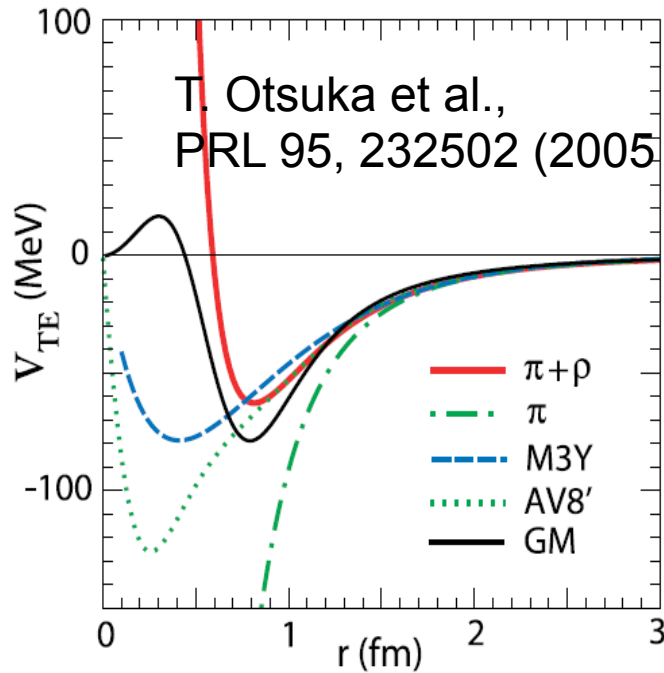
$$V_{\omega} \approx \frac{g_{\omega}^2}{\vec{q}^2 + m_{\omega}^2} \left[+1 - 3 \frac{\vec{L} \cdot \vec{S}}{2M^2} \right]$$

short-ranged, repulsive central force plus strong LS force

ρ (770)

$$V_{\rho} = \frac{f_{\rho}^2}{12M^2} \frac{\vec{q}^2}{\vec{q}^2 + m_{\rho}^2} [-2\vec{\sigma}_1 \cdot \vec{\sigma}_2 + S_{12}(\hat{q})] \vec{r}_1 \cdot \vec{r}_2$$

short-ranged tensor force, opposite to pion



T. Otsuka et al.,
 PRL 95, 232502 (2005)

What are Short Range Correlations in nuclei ? (taken from Eli Piassetzky)

SRC $\sim R_N$

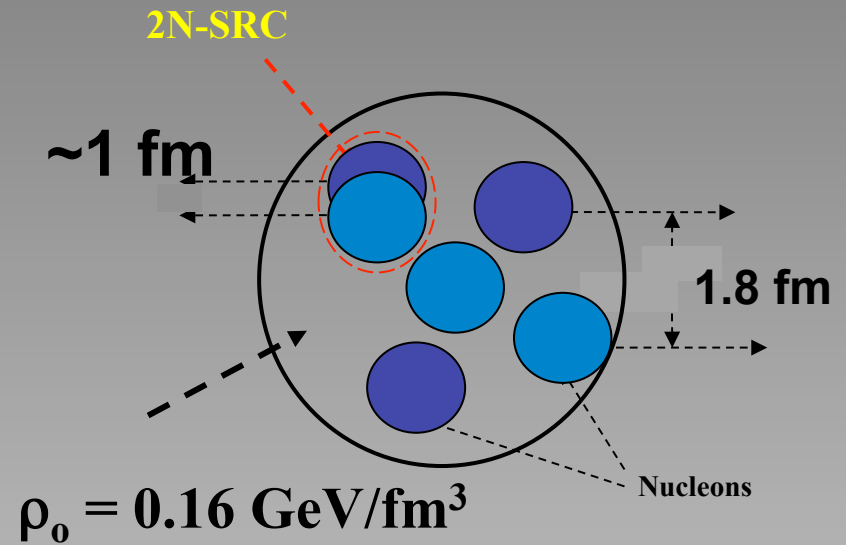
LRC $\sim R_A$

$k_F \sim 250 \text{ MeV}/c$

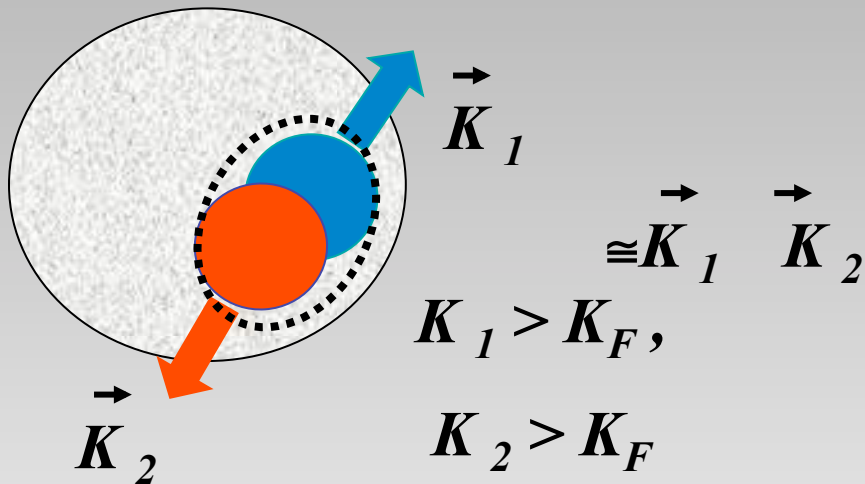
High momentum tail:

300-600 MeV/c

$1.5 K_F - 3 K_F$



In momentum space:



A pair with large relative momentum between the nucleons and small CM momentum.

Tensor force induced (1) high-momentum tail in single-particle momentum distribution and (2) isospin dependence of NN correlation

Theory of Nuclear matter

H.A. Bethe

Ann. Rev. Nucl. Part. Sci., 21, 93-244 (1971)

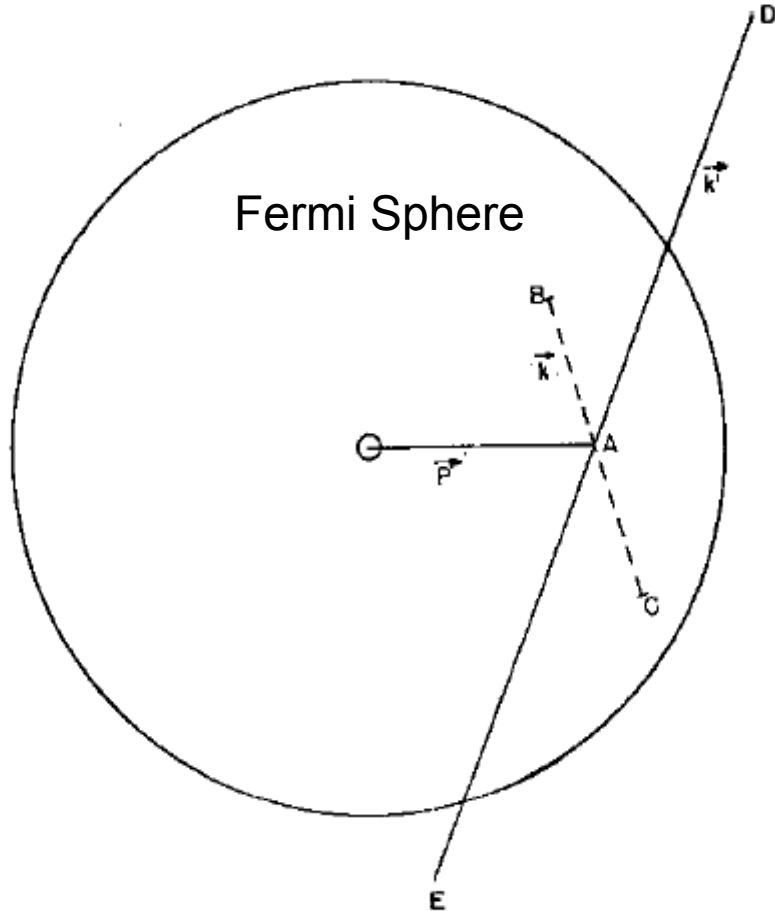
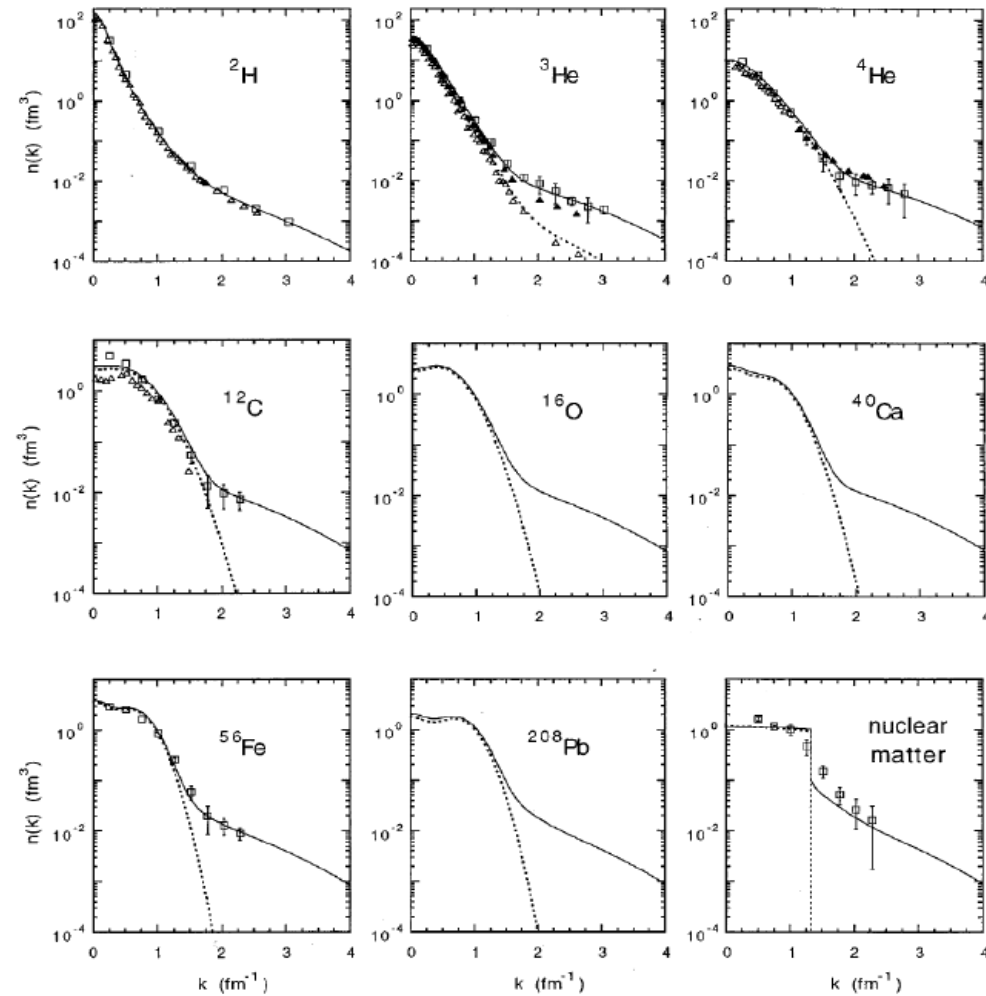


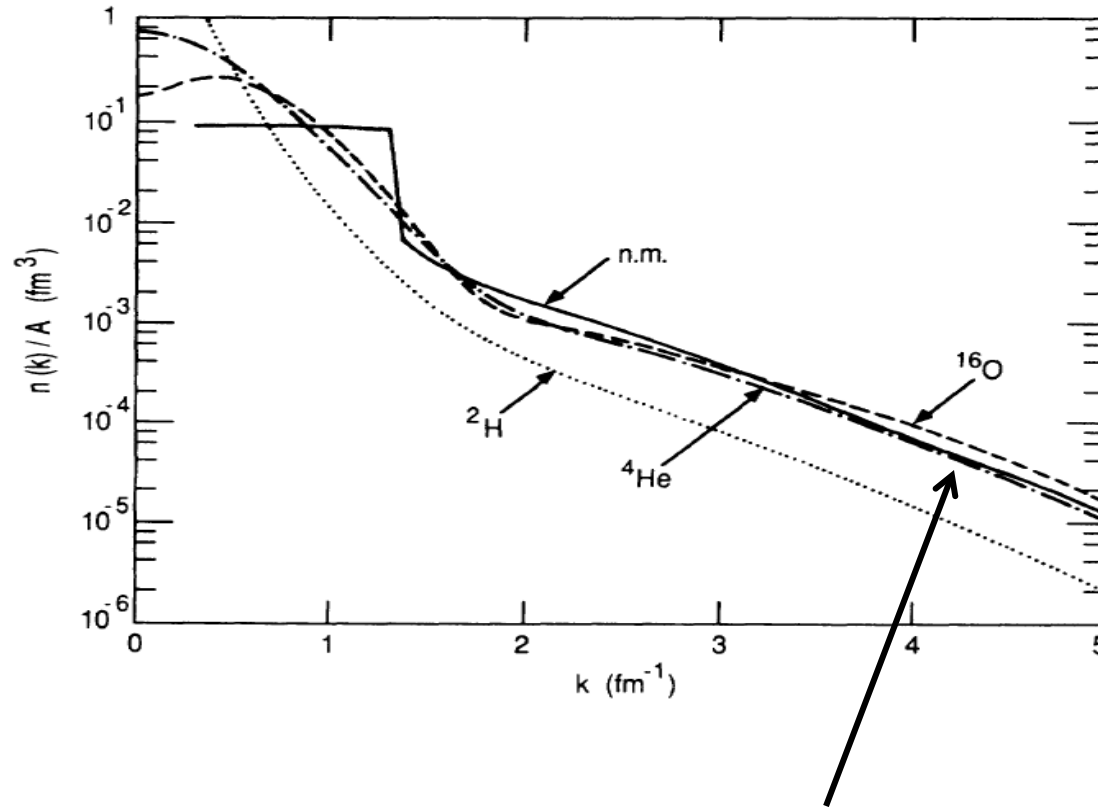
FIGURE 10. Two nucleons are initially in states B and C , having average momentum P and relative momentum k . When they interact they are shifted to states D and E outside the Fermi sphere, with relative momentum k' . If they are initially in a 1S state and interact by tensor force, then they are in a 3D_1 state in DE .



S. Fantoni and V. R. Pandharipande, Nucl. Phys. A **427**, 473 (1984).

C. Ciofi degli Atti and S. Simula, Phys. Rev. C **53**, 1689 (1996).

Scaling of the high-momentum tail due to tensor force



Universal shape of high-momentum tail

→ due to short-range interaction of two nearby nucleons

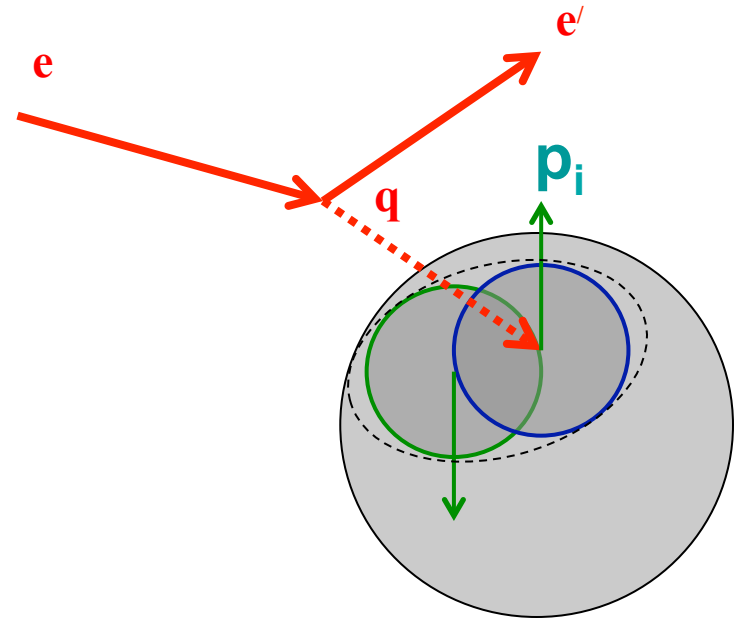
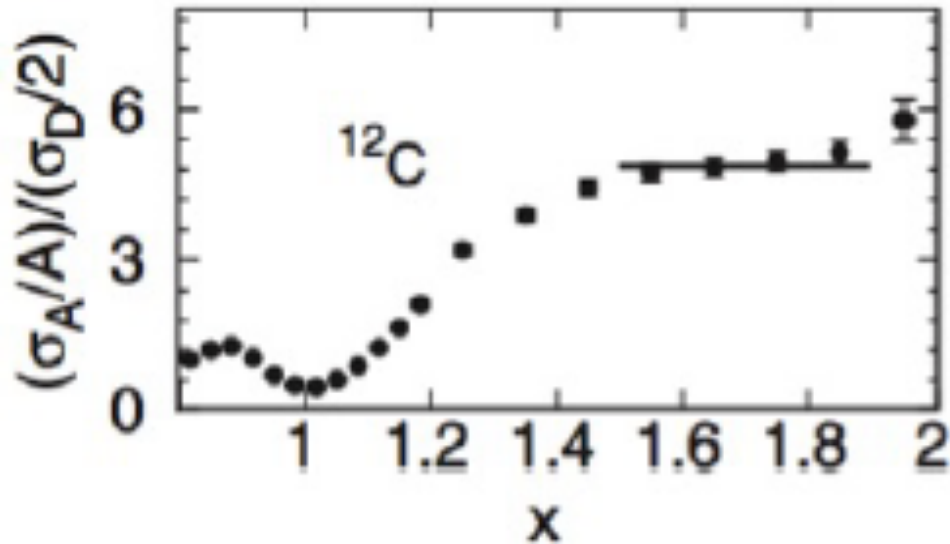
→ scaling of weighted (e, e') inclusive xsections from light to heavy nuclei: the ratio of weighted xsection should be independent of the scattering variables

The inclusive $A(e,e')$ measurements

K. Sh. Egiyan et al. PRC 68, 014313 (2003)

K. Sh. Egiyan et al. PRL. 96, 082501 (2006)

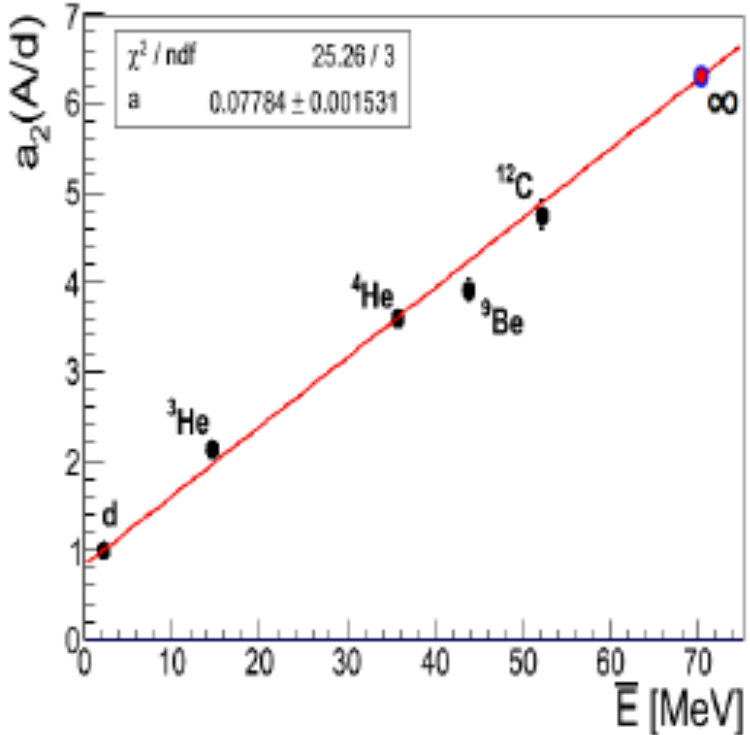
N. Fomin et al. PRL 108, 092502 (2012)



$$x_B = \frac{Q^2}{2m\omega} \quad Q^2 = \vec{q}^2 - \omega^2$$

Relative probability of SRC in nucleus A with respect to that in deuteron

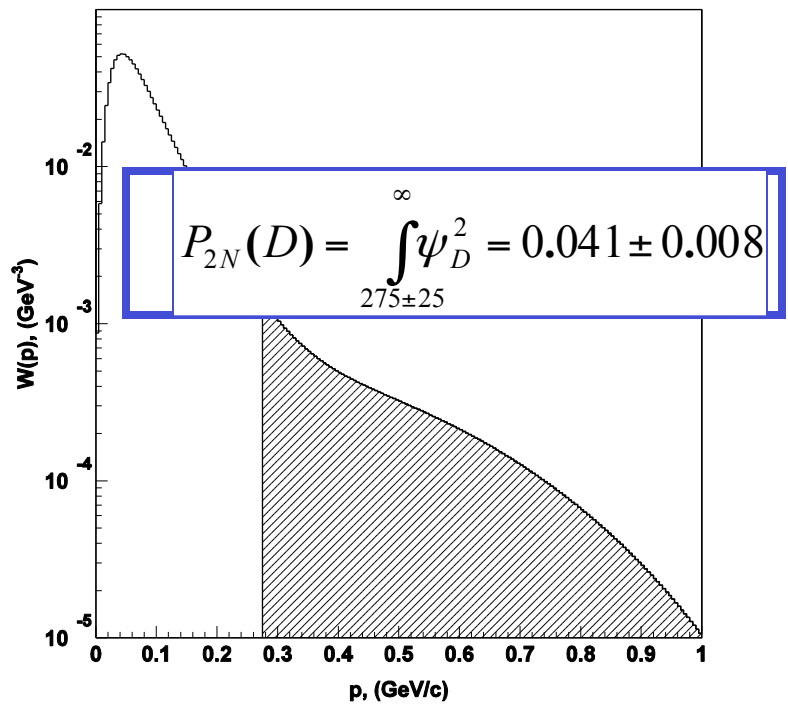
$a_2(A/d)$ extrapolated to infinite SNM



Nucleon removal energy

$$\bar{E} = \bar{T} \frac{A-2}{A-1} - \frac{E_0}{A}$$

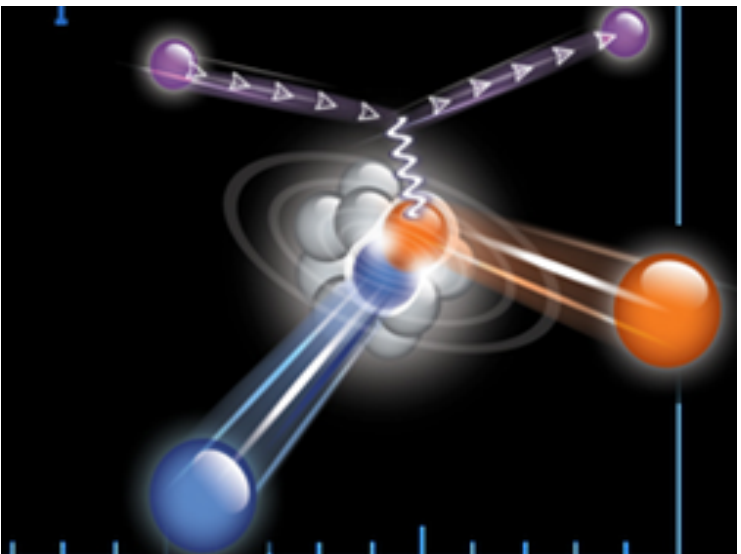
Greens Function Monte-Carlo (GFMC)



$$P_{2N}(A) = a_{2N}(A/d) \cdot P_{2N}(D)$$

$$P_{2N}(\infty) \approx 20-30\%$$

Triple coincidence measurement of the isospin dependence of SRC

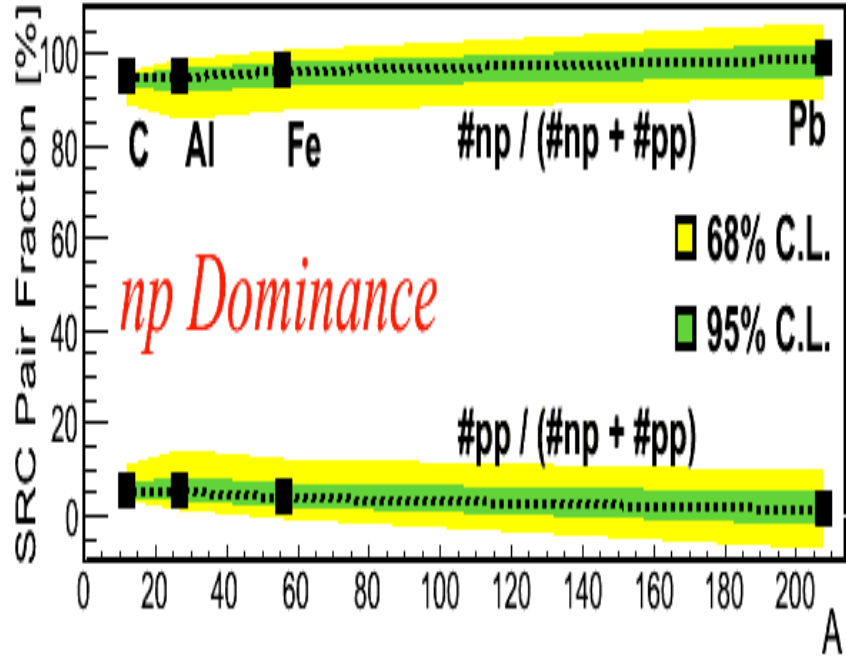
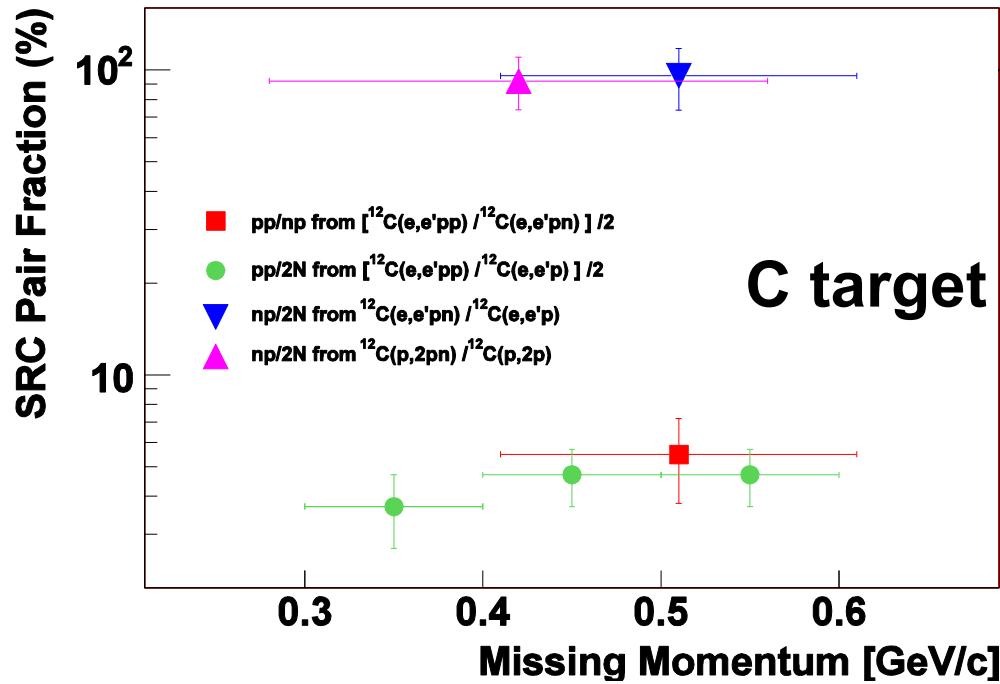


R. Subedi et al., Science 320, 1476 (2008)

Or Hen et al., sub. to Science (2014)

1. $np/pp \approx 18$

2. High momentum tail in pure neutron matter is about 1-2%



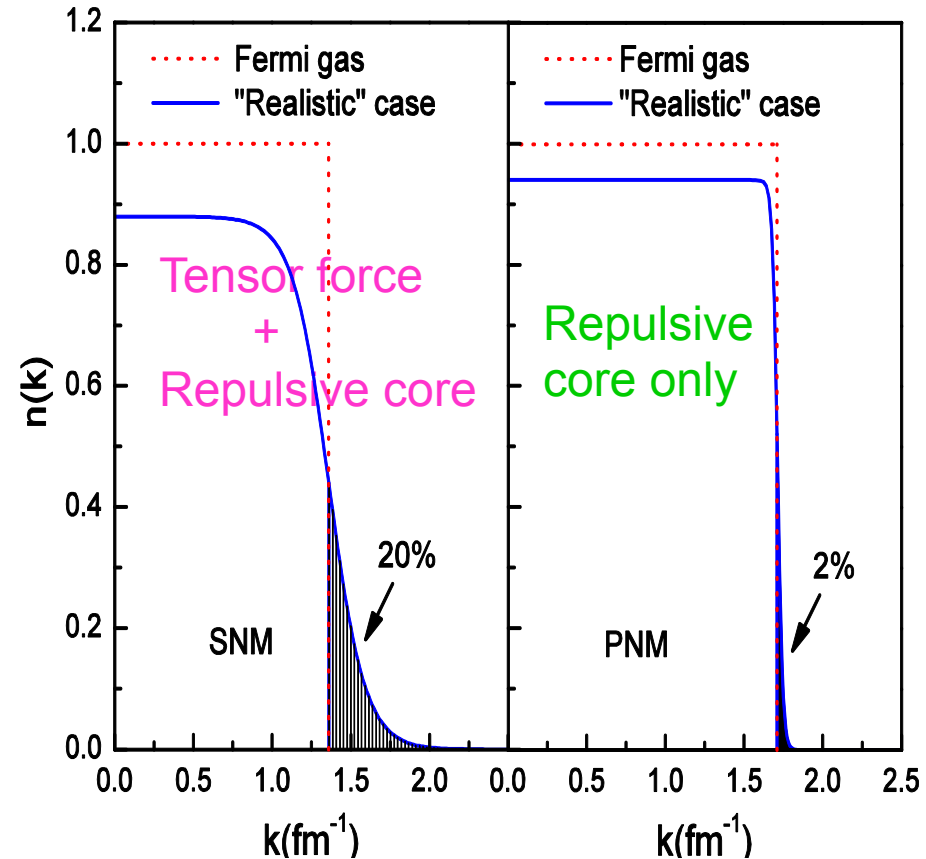
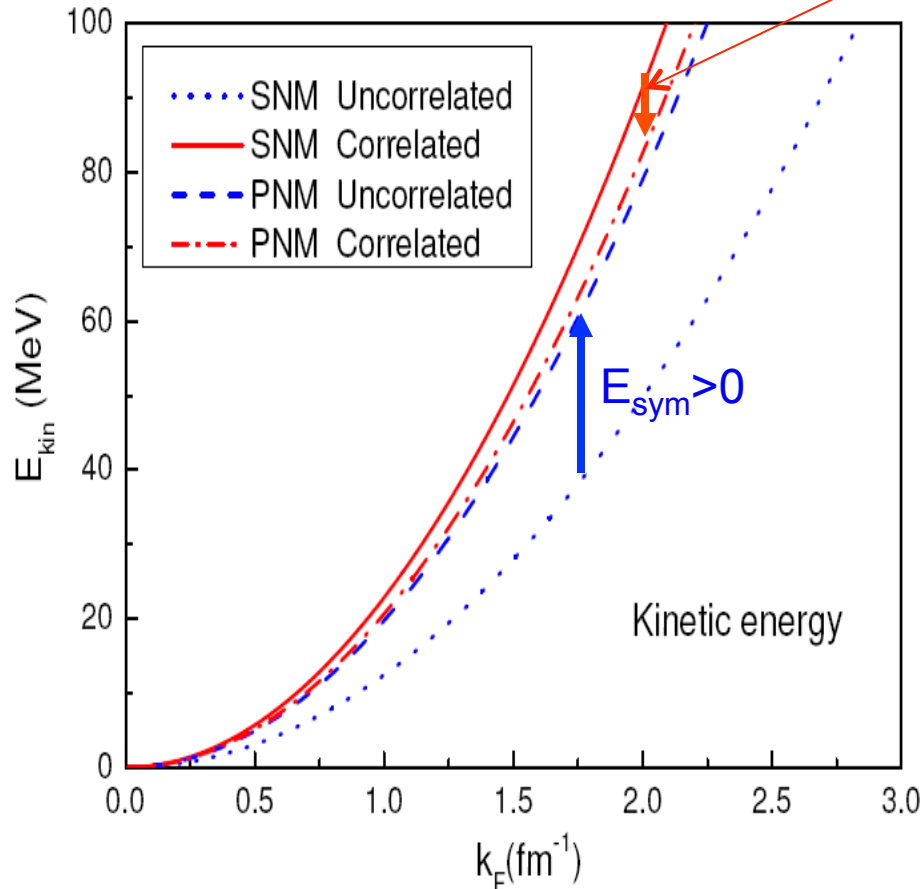
How does the tensor force affect the kinetic symmetry energy?

Chang Xu and Bao-An Li, [arXiv:1104.2075](https://arxiv.org/abs/1104.2075)

Chang Xu, Ang Li and Bao-An Li,
JPCS 420, 012190 (2013).

$$E_{kin} = \alpha \int_0^\infty \frac{\hbar^2 k^2}{2m} n(k) k^2 dk,$$

$$E_{sym}^{kin} = E_{PNM}^{kin} - E_{SNM}^{kin} < 0$$



Confirmation by Microscopic Many-Body Theories

1. [Isaac Vidana](#), [Artur Polls](#), [Constanca Providencia](#)

PRC84, 062801(R) (2011)

Brueckner--Hartree--Fock approach using the Argonne V18 potential plus the Urbana IX three-body force

2. [Arianna Carbone](#), [Artur Polls](#), [Arnau Rios](#), *EPL 97, 22001 (2012)*

Self-Consistent Green's Function Approach with Argonne Av18, CDBonn, Nij1, N3LO interactions

3. [Alessandro Lovato](#), [Omar Benhar](#) et al., extracted from results already published in

Phys. Rev. C83:054003,2011

Using Argonne V'_{α} interaction

Fermi-Hyper-Netted-Chain (FHNC)

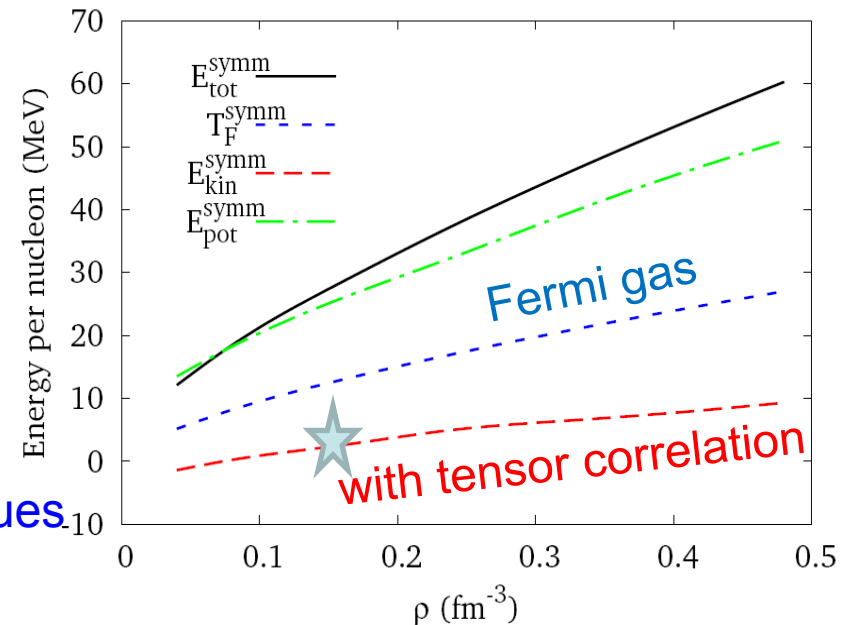
Single Operator Chains (SOC)

4. [A. Rios](#), [A. Polls](#), [W. H. Dickhoff](#)

PRC 89, 044303 (2014).

Ladder Self-Consistent Green Function

They all included the tensor force and many-body correlations using different techniques.



The universal shape of the high-momentum tail in all symmetric 2-component fermion systems with strong contact/interaction between unlike particles if

$$a \gg d \gg r_{\text{eff}}$$



With large scattering length a between different Fermions of separation d And short-range (r_{eff}) interaction

$$n(k) = C / k^4$$

C is the Tan's contact term

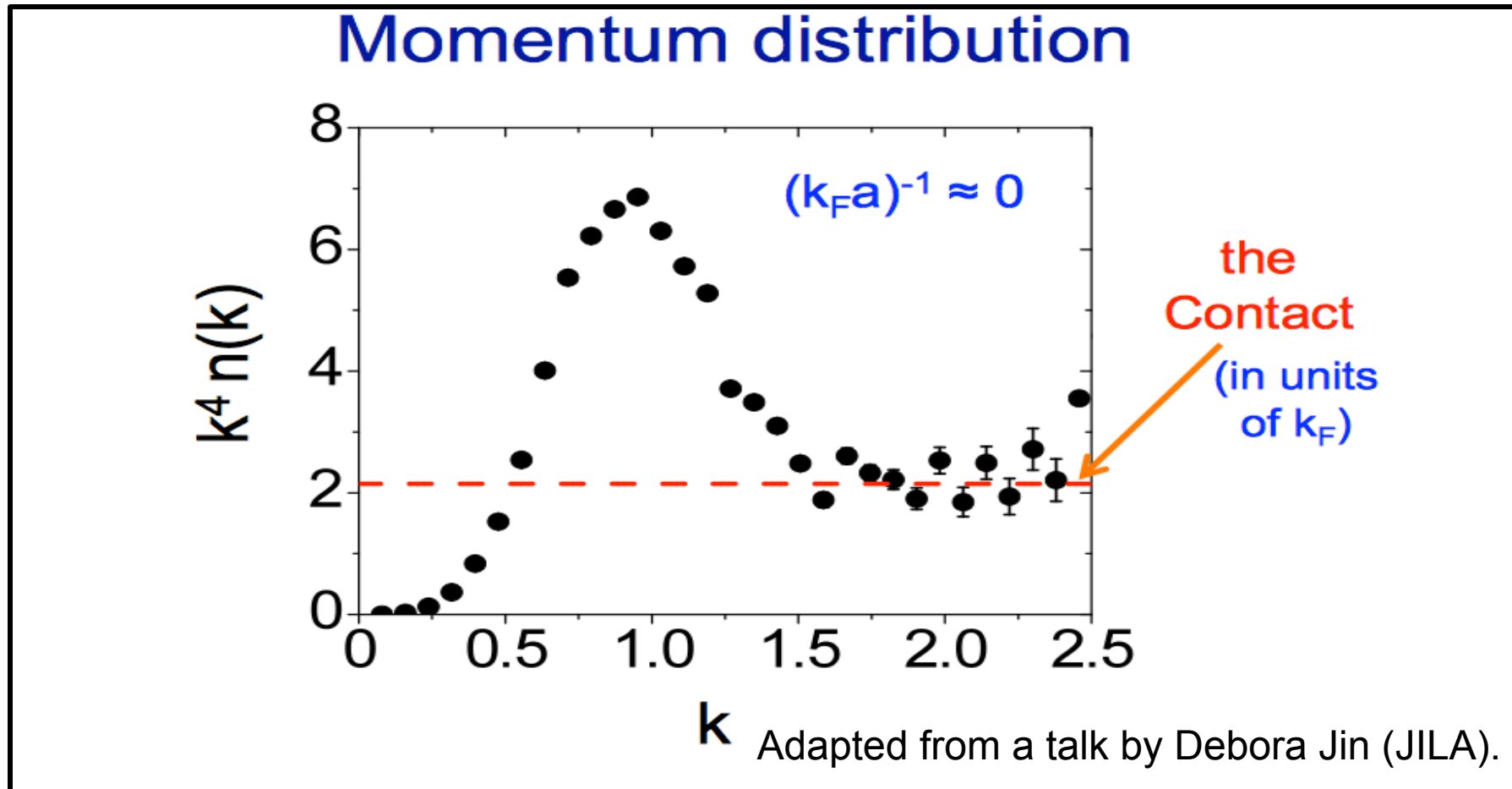
B.S. Tsinghua, 1997, Ph.D. U. of Chicago 2006

Thermodynamics can be describe by thr single parameter C

Shina Tan, Annals of Physics 323 (2008) 2952, ibid 2971, ibid 2987

(George E. Valley Prize, American Physical Society 2010)

Experiments with two spin-state mixtures of ultra-cold ^{40}K and ^6Li atomic gas systems extracted the contact term and verified the universal relations



What about nuclear contact ?

(Eli Piassetzky)

$$a \gg d \gg r_{eff} \quad ?$$

$$d = \rho^{-1/3} \approx 1.8 \text{ fm}$$

$$r_{eff} \approx \frac{\hbar}{2 \cdot m_{\pi} \cdot c} \approx 0.7 \text{ fm} \quad \text{Tensor force}$$

The high- momentum tail is predominantly:

J=1 S and D pairs :

$$T=0 \quad \mathbf{S=1} \quad L=0 \quad {}^3S_1$$

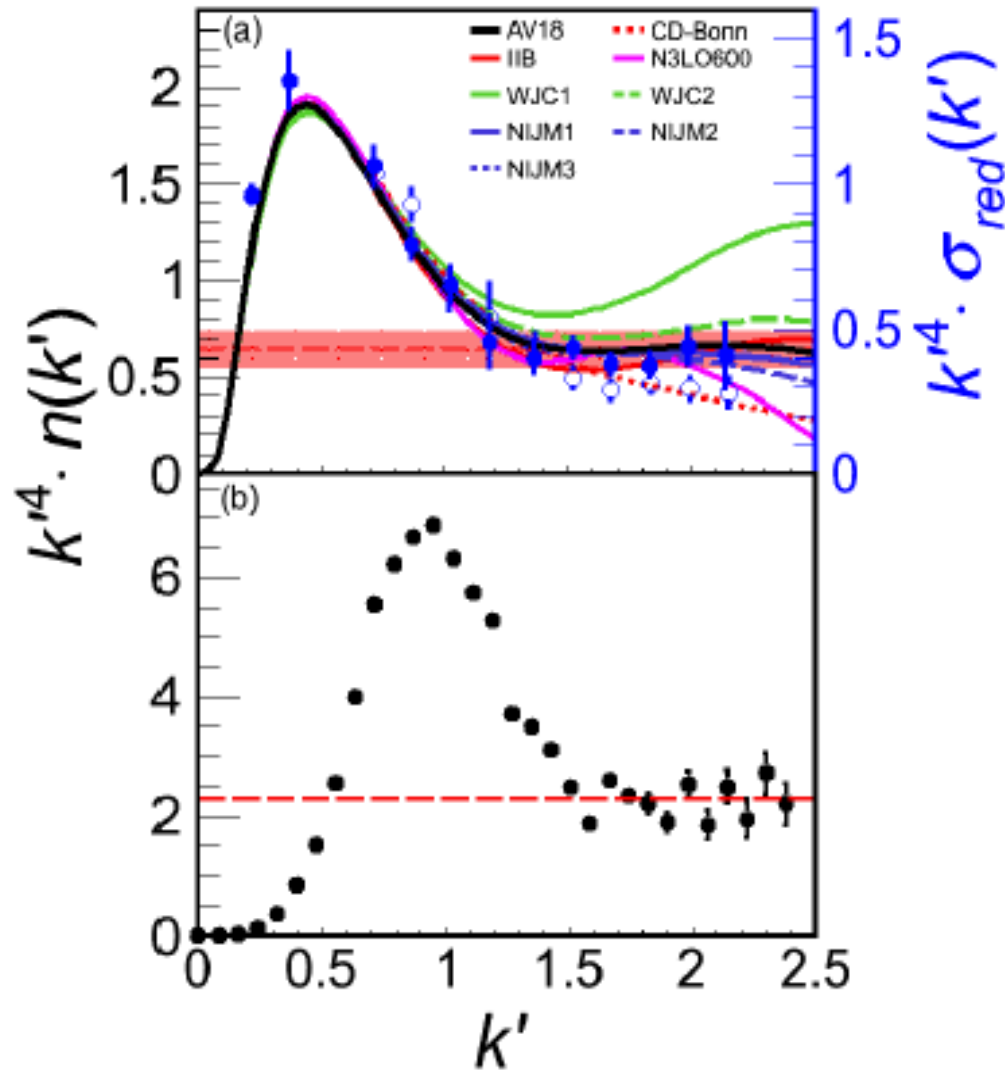
$$T=0 \quad \mathbf{S=1} \quad L=2 \quad {}^3D_1$$

$$a({}^3S_1) = 5.424 \pm 0.003 \text{ fm}$$

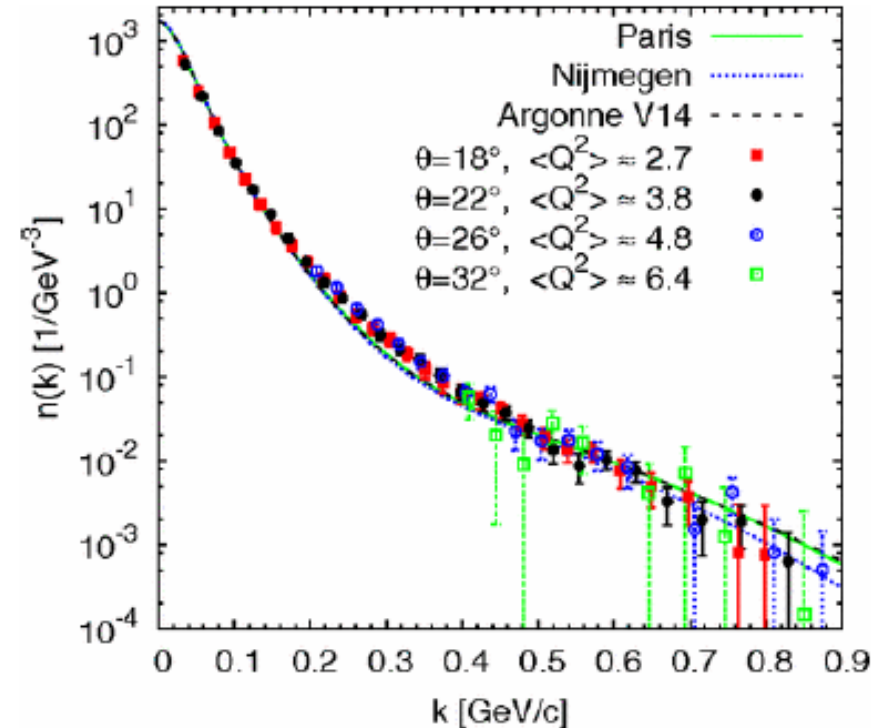
$$a(\approx 5.4 \text{ fm}) > d(1.8 \text{ fm}) > r_{eff}(0.7 \text{ fm})$$

The high-momentum tail in deuteron scales as $1/K^4$

O. Hen, L. B. Weinstein, E. Piassetzky, G. A. Miller, M. M. Sargsian, [arXiv:1407.8175](https://arxiv.org/abs/1407.8175)



$$R_d = 0.64 \pm 0.10$$



Kinetic symmetry energy of correlated fermions

$$n_{SNM}^{SRC}(k) = \begin{cases} A_0 & k < k_F \\ C_\infty/k^4 & k_F < k < \lambda k_F^0 \\ 0 & k > \lambda k_F^0 \end{cases} \quad R_d = (k/k_F)^4 \cdot n_d(k/k_F) \quad 1.3 \leq k/k_F \leq 2.5$$

$$E_{sym}^{kin}(\rho) = E_{sym}^{kin}(\rho)|_{FG} - \Delta E_{sym}^{kin}(\rho) \quad (7)$$

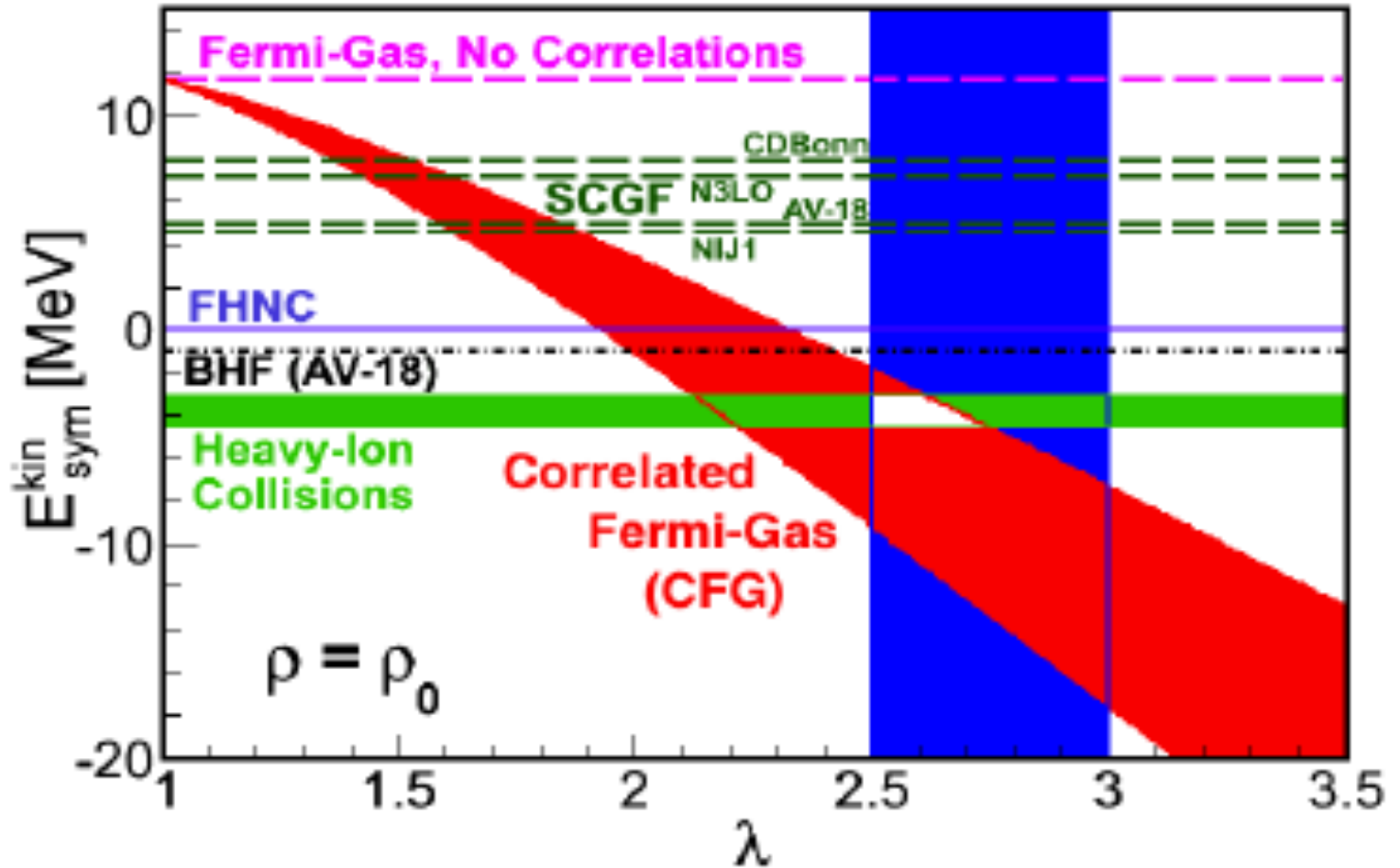
where the SRC correction term is:

$$\Delta E_{sym}^{kin} \equiv \frac{E_F^0}{\pi^2} c_0 \left[\lambda \left(\frac{\rho}{\rho_0} \right)^{1/3} - \frac{8}{5} \left(\frac{\rho}{\rho_0} \right)^{2/3} + \frac{3}{5} \frac{1}{\lambda} \left(\frac{\rho}{\rho_0} \right) \right]. \quad (8)$$

$$C_0 = R_d^* a_2(\infty)$$

Kinetic symmetry energy of correlated fermions

Or Hen, Bao-An Li, Wen-Jun Guo, L.B. Weinstein, Eli Piassetzky, [arXiv:1408.0772](https://arxiv.org/abs/1408.0772)

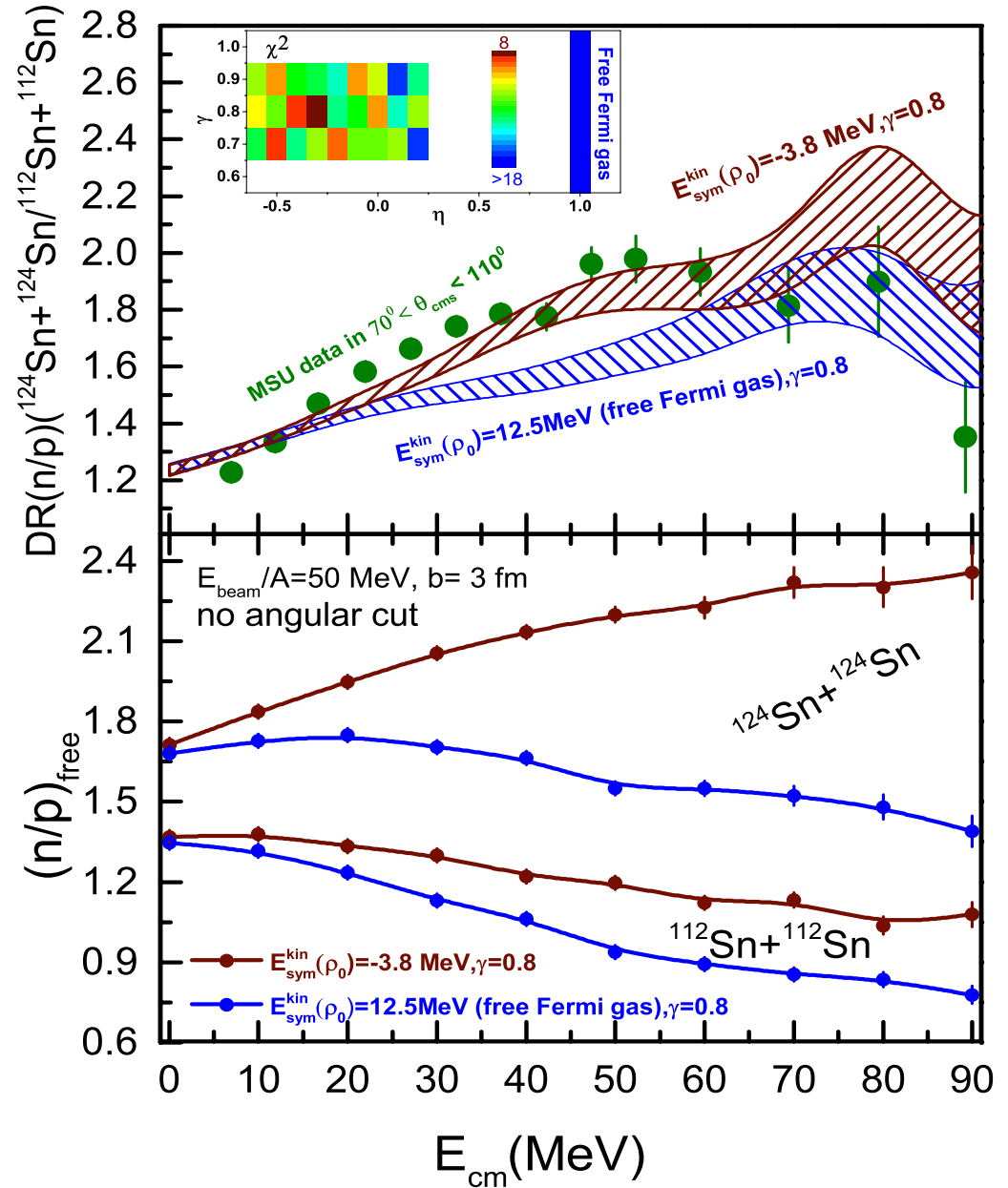


Width of the high momentum tail above the Fermi momentum

Effects of reduced kinetic symmetry energy on heavy-ion collisions

$$E_{sym}^{pot}(\rho) = [E_{sym}(\rho_0) - \eta \cdot E_{sym}^{kin}(\rho_0)]_{FG} \cdot (\rho/\rho_0)^\gamma.$$

$$V_{sym}^{n/p}(\rho, \delta) = [E_{sym}(\rho_0) - \eta \cdot E_{sym}^{kin}(\rho_0)]_{FG} (\rho/\rho_0)^\gamma \times [\pm 2\delta + (\gamma - 1)\delta^2]. \quad (10)$$



Summary

Where does the $S_0 = E_{\text{sym}}(\rho_0) \approx 31$ MeV come from?

It is probably all coming from the potential contribution due to the NN SRC-induced reduction of the kinetic symmetry energy

$$E_{\text{sym}}^{\text{kin}}(\rho) \neq \frac{1}{3} E_F(\rho_0) (\rho / \rho_0)^{2/3} \approx 12.5 \text{ MeV at } \rho_0$$

$$S(\rho) = \frac{C_{s,k}}{2} \left(\frac{\rho}{\rho_0}\right)^{2/3} + \frac{C_{s,p}}{2} \left(\frac{\rho}{\rho_0}\right)^{\gamma_i}$$

Kinetic Potential

$$C_{s,k} \neq 25 \text{ MeV}$$

$$C_{s,p} = 2S_0 - C_{s,k}$$

The MSU double n/p ratio data strongly indicate the need of a SRC-reduced kinetic symmetry energy